

ECO 300 – MICROECONOMIC THEORY – FALL 2005  
SOLUTIONS FOR PRECEPTS WEEK 11 – DECEMBER 7

## GAME THEORY

(a) Vizzini does not see the move Westley has made. Therefore even though he moves later, he cannot respond to what Westley has actually done. (If he could, he would have a tremendous “second-mover advantage”). And therefore Westley cannot look forward to this response and roll back to solve the game. Therefore the moves are as if they were simultaneous. They are called strategically simultaneous even though they are temporally sequential.

(b) This is a constant sum game; the two payoffs in each cell sum to 1.

(c) No; there is no Nash equilibrium in pure strategies.

(d) Suppose Westley is choosing to poison cup A with probability  $p$  and B with probability  $(1-p)$ . Against this strategy, Vizzini gets expected payoff  $0 * p + 1 * (1-p) = 1-p$  by choosing A, and  $1 * p + 0 * (1-p) = p$  by choosing B. For Vizzini to be mixing between his own two choices of which cup to drink from, he must be indifferent between the two, that is, he must be getting equal expected payoff from the two. That means  $1-p = p$ , or  $p = 1/2$ . Similarly, Vizzini’s own mixture probabilities are found to be  $1/2$  for each of his two strategies.

(e) Vizzini does not know that Westley’s payoff is 1 no matter what; the game becomes one with a kind of asymmetric information. Actually it is even worse than the moral hazard or adverse selection situations we saw in the theory. There at least the uninformed party knew the way in which the informed party’s action affected the outcome (moral hazard), or what different types of the informed party could exist, for example different levels of skill or different qualities of cars, and the probabilities of these different types (adverse selection). Here Vizzini has a much bigger and perhaps even qualitatively different information disadvantage – he does not even know what is possible for Westley.

There is a qualitative difference between putting a low probability on something, and regarding it as inconceivable or impossible and therefore omitting it altogether from your calculation. Examples of this often occur in military conflicts. In June 1967, Israel destroyed the Egyptian air force by a preemptive attack; their planes flew a long hook-shaped route and attacked Egyptian airfields and airplanes from the rear. The general reaction was: “Who would have thought they could do that?” But should it have been anticipated? And in June 1940, did the French put a very low probability on a German attack through the Ardennes forest, or did they simply disregard it because they thought it was impossible?

Game theory has not yet found a way to treat such situations in a mathematically rigorous formal way. But there is some vague intuitive way the uninformed player can make a general allowance for such qualitative absence of information. The mere fact that Westley is offering to play a game of life and death where Vizzini’s decision will control the outcome, should lead Vizzini to suspect that Westley must have somehow stacked the odds at far better than 50:50 in his own favor, so Vizzini should simply refuse to play such a game. A related example occurs in the movie *Guys and Dolls*. The gambler Sky Masterson quotes the advice from his father: “Son, no matter how far you travel, or how smart you get, always remember this: Some day, somewhere, a guy is going to come to you and show you a nice brand-new deck of cards on which the seal is never broken, and this guy is going to offer to bet you that the jack of spades will jump out of this deck and squirt cider in your ear. But son, do not bet him, for as sure as you do you are going to get an ear full of cider.”

## MORAL HAZARD AND THE DESIGN OF INCENTIVES

### 1. Course grades

The indicators on which incentive payments (grades) are based must be observable. Therefore assertions of “I worked really hard” and “I know the material really well” won’t work. There are several direct indicators of effort – coming to class and precepts, doing the work for the precepts in advance (attempting the assigned problems or thinking about the discussion topics) which are better observable. But in a large class it would take too much time to

keep a formal record of the attendance, and Martin cannot simultaneously conduct the discussion and remember all the details of who was well prepared and who contributed how much to the discussion. Therefore such indicators are used only a little, for example if a student is right at a borderline between two grades, these considerations may decide whether he/she goes up or down. Doing all eight problem sets even when two freebies are allowed can be a similar indicator useful at the margins.

The main indicators have to be the problem sets and exams. For them to be good indicators: [1] They should be comprehensive. That is, they should cover as much of the material as possible, so it is not possible to get a good grade by skipping a lot of the material in your review. [2] They should have low error of observation. Essay type questions are bad in this regard, because their grading has some unavoidable subjective element. Multiple choice questions are better, as are math problems. But there are offsetting considerations. Essays enable students to display their understanding in a less tightly constrained way and even exhibit some extra imagination and creativity. Multiple choice questions do leave some ambiguity. And in math problems, if you make a careless mistake in the early part, “conditionally correct” later work still unavoidably gets only partial credit at best. So the optimum is highly constrained, and must balance all these considerations.

P.S. on the grades question:

In the precept, Rada Leenders raised an interesting point: That whole discussion was based on the assumption that students are motivated by grade maximization rather than the pursuit of knowledge. In reality there are students that try to maximize knowledge rather than grades: good grades and prizes are then a natural consequence rather than motivation for action. In such cases, the professor’s elaborate incentive schemes based on grades may have the perverse effect of leading even these students to behave as grade-maximizers, against their true nature and original interest is knowledge maximization.

This is an excellent point and I fully agree. This is the case of what I called “motivated agents” (see the handout dated November 29, p. 8) – those share (at least partly) the aims of the principal and therefore are easier to incentivize. And such agents may be put off by the existence of material incentives that they regard as unworthy, and actually exert less effort as a result. This conflict between “intrinsic” and “extrinsic” incentives has been studied by psychologists, and more recently taken up by economists. Our own professor Roland Benabou is an expert in this area. See his paper “Intrinsic and Extrinsic Motivation,” with Jean Tirole, *Review of Economic Studies*, 70(3), July 2003, pp. 489-520.

I am sure that each and every one of you is exactly the kind of student that Rada describes ☺ . However, I should add that in my Princeton experience there are too many students exactly of the kind I assumed in the precept question ☹ .

## 2. Payment for proofreading

I want the student to catch all the typesetting errors there may be, but I cannot be sure of that unless I do the checking myself, which defeats the whole purpose of hiring the student. The student’s effort is unobservable - she takes away the materials and comes back a week later to tell me of the errors she has found. What is worse, even the outcome cannot be observed immediately. I will find out about any error she failed to catch only when some other reader tells me about them. So the student has the temptation to shirk — just hold on to the materials for a few days and then tell me that there are no errors. So I cannot offer her a fixed flat sum for the job. But if I offer a piece rate (so much per error she finds), she may worry that the typesetter has done a perfect job, so she may have to spend a week or more on the work and get no money at the end of it. She will be reluctant to take the job on these terms.

Therefore the compensation scheme has to be a compromise between the two extremes — a flat sum plus a bonus per error she discovers. This should give her enough assurance of the total compensation to make the job attractive enough, and give her enough incentive to attempt a thorough reading. My solution was a dollar per page (\$600 total) as a flat sum, plus \$1 per error found (there were 274). I don’t claim it was fully optimal or the best deal I could have got. And the job was not perfect either: about 50 errors she missed have come to light since. But the principle that a mixture is optimal is much more generally valid.

Some further points. [1] If there is a longer-term interaction, for example the student may need letters of recommendation from me in the future for jobs where similar skills are relevant, then an immediate cash incentive become less important. The student will do a good job for the sake of the implicit future rewards such as good letters of recommendation (career concerns). [2] If I do not know the quality of the student in advance, there is the possibility of adverse selection – a student who is willing to do poorly paid proofreading work instead of lucrative work as a software or web consultant for local businesses may not be very good. However, selection does not have to be adverse. A student willing to do an academic-related job on the campus for relatively low payment may be more genuinely interested in the subject and therefore likely to do a better job, and also more likely to be aware of and motivated by the “career concerns” mentioned above. [3] I may be able to reduce the uncertainty which exists due to the fact that he cannot perfectly monitor the student’s effort level, by hiring two students (that do not know each other) and split the work between them, but with some overlap of pages to be corrected by both of them. In this case, a student who finds only a few errors in this segment is shown to be slacking if the other finds many more. So the payment can be based on the “relative performance” on the overlap, to sharpen the incentives. Of course I should not tell each student who the other is (else they can collude), nor should I tell either what pages overlap between them (else they will be careful with that part and careless with the rest).