

Solution Handout for Precepts Week 2: September 27, 28

Elasticities for Empirically Estimated Demand Function

F. Gasmi, J.-J. Laffont, and Q. Vuong (Journal of Economics and Management Strategy, Summer 1992) estimated the following demand functions for Coke and Pepsi:

$$\begin{aligned}Q_C &= 26.17 - 3.98 P_C + 2.25 P_P + 2.60 (A_C)^{1/2} - 0.62 (A_P)^{1/2} + 0.99 I + 9.58 S \\Q_P &= 17.48 + 1.40 P_C - 5.48 P_P - 4.81 (A_C)^{1/2} + 2.83 (A_P)^{1/2} + 1.92 I + 11.98 S\end{aligned}$$

The symbols are as follows:

- Q_C = quantity of Coke per quarter (units of ten million cases)
- Q_P = quantity of Pepsi per quarter (units of ten million cases)
- P_C = price of Coke (1986 dollars per ten cases)
- P_P = price of Pepsi (1986 dollars per ten cases)
- A_C = quarterly advertising expenditure by Coke (millions of 1986 dollars)
- A_P = quarterly advertising expenditure by Pepsi (millions of 1986 dollars)
- I = average per capital disposable income (thousands of 1986 dollars)
- S = “dummy variable” = 1 for summer and spring quarters, 0 for fall and winter

The average values of the independent variables in the data were as follows:

$$P_C = 12.96, \quad P_P = 8.16, \quad A_C = 34.69, \quad A_P = 27.88, \quad I = 20.63$$

We are to consider the case of spring and summer seasons ($S = 1$).

We can find the quantities by substitution; this yields $Q_C = 34.99$, $Q_P = 29.11$. Using all this information, we can calculate various elasticities of demand. Care is needed in calculating the derivatives and then the elasticities of demand with respect to advertising. For example

$$\frac{\partial Q_C}{\partial A_C} = 2.60 \left[\frac{1}{2} (A_C)^{(1/2)-1} \right] = 1.30 (A_C)^{-1/2}$$

Then the elasticity is

$$\frac{A_C}{Q_C} \frac{\partial Q_C}{\partial A_C} = 1.30 \frac{A_C}{Q_C} (A_C)^{-1/2} = 1.30 \frac{(A_C)^{1/2}}{Q_C}$$

Here are the numbers I got (please check them; there may be numerical errors):

Elasticity with respect to	Elasticity of Demand for	
	Coke	Pepsi
Own price	-1.47	-1.54
Other's price	0.52	0.62
Income	0.58	1.36
Own Advertising	0.219	0.257
Other's Advertising	-0.047	-0.487

We can discuss whether these are reasonable numbers. The own and cross price elasticities look reasonable in sign and magnitude. The income elasticities are highly unequal; it is hard to believe that Coke is a necessity and Pepsi a luxury good! And the cross advertising effects are surprisingly unequal.

Preferences Yielding Constant Expenditure Shares

In class we represented the indifference map of rectangular hyperbolas by the utility function

$$U(X, Y) = X Y,$$

and maximized it by analogy with the area enclosing problem. Here we consider the general budget constraint

$$P_X X + P_Y Y = I.$$

Use the substitution method. Solve for Y :

$$Y = \frac{I - P_X X}{P_Y} = \frac{I}{P_Y} - \frac{P_X}{P_Y} X$$

Substitute this in the expression for utility to express it as a function of X alone:

$$\begin{aligned} U(X) &= X \left[\frac{I}{P_Y} - \frac{P_X}{P_Y} X \right] \\ &= \frac{I}{P_Y} X - \frac{P_X}{P_Y} X^2 \end{aligned}$$

To maximize this, set the derivative equal to zero:

$$\frac{dU}{dX} = \frac{I}{P_Y} - \frac{P_X}{P_Y} 2X = 0$$

and solve for X :

$$X = \frac{I/P_Y}{2 P_X/P_Y} = \frac{I}{2 P_X}$$

Then use the budget constraint to find the solution for Y :

$$Y = \frac{I - P_X X}{P_Y} = \frac{I - I/2}{P_Y} = \frac{I}{2 P_Y}$$

By the way, check the second-order condition:

$$\frac{d^2 U}{dX^2} = -2 \frac{P_X}{P_Y}$$

which is always negative, so we are OK.

