# Question for Precepts Week 9: November 23

A small town has just one pizza-maker who sells a large pie, and one beer store that sells sixpacks. Call pizza good 1 and beer good 2. It is known that when the prices of the two goods are $P_1$ and $P_2$, the demands facing the two are

$$Q_1 = 36 - 2\ P_1 - P_2, \qquad Q_2 = 36 - P_1 - 2\ P_2\ .$$

The cost of making each pizza is \$2, and the cost of each six-pack is also \$2. Any fixed costs are sunk and can be ignored for the purpose of this question. The stores are price-setters and profit-maximizers.

(a) Are the goods substitutes or complements?

(b) Write down the algebraic formulas expressing the profits $\Pi_1$ and $\Pi_2$ of the two stores in terms of their prices $P_1$ and $P_2$.

(c) Suppose the two choose their prices simultaneously and independently (Bertrand competition). Find the equations for the best response functions (reaction curves), and graph these showing $P_1$ on the horizontal axis and $P_2$ on the vertical axis. Find the Bertrand equilibrium, that is, the Nash equilibrium of the price-setting game. Find the prices, quantities, and profits of the two stores in this equilibrim.

(d) Now suppose the firms get together and choose $P_1$ and $P_2$ collusively to maximize the total profit $\Pi = \Pi_1 + \Pi_2$. To find this, you must maximize $\Pi$ with respect to each price. To do so, obtain two equations for $P_1$ and $P_2$ by setting $\partial\Pi/\partial P_1 = 0$ and $\partial\Pi/\partial P_2 = 0$, and then solve the resulting pair of equations for $P_1$ and $P_2$. Also draw the graphs of these two equations, either in the same graph as the one you drew for (c) above, or a separate one. Find the joint-profit-maximizing prices and quantities, and the resulting profit of each firm.

(e) Compare the prices, quantities, and profits in (c) and (d) above. You will find that the prices in (d) when firms are colluding are lower, not higher, than those in (c) where they are choosing prices independently. Give an economic intuition for this findings. What are the implications for antitrust policy?

# Solution Handout for Precepts Week 9: November 23

## Solution:

(a) Complements.
(b) Profit expressions:

$$\Pi_1 = (36 - 2\,P_1 - P_2)\ (P_1 - 2), \qquad \Pi_2 = (36 - P_1 - 2\,P_2)\ (P_2 - 2)$$

(c) For firm 1's best response function, choose $P_1$ to maximize $\Pi_1$ given $P_2$:

$$\partial \Pi_1 / \partial P_1 = (36 - 2\,P_1 - P_2) - 2\ (P_1 - 2) = 40 - 4\,P_1 - P_2 = 0$$

Solving for $P_1$ as a function of $P_2$:

$$P_1 = 10 - \tfrac{1}{4}\ P_2$$

This equation is labeled $BR_1$ in the graph at the top of the next page.

Similarly firm 2's best response function is

$$40 - P_1 - 4\,P_2 = 0 \qquad \text{or} \qquad P_2 = 10 - \tfrac{1}{4}\ P_1$$

This equation is labeled $BR_2$ in the graph below.

To solve the two best response equations jointly, substitute from one into the other to write

$$P_1 = 10 - \tfrac{1}{4}\ \left[\ 10 - \tfrac{1}{4}\ P_1\ \right] = 7.5 + \tfrac{1}{16}\ P_1$$

Therefore

$$\tfrac{15}{16}\ P_1 = \tfrac{15}{2} \qquad \text{or} \qquad P_1 = 8$$

Then $P_2 = 10 - \frac{8}{4} = 8$ also. This is the Bertrand equilibrium shown at the point B (for Bertrand) in the graph below.

Then $Q_1 = Q_2 = 36 - 16 - 8 = 12$, and $\Pi_1 = \Pi_2 = 12 * (8 - 2) = 72$.

