ECO 300 – MICROECONOMIC THEORY
Fall Term 2005
PROBLEM SET 4 –ANSWER KEY

The distribution of scores was as follows:

100	4
90-99	53
80-89	7
70-79	10
< 70	4

QUESTION 1: (Total 30 points. 8 for the calculation in each case, 6 for the comments.)

GENERAL COMMENTS - The calculations were mostly correctly done but few people had good intuitions or interpretations about correlation etc. Some people tried to calculate variances of sums using the formula involving covariance. This works if done correctly, but it is easy to make mistakes. A much simpler method, followed below, is just to list all possible outcomes of the uncertainty (scenarios), and then calculate the means and variances directly using the definitions.

CASE A – Two scenarios: Rain and Drought, probabilities ½ each.

Total income is $200 if Rain, $100 if Drought. Each gets half of this. So

$E[I] = 0.5 * 100 + 0.5 * 50 = 75$

$V[I] = 0.5 * (100-75)^2 + 0.5 * (50-75)^2 = 0.5 * 625 + 0.5 * 625 = 625$

$SD[I] = 25$

CASE B – Two scenarios, Rain North and Rain South, probabilities ½ each.

Total income is 100 + 50 = 150 if Rain North, 50 + 100 = 150 if Rain South. The two owners share this equally. So each gets 75 in each scenario.

$E[I] = 0.5 * 75 + 0.5 * 75 = 75$

$V[I] = 0.5 * (75-75)^2 + 0.5 * (75-75)^2 = 0$

$SD[I] = 0$

CASE C – Four scenarios: Rain Both, Rain North, Rain South, and Rain Neither. Probabilities ¼ each.

Total incomes 200, 150, 150, and 100 in the four scenarios. Each gets half. So after the risk-sharing the income of each in the four scenarios is 100, 75, 75, and 50.

$E[I] = 0.25 * 100 + 0.25 * 75 + 0.25 * 75 + 0.25 * 50 = 75$

$V[I] = 0.25 * (100-75)^2 + 0.25 * (75-75)^2 + 0.25 * (75-75)^2 + 0.25 * (50-75)^2$
$= 0.25 * 625 + 0 + 0 + 0.25 * 625 = 0.5 * 625$

$SD[I] = 25 / \sqrt{2} = 17.68$

INTERPRETATION: In Case A, the two owners' risks are perfectly positively correlated – either both have high income or both have low income. Therefore there is no risk-reduction available by sharing. In Case B, the two risks are perfectly negatively correlated. Therefore the risk can be eliminated entirely by sharing. In Case C the two risks are independent. Therefore some reduction but not total elimination can be achieved by sharing.

QUESTION 2: (Total 45 points)

GENERAL COMMENTS: [1] In the graph, you should follow the economics convention and show p on the vertical axis and x on the horizontal axis. Also you should show the demand/supply functions for Sam and Nancy on the same graph so you can show the equilibrium. This time no points were taken off for these matters. [2] Appropriate partial credit was given for correct intermediate results, but almost no credit was given when later results were wrong due to previous mistakes.

Using the formulas for the final incomes of the two people in the two scenarios:

$$E[I(Sam)] = 0.5 * (50 – p\,x + x) + 0.5 * (100 – p\,x) = 75 – p\,x + 0.5\,x$$

$$\begin{aligned} V[I(Sam)] &= 0.5 * [(50 – p\,x + x) – (75 – p\,x + 0.5\,x)]^2 \\ &\quad + 0.5 * [(100 – p\,x) – (75 – p\,x + 0.5\,x)]^2 \\ &= 0.5 * [– 25 + 0.5\,x]^2 + 0.5 * [25 – 0.5\,x]^2 \\ &= [25 – 0.5\,x]^2 = 625 – 25\,x + 0.25\,x^2 \end{aligned}$$

And

$$E[I(Nancy)] = 0.5 * (50 + p\,x – x) + 0.5 * (100 + p\,x) = 75 + p\,x – 0.5\,x$$

$$\begin{aligned} V[I(Nancy)] &= 0.5 * [(50 + p\,x – x) – (75 + p\,x – 0.5\,x)]^2 \\ &\quad + 0.5 * [(100 + p\,x) – (75 + p\,x – 0.5\,x)]^2 \\ &= 0.5 * [– 25 – 0.5\,x]^2 + 0.5 * [25 + 0.5\,x]^2 \\ &= [25 + 0.5\,x]^2 = 625 + 25\,x + 0.25\,x^2 \end{aligned}$$

(20 points for the calculations up to this point; 4 for each of the two expected values, 6 for each of the two variances.)