Next let us find the elasticities of these demand functions. First hold prices constant and let income alone vary. The derivative (written with the partial derivative signs ∂ to signify that things other than income are being held constant) is

$$\frac{\partial X}{\partial I} = \frac{1}{2 P_X}$$

Then the elasticity is

$$\frac{I}{X} \frac{\partial X}{\partial I} = \frac{I}{I/(2 P_X)} \frac{1}{2 P_X} = 1$$

Next hold income and the price of Y constant and change the price of X . The formula for X can be written

$$X = \frac{I}{2} (P_X)^{-1}$$

Therefore the derivative is

$$\frac{\partial X}{\partial P_X} = -\frac{I}{2} (P_X)^{-2}$$

Then the “own-price” elasticity of demand for X is

$$\frac{P_X}{X} \frac{\partial X}{\partial P_X} = -\frac{P_X}{I/(2 P_X)} \frac{I}{2} (P_X)^{-2} = -1$$

Finally, change the price of Y while holding income and the price of X constant. We see that P_Y appears nowhere in the formula for X , so the derivative is

$$\frac{\partial X}{\partial P_Y} = 0$$

Then the “cross-price” elasticity of demand for X is also zero.

Next consider a slightly more general utility function

$$U(X, Y) = X^k Y$$

where k is positive, but does not have to be an integer. Again solve for Y using the budget constraint and substitute

$$Y = \frac{I - P_X X}{P_Y} = \frac{I}{P_Y} - \frac{P_X}{P_Y} X$$

so

$$\begin{aligned} U(X) &= X^k \left[\frac{I}{P_Y} - \frac{P_X}{P_Y} X \right] \\ &= \frac{I}{P_Y} X^k - \frac{P_X}{P_Y} X^{k+1} \end{aligned}$$

To maximize this, set the derivative equal to zero:

$$\begin{aligned} \frac{dU}{dX} &= \frac{I}{P_Y} k X^{k-1} - \frac{P_X}{P_Y} (k+1) X^k = 0 \\ &= X^{k-1} \left[\frac{I}{P_Y} k - \frac{P_X}{P_Y} (k+1) X \right] \end{aligned}$$

and solve for X :

$$X = \frac{(I/P_Y) k}{(P_X/P_Y) (k+1)} = \frac{k I}{(k+1) P_X}$$

Then use the budget constraint to find the solution for Y :

$$Y = \frac{I - P_X X}{P_Y} = \frac{I - k I/(k+1)}{P_Y} = \frac{I}{(k+1) P_Y}$$

We leave it as practice work you to check the second-order condition for maximization, and to show that the elasticities of demand for each of the goods are as follows: (i) income elasticity = 1, (ii) own-price elasticity = -1, (iii) cross-price elasticity = 0.

Finally, consider the most general Cobb-Douglas form:

$$U(X, Y) = X^a Y^b$$

where a and b are positive numbers (not necessarily integers). Observe that we can write

$$U(X, Y) = \left(X^{a/b} Y \right)^b$$

Maximizing this is the same as maximizing

$$V(X, Y) = X^{a/b} Y$$

so we can use the previous formulas with a/b instead of k . This gives demand functions

$$X = \frac{(I/P_Y) (a/b)}{(P_X/P_Y) [(a/b) + 1]} = \frac{a I}{(a+b) P_X}, \quad Y = \frac{I}{[(a/b) + 1] P_Y} = \frac{b I}{(a+b) P_Y}$$

The proportions of income spent on the two goods are then

$$\frac{P_X X}{I} = \frac{a}{a+b}, \quad \frac{P_Y Y}{I} = \frac{b}{a+b}$$

This is done differently (using Lagrange multipliers) in the textbook (Appendix to Chapter 4, pp. 148-9).