ECO 300 – MICROECONOMIC THEORY
Fall Term 2005
PROBLEM SET 3 – DRAFT ANSWER KEY

The distribution of scores was as follows:

100	5
90-99	63
80-89	9
< 80	2

Very good job; congratulations all round! Try to keep this up.

There are only a few general comments to do with economic interpretations.

QUESTION 1 (Total 35 points)

(a) (5 points) With I = 20, $P_X = 2$, and $P_Y = 1$, by the standard Cobb-Douglas formula the optimal choices are X = ½ 20 / 2 = 5, Y = ½ 20 / 1 = 10. The resulting utility level is 50.

(b) (5 points) With I = 20, $P_X = 1$, and $P_Y = 1$, again by the Cobb-Douglas formula the optimal choices are X = ½ 20 / 1 = 10, Y = ½ 20 / 1 = 10. The resulting utility level is 100.

(c) (10 points) To calculate the compensating variation, suppose your income is Z at the new prices. You choose X = ½ Z / 1 = Z / 2, Y = ½ Z / 1 = Z / 2, and get utility Z^2 / 4 . To keep this equal to the old utility 50, we take $Z^2 = 4 * 50 = 200$, or $Z = 10 \sqrt{2}$ = 14.14 . So 20 – 14.14 = 5.86 could be taken away from you and leave you with the same utility as before. This is the compensating variation.

(d) (10 points) To calculate the equivalent variation, suppose your income is Z at the old prices. You choose X = ½ Z / 2 = Z / 4, Y = ½ Z / 1 = Z / 2, and get utility Z^2 / 8 . To keep this equal to the new utility 100, we take $Z^2 = 8 * 100 = 800$, or $Z = 20 \sqrt{2}$ = 28.28 . So 28.28 – 20 = 8.28 should be given to you in order to yield you the same utility as you got from the price decrease. This is the equivalent variation.

(e) (3 points) Here EV > CV. This is a situation where the consumer gains from the change (the decrease in the price of a good). Since the good X is normal, he buys more of it in the new situation. To enable him to get the new higher utility without the price decrease requires giving him more income, than the amount of income he can give up while preserving the old lower utility. . Also see the class handout of Oct. 4, p. 4 bottom. (This is a little tricky, and for that reason your answers were graded relatively generously.)

(f) (2 points) This is exactly the reverse of the comparison we did in class. There P_X increased from 1 to 2; here it drops from 2 to 1. The status quo and the changed situation reverse roles. Therefore the CV there becomes the EV here and vice versa.

QUESTION 2 (Total 35 points)

(a) (5 points) For the saver, C(2005) = 200 – S , C(2006) = 220 + 1.1 S

(b) (5 points) For the borrower, C(2005) = 200 + B , C(2006) = 220 – 1.1 B

(c) (3 points) Write the saver's equation as C(2006) / 1.1 = 220 / 1.1 + S. Also C(2005) = 200 – S. Adding the two equations, we have C(2005) + C(2006) / 1.1 = 200 + (220 / 1.1) = 200 + 200 = 400.

Similarly, write the borrower's equation as C(2006) / 1.1 = 220 / 1.1 - B. Also C(2005) = 200 + B. Adding the two equations, we have C(2005) + C(2006) / 1.1 = 200 + (220 / 1.1) = 200 + 200 = 400.

Interpretation: (2 points) The left hand side of the common equation is the present discounted value (PDV) or net present value (NPV) of the consumption amounts in the two years. The right hand side is the PDV or NPV of the income amounts in the two years. So the budget constraint says that the PDV of consumption expenditures must equal the PDV of the income stream, that is, the wealth. This is an "intertemporal budget constraint," and the 1/1.1 thought of as the "price of tomorrow's dollar relative to today's dollar." Future dollars are valued less because they are later; this is discounting.

(d) (10 points for each of Ina and Vera) Mathematically, the budget constraint is just like the standard $P_X X + P_Y Y = I$, where now X = C(2005), Y = C(2006), $P_X = 1$, $P_Y = 1/1.1$, and I = 400. Using the Cobb-Douglas formulas:

For Ina, $P_X X / I = 0.6$, so C(2005) = 0.6 * 400 = 240. Ina borrows $40,000

For Vera, $P_X X / I = 0.3$, so C(2005) = 0.3 * 400 = 120. Vera saves $80,000

(10 points each)

QUESTION 3: (total 30 points)

(a) (2 points) Y = w L – S

(b) $U = (20 - L) * (w L - S) = - 20 S + (20 w + S) L - w L^2$ (8 points)

(c) To maximize this, dU/dL = (20 w + S) – 2 w L = 0. Solving for L, we get L = 10 + S / (2w). This is Joe's "labor supply function". (8 points for math, 2 for name). Some people had this wrong, and called it "marginal utility of labor" etc.

(d) (5 points) When S decreases, L decreases. This is like a pure income change in standard consumer theory, and leisure is a normal good.

(e) (5 points) When w increases, L decreases. This is like a price change in standard consumer theory, and it has an income effect and a substitution effect. The pure substitution effect of an increase in w causes Joe to want to supply more labor. (You can think of this as saying that the opportunity cost of taking leisure has gone up). But the increase in w makes Joe better off, so the income effect wants him to take more leisure and so supply less labor. So the two effects oppose each other. In general the balance could go either way; in our specific problem we saw that L decreased when w increased. So in our specific problem the income effect outweighs the substitution effect. (5 points)

Many people had this wrong. They thought of "income-targeting behavior," which will also produce a backward-bending labor supply curve but that is not the nature of the preferences here. (If there were a target income say I^* after the debt was paid off, then the labor supply would be $(I^*+S) / w$.) Some people said "now he can achieve the same utility by working less." True, but the objective is not that of achieving the same utility; the objective is to maximize utility.