Therefore Sam's objective function (here called "utility", and you were told not to confuse it with the "expected utility theory") is

$$U(Sam) = 75 – p\,x + 0.5\,x – 0.006\,[625 – 25\,x + 0.25\,x^2]$$

Choosing x to maximize this, we set dU(Sam)/dx = 0, so

$$– p + 0.5 – 0.006 [– 25 + 0.5\,x] = 0$$
$$– p + 0.5 + 0.15 – 0.003\,x = 0$$
$$x = [0.65 – p] / 0.003 = 1000 [0.65 – p] / 3$$

And similarly Nancy's objective function (utility) is

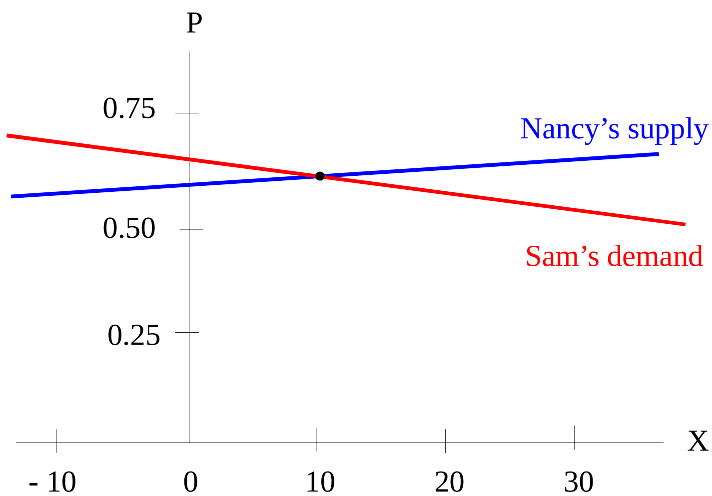
$U(Nancy) = 75 + p\,x - 0.5\,x - 0.004\,[625 + 25\,x + 0.25\,x^2]$

Choosing x to maximize this, we set dU(Nancy)/dx = 0, so

$$p - 0.5 - 0.004\,[\,25 + 0.5\,x\,] = 0$$
$$p - 0.5 - 0.1 - 0.002\,x = 0$$
$$x = [\,p - 0.6\,] / 0.002 = 500\,[\,p - 0.6\,]$$

(7 points for each person's demand/supply function.)

Graphs of these are shown in the figure: (6 points for the graph)



From the graph, or by solving the two equations, we find

$$1000\,[\,0.65 - p\,] / 3 = 500\,[\,p - 0.6\,]$$
$$1000\,[\,0.65 - p\,] = 1500\,[\,p - 0.6\,]$$
$$650 - 1000\,p = 1500\,p - 900$$
$$2500\,p = 1550$$
$$p = 0.62$$

and then $x = 1000\,[\,0.65 - 0.62\,] / 3 = 10$. (5 points for finding the equilibrium)

NOTES: Here I did not ask for an interpretation because it is tricky. But read the following; it will help you understand the material better:

1. Nancy's supply curve and Sam's demand curve both have portions with negative x. You need not have shown these because they are not in the relevant region. However, they are not without economic interest. If the price of insurance is low enough, Nancy will buy rather than sell some ($x < 0$ for her), whereas if the price of insurance is high enough, even Sam will want to sell some ($x < 0$ for him). There are situations of this kind where one cannot tell in advance whether some trader is on the buying side or the selling side of a market. For example, whether a country is an importer or an exporter of a good depends on its world price in relation to the cost of producing the good in that country; this can change over time.