(d) Expression for total profit

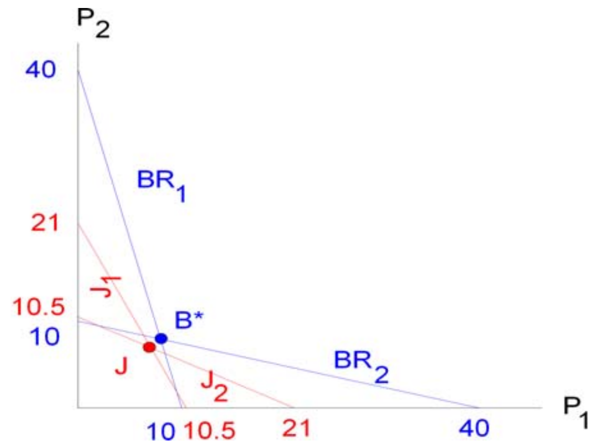
$$\Pi = \Pi_1 + \Pi_2 = (36 - 2\,P_1 - P_2)\ (P_1 - 2) + (36 - P_1 - 2\,P_2)\ (P_2 - 2)$$

To maximize this,

$$\partial \Pi / \partial P_1 = (36 - 2\,P_1 - P_2) - 2\ (P_1 - 2) - (P_2 - 2) = 42 - 4\,P_1 - 2\,P_2 = 0$$

This equation is labeled $J_1$ (for joint-profit-maximization condition with respect to $P_1$) in the graph below.

Similarly with respect to $P_2$ we have $42 - 2\,P_1 - 4\,P_2 = 0$. This equation is labeled $J_2$ in the graph below.




The two maximization conditions together determine the collusively optimal $P_1$ and $P_2$. To solve them as a pair of simultaneous equations, subtract one from the other to get $2\,P_1 - 2\,P_2 = 0$ so $P_1 = P_2$, and then $42 - 6\,P_1 = 0$ so $P_1 = P_2 = 7$. This is the point J in the graph below.

Therefore $Q_1 = Q_2 = 36 - 14 - 7 = 15$, and $\Pi_1 = \Pi_2 = 15 * (7 - 2) = 75$.

Joint graph shown on the next page:

(e) Comparisons: When the firms collude, they charge lower prices, sell larger quantities, and make more profits.

Intuition: It may seem surprising that when firms collude and therefore can monopolize the industry, they charge lower prices than when they compete (Bertrand). The reason is that unlike the usual situation where the firms are selling substitute products, here they are selling complements. If one firm lowers its price, that shift the demand curve facing the other firm outward, not inward. That increases the profit the other firm can make. In other words, one firm by lowering its price would be conveying a kind of "positive externality" on the other. When the firms are acting independently, each one being concerned with its own profit does not take into account this benefit it confers on the other firm. Therefore each cuts its price too little (keeps its price too high) as judged from the perspective of their joint profits. When the firms collude (or merge), they do take into account this benefit that lowering one firm's (or branch's) price would convey on the other firm (or branch); therefore they do lower both prices to a new optimal point.

Incidentally, in this situation the consumers also benefit form the collusion or merger: a cartel or a monopoly is better for consumers than duopoly! It is a win-win situation; regulators or antitrust policy makers should not try to break up such a cartel; in fact they should encourage cartelization of complements.

So are operating systems and other software like browsers complements? If you think they are complements, offer your consulting services to Microsoft!

Additional line of thought:

Now take this intuition back to the case of substitutes, and explain the usual finding that collusion or merger will lead to higher prices in terms of a "negative externality" between the firms.

An additional point:

If you look carefully, you will notice that the $BR_2$ and $J_2$ lines cross close to the vertical axis, that is, for very small values of $P_1$, the joint-profit-maximizing $P_2$ is higher than the non-cooperative Bertrand best response $P_2$. Can we reconcile this with the above intuition which says that because the products are complements, for given $P_1$, the joint-profit-maximizing $P_2$ should be lower than the Bertrand best response $P_2$?

Note that $BR_2$ and $J_2$ cross when $40 - P_1 - 4\,P_2 = 0$ and $42 - 2\,P_1 - 4\,P_2 = 0$. These yield solutions $P_1 = 2$ and $P_2 = 9.5$. But when $P_1 < 2$, firm 1's price is below its marginal cost. In this situation, the fact that a decrease in $P_2$ increases the quantity sold by firm 1 (the products are complements) in fact hurts firm 1's profits. Joint decision-making takes this into account whereas separate price-setting does not. That is why the jointly optimal $P_2$ is higher than the Bertrand best response $P_2$ when $P_1 < 2$.