

ECO 300 – MICROECONOMIC THEORY  
Fall Term 2005  
PROBLEM SET 1 – ANSWER KEY

The distribution of scores was as follows:

90-99	56
80-89	18
70-79	3
< 80	2

This was an excellent start; try to keep it up. But past experience indicates that the scores on some later problem sets are lower, and those on exams are much lower ☹

When the overall performance is this good, there are very few common errors to point out. One was sloppiness about labeling axes, curves, and points properly in Questions 2-4. Please read the answer key carefully, to see how labeling in figures helps the verbal argument: you can refer to the figure more easily and clearly.

A few other points are noted below; Martin has noted some individual errors on the graded answers being returned.

Read the “additional information” in the key to Question 5; it will prove useful in later problem sets and exams.

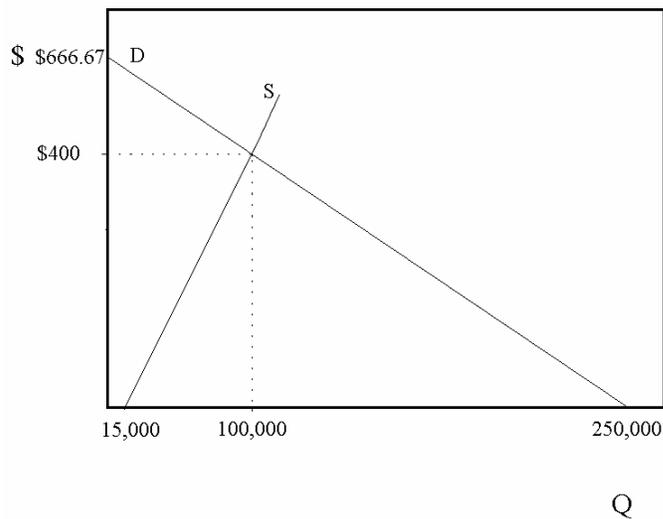
These practices – brief discussion of common errors and occasional additional information in answer keys, and individual-specific remarks on graded answers – will continue for all problem sets and exams.

QUESTION 1: (Total 12 points; 1 each for nos. 1-8, 2 each for nos. 9, 10)

1. c
2. b
3. e
4. b
5. c
6. b
7. a
8. a
9. a
10. a

QUESTION 2: (Total 20 points, 10 for each of parts a and b)

(a)



The equilibrium price can be found by equating Demand and Supply (graphically, this is where the Demand and Supply curves D and S intersect). We can then solve for equilibrium price; it is \$400. At this price, quantity supplied and quantity demanded are each equal to 100,000.

(b) At the endpoints of the arc (for the given prices 300 and 500), calculate from the given equations the quantities demanded and supplied. They are:

Price	Quantity demanded	Quantity supplied
300	137500	78750
500	62500	121250

At the midpoint, the price is 400, the equilibrium price, so both quantities are 100000. Using the formula, the arc elasticity of demand (in numerical value) is

$$\frac{(137500 - 62500) / 100000}{(500 - 300) / 400} = 1.5$$

And that of supply is

$$\frac{(121250 - 78750) / 100000}{(500 - 300) / 400} = 0.85$$

At the equilibrium point, using the point elasticity formula, the elasticity of demand (in numerical value) is

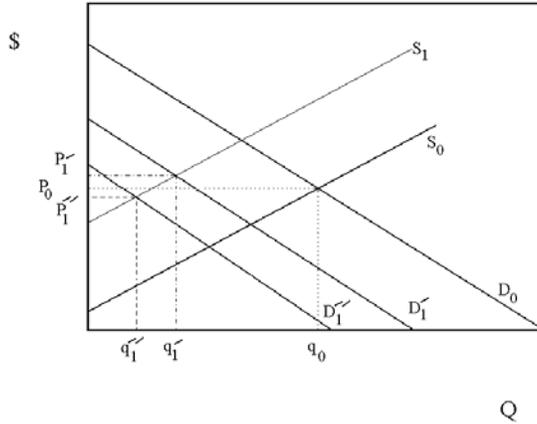
$$\frac{400}{100000} \cdot 375 = 1.5$$

And that of supply is

$$\frac{400}{100000} \cdot 212.5 = 0.85$$

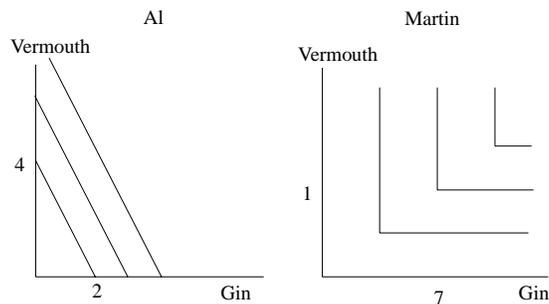
(For your extra information: The finding that the arc elasticity over a range of prices equals the point elasticity at the midpoint of that range is a special property of straight line demand and supply curves. It does not hold more generally.)

QUESTION 3: (Total 20 points)



The increase in the cost of production of steel will shift the supply curve to the left. This effect alone on the market will influence the market price to rise while the market quantity will fall. This is shown above by a movement from the original supply curve  $S_0$  to a new supply curve such as  $S_1$ . The decrease in demand will cause the demand curve to shift to the left. This effect alone on the market will influence the market price and quantity of steel to fall. Note that the supply and demand effects on price work in opposite directions. If the supply effect dominates the demand effect, the equilibrium prices will rise. This is exhibited by the decrease in demand to  $D_1'$ . On this demand curve, the net effect is for price to rise from  $P_0$  to  $P_1'$ . On the other hand if the demand effect dominates, equilibrium price will fall. This is exhibited by the decrease in demand to  $D_1''$ . On this demand curve, the net effect is for price to fall from  $P_0$  to  $P_1''$ . As we don't know given the current information which effect dominates, we can't perfectly predict the change in price. The quantity is unambiguously decreased.

QUESTION 4: (Total 13 points, 5 for each of a and b, 3 for c)



(a) and (b) The indifference maps are shown above; note that the scales are different for the two. The amount of alcohol Al gets is  $0.4 G + 0.2 V$ , where  $G$  is the quantity of Gin and  $V$  the quantity of Vermouth, and we can take this to be Al's utility function  $U(G,V)$ . Alternatively,  $U = 2 G + V$ , or  $\log(2G+V)$ , or  $\sqrt{(2G+V)}$ , any other increasing function of  $2 G + V$  will do just as well; utility is ordinal.

(c) Al's preferences are the case of perfect substitutes; Martin's the case of perfect complements. (NOT compliments, a common spelling error.)

QUESTION 5: (35 points)

(a) The budget constraint is  $M + F = 30$  (or  $M + F \leq 30$  if you want to be pedantic)

(b) Substituting  $M = 30 - F$ , we have

$$G = 0.1 (30 - F) + 0.2 F - 0.01 F^2 = 3.0 + 0.1 F - 0.005 F^2$$

To maximize G: First-order condition

$$dG/dF = 0.1 - 0.005 * 2 F = 0.1 - 0.01 F = 0$$

Second-order condition is satisfied:  $dG/dF = -0.01 < 0$ .

Therefore  $F = 0.1 / 0.01 = 10$

Then  $M = 30 - 10 = 20$ ,

and  $G = 0.1 * 20 + 0.2 * 10 - 0.005 * 100 = 2.0 + 2.0 - 0.5 = 3.5$

(c) The Lagrangian is: as in P-R p. 146

$$L = (0.1 M + 0.2 F - 0.005 F^2) - \lambda (M + F - 30)$$

or as in Lecture handout of Sep. 20, p. 7

$$L = (0.1 M + 0.2 F - 0.005 F^2) - \lambda (M + F)$$

where  $\lambda$  is the Lagrange multiplier to be determined as part of the solution.

(Since the 30 does not depend on M or F, the first-order conditions with respect to these variables are not affected by your choosing the one or the other. If you use the P-R method, you will write the third condition as  $\partial L / \partial \lambda = 30 - M - F = 0$ ; by the way P-R make a sign mistake in their expression for  $\partial L / \partial \lambda$ , but since the condition sets it equal to zero, the sign does not matter. If you use the lecture handout method, you will have to remember separately that the third condition is just the constraint.)

First-order conditions are

$$\partial L / \partial M = 0.1 - \lambda = 0$$

$$\partial L / \partial F = 0.2 - 0.005 * 2 F - \lambda = 0.2 - 0.01 F - \lambda = 0$$

So  $\lambda = 0.1$ , therefore  $0.01 F = 0.2 - 0.1 = 0.1$ , so  $F = 10$ , and  $M = 30 - 10 = 20$

$$G = 0.1 * 20 + 0.2 * 10 - 0.005 * 100 = 3.5$$

And the Lagrangian is  $L = G - \lambda (M + F - 30) = 3.5 - 0.1 (20 + 10 - 30) = 3.5$

Incidentally, the fact that at the optimal values, the Lagrangian equals the value of the maximand (so the added term  $-\lambda (M + F - 30)$  equals zero, is of mathematical significance; so just remember this in case you encounter it in a subsequent applied course. It won't matter in this course.

(d) If the constraint changes to  $M + F = 35$ , retracing the above steps shows that in the new solution we still have  $F = 10$  and  $\lambda = 0.1$ , but now  $M = 25$  and

$$G = 0.1 * 25 + 0.2 * 10 - 0.005 * 100 = 2.5 + 2.0 - 0.5 = 4.0 \text{ ☺}$$

(e) (i) The solutions in parts (b) and (c) are exactly the same, as they had better be since they are different methods of solving the same problem.

(ii) Comparing parts (c) and (d):

The optimal value of F is unchanged but that of M changes – all the increase in available time is spent on studying for Mathematical Methods. The brief intuitive reason is as follows. In the formula for G, the marginal product of M stays constant at 0.1. The marginal product of F is  $\partial G/\partial F = 0.2 - 0.005 * 2F = 0.2 - 0.01 * F$ . This decreases as F increases. At F = 10, it is  $0.2 - 0.1 = 0.1$ , the same as the marginal product of M. In fact equalization of the marginal products is exactly what makes F = 10 and M = 20 the optimum. If more hours are available, spending them on F will lower the marginal product of the F-effort below that of the M-effort, and that would not be optimal. The equality of the marginal products is maintained by keeping F = 10 and increasing M. (For a fuller explanation, see the “additional information” below.)

Some of you thought that the effort spent on F does not increase because any further increase would decrease GPA. That is not true. To drive the marginal product of F negative would require  $F > 20$ , as you can see from the formula for the marginal product. But negative marginal product is not needed to make it non-optimal to spend extra hours on F; all that is needed is that the marginal product be less than that of spending those extra hours on M.

The value of G increases by 0.5, which equals the Lagrange multiplier  $\lambda = 0.1$  times the amount of increase in the time,  $35 - 30 = 5$ . This confirms the economic interpretation of the Lagrange multiplier as the marginal value of relaxing the constraint.

#### ADDITIONAL INFORMATION:

You were not asked for this and no points attach to it. But you should read it now; it will help you understand some later material better and some of this may appear on exams.

When you are asked to do two or more calculations with different numbers, it is often easier to do them just once using an “algebraic constant” or “parameter,” and then calculate the results for the stated numerical values of this constant. In this question, suppose you have a total of H hours of time for study. Let us use the method in part (b); the calculations in part (c) are similar.

The budget constraint becomes  $M + F = H$ . Substituting out M, the expression for G as a function of F alone is

$$G = 0.1 (H - F) + 0.2 F - 0.01 F^2 = 0.1 H + 0.1 F - 0.005 F^2$$

To maximize G: First-order condition

$$dG/dF = 0.1 - 0.005 * 2 F = 0.1 - 0.01 F = 0$$

Second-order condition is satisfied:  $dG/dF = -0.01 < 0$ .

Therefore  $F = 0.1 / 0.01 = 10$ . Then  $M = H - 10$ , and

$$G = 0.1 * H + 0.1 * 10 - 0.005 * 100 = 0.1 H + 1.0 - 0.5 = 0.1 H + 0.5$$

Now you can evaluate these expressions for  $H = 30$  and  $H = 35$ .

But the use of the algebraic constant  $H$  tells us something more. The solution for  $F$  is completely independent of  $H$ . You should optimally choose  $F = 10$ , and devote all the extra time you have to studying for  $M$ .

This is again a consequence of the fact that the marginal value of time spent on  $M$  remains constant, while that of time spent on  $F$  falls as  $F$  increases. If you regard this as a standard consumer problem with  $M$  and  $F$  as the two goods and  $G$  as the utility, the utility function is linear in  $M$ , but non-linear (in fact concave) as a function of  $F$ . Such a utility function is called *quasi-linear*. In the consumer interpretation, an increase in  $H$  is like an increase in income, so its effects on  $F$  and  $M$  are like pure income effects. And the result is that with quasi-linear utility, the pure income effect is 1 for the good in which utility is linear and 0 for the good in which utility is concave (has diminishing marginal utility).

However, there is a limit to this. If  $H < 10$ , the optimal solution for  $M$ , namely  $H - 10$ , becomes negative, which is not possible. Then the true optimum is to set  $M = 0$ , and give all of your time to  $F$ . To see the sense of that, take the formula for the GPA,

$$G = 0.1 M + 0.2 F - 0.005 F^2$$

and calculate the marginal effect of  $F$  on  $G$ ,

$$\partial G / \partial F = 0.2 - 0.005 * 2 F = 0.2 - 0.01 F = 0.01 (20 - F)$$

When  $F < 10$ ,  $\partial G / \partial F > 0.1$ . But  $\partial G / \partial M = 0.1$ . Therefore when your time is really scarce, the marginal value of devoting it to  $F$  is higher than that of diverting any of it to  $M$ . That is why  $M = 0$  is optimal in those circumstances. Similarly in the consumer interpretation, if your preferences are quasi-linear, when your income is sufficiently low, it becomes optimal to spend all of it on the good that has diminishing marginal utility and none on the good that has constant marginal utility. The flip side of diminishing marginal utility is that when the consumption quantity is small, the marginal utility is high.