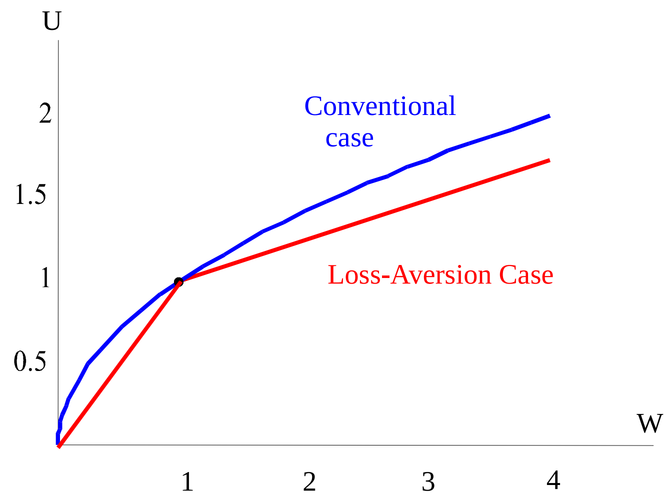
The total insurance premium paid is p x = 6.2. Now calculate the final income amounts of the two owners in the two scenarios:
In Rain scenario, Sam 100 – 6.2 = 93.8, Nancy 100 + 6.2 = 106.2
In Drought scenario, Sam 50 – 6.2 + 10 = 53.8, Nancy 50 + 6.2 – 10 = 46.2

For Sam, without insurance, as in Question 1 or from the formulas above,
E[I(Sam)] = 75, V[I(Sam)] = 625, U(Sam) = 75 – 0.006 * 625 = 71.25
With insurance,
E[I(Sam)] = 0.5 * 93.8 + 0.5 * 53.8 = 73.8
V[I(Sam)] = 0.5 * $(93.8 – 73.8)^2$ + 0.5 * $(73.8 – 53.8)^2$ = 400
U(Sam) = 73.8 – 0.006 * 400 = 73.8 – 2.4 = 71.4 > 71.25

For Nancy, without insurance, as in Question 1 or from the formulas above,
E[I(Nancy)] = 75, V[I(Nancy)] = 625, U(Nancy) = 75 – 0.004 * 625 = 72.5
With insurance,
E[I(Nancy)] = 0.5 * 106.2 + 0.5 * 46.2 = 76.2
V[I(Nancy)] = 0.5 * $(106.2 – 76.2)^2$ + 0.5 * $(76.2 – 46.2)^2$ = 900
U(Sam) = 76.2 – 0.004 * 900 = 76.2 – 3.6 = 72.6 > 72.5

Both have gained from their “trade in risk”. The gains may look small but remember that utility numbers don’t have any special significance so we can’t jump to this conclusion.

QUESTION 3: (total 25 points; 3 for graphs, 2 for each cell, 2 for comments)



R	U(W) = √W	U(W) = 1 + 0.2 * (W-1) if W > 1 U(W) = W if W < 1
0.5	0.0670	0.20
0.4	0.0417	0.16
0.3	0.0230	0.12
0.2	0.0101	0.08
0.1	0.0025	0.04

You can work out formulas in each case, and then substitute the numbers. For the square root function, $\sqrt{(1-P)} = ½ \sqrt{(1+R)} + ½ \sqrt{(1-R)}$, therefore

$$(1-P) = [(1+R)+(1-R) +2\sqrt{(1+R)(1-R)}]/4, \text{ or } P = 1 – [1+\sqrt{(1-R^2)}]/2 = [1 – \sqrt{(1-R^2)}]/2.$$

For the function with the kink, you must use the appropriate formula for utility on each side of 1. Therefore

$$(1-P) = ½ [1+0.2 * (1+R-1)] + ½ (1-R) = 1 – 0.4 R, \text{ so } P = 0.4 R.$$

Observe how it is much simpler to calculate the algebraic formula for P as a function of R, and then plug in the various values of R, than it would be to do each case numerically from scratch.

INTERPRETATION: With the kinked utility function, the risk premium is larger, and goes down only linearly with the magnitude of the risk, whereas with the conventional smooth function, the risk premium is smaller and decreases more rapidly as the size of the risk diminishes (roughly, risk premium is quadratic in the size of the risk). Note that even the "conventional" consumer is risk-averse (concave utility function, with U'' < 0); a few of you did not realize this.

Here are a couple more extras for those interested. For the calculus experts among you, I should qualify that the exact linearity in the kink case is a consequence of the utility function being linear in its separate pieces for W < 1 and W > 1. More generally, the linearity will be only "local", for small levels of risk. The non-experts can ignore this subtlety.

Additional information: Here is the full graph the risk premium function found for the conventional square root utility function case, drawn using Mathematica: