ECO 300 – Fall 2005
Microeconomic Theory
Problem Set 5 – Answer Key

A generally excellent performance. The distribution was as follows:

Range	Number
100	6
90–99	35
80–89	17
70–79	2
60–69	3
< 60	3

and several freebies. Be careful how you use freebies; you may need them even more urgently later. (For the general principle of making such irreversible choices, read Avinash Dixit, "Investment and Hysteresis," Journal of Economic Perspectives, 6(1), Winter 1992, 107-132.)

Question 1: (Multiple choice, total 20 points, 4 each)

1. (e)
2. (d)
3. (c)
4. (a)
5. (d)

Question 2: (Total 80 points, divided as indicated in the question)

A few common errors: In Part (c) often the quantity-0 part was missing in graph or algebraic expression. Some students expressed the supply function showing price as a function of quantity, and then made errors in adding over firms: multiplying p(Q) by the number of firms would be vertical summation of the firms' supply curves, not horizontal. In Part (h) supply is infinitely elastic in the long run, one student had a graph with supplies with varying elasticities.

(a) (7 points) Each consumer maximizes

$$U(q, y) = y + 10\ q - 5\ q^2\,,$$

subject to the budget constraint

$$p\,q + y = I\,.$$

Substituting out y, the objective is

$$F(q) \equiv I - p\,q + 10\ q - 5\ q^2\,.$$

To maximize this as a function of q, we have the first-order condition

$$F'(q) = 10 - p - 10\, q,$$

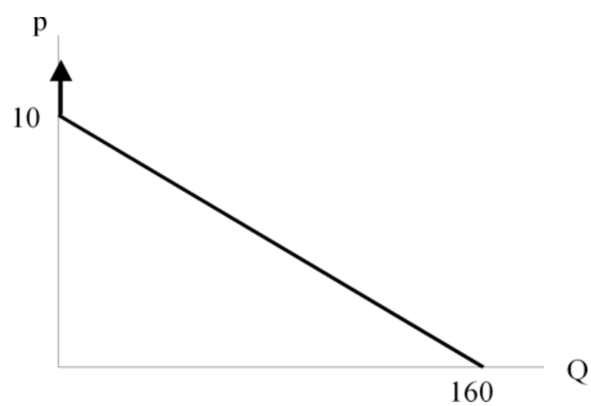
(and the second-order condition is satisfied: $F''(q) = -\,10 < 0$, bonus +1 point for checking this)

Solving the first-order condition yields $q = (10 - p)/10 = 1 - p/10$, which is positive so long as $p < 10$. If $p > 10$, we see that $F'(q) < 0$ for all $q > 0$, so $q = 0$ is optimal (no points lost for not saying this rigorously). Thus the individual demand function is

$$\begin{aligned} &\text{If } p \geq 10, \text{ then } q = 0 \\ &\text{If } p < 10, \text{ then } q = 1 - p/10. \end{aligned}$$

(b) (3 points) The market demand function just the sum over 160 consumers:

$$\begin{aligned} &\text{If } p \geq 10, \text{ then } Q = 0 \\ &\text{If } p < 10, \text{ then } Q = 160 - 16\, p. \end{aligned}$$



(c) (26 points) Other than the $LRTC$ which is as stated, the other costs as functions of q are meaningful only for $q > 0$. The expressions are :

$$\begin{aligned} LRTC(q) &= \begin{cases} 0 & \text{if } q = 0 \\ 4 + q^2 & \text{if } q > 0 \end{cases} \\ LRAC(q) &= 4/q + q \\ SRTAC(q) &= 1 + q^2 \\ SRAAC(q) &= 1/q + q \\ MC(q) &= 2\, q \end{aligned}$$

$LRAC'(q) = -4/q^2 + 1$, $LRAC''(q) = 8/q^3 > 0$. So $LRAC(q)$ is minimized at $q = 2$ and the minimum is 4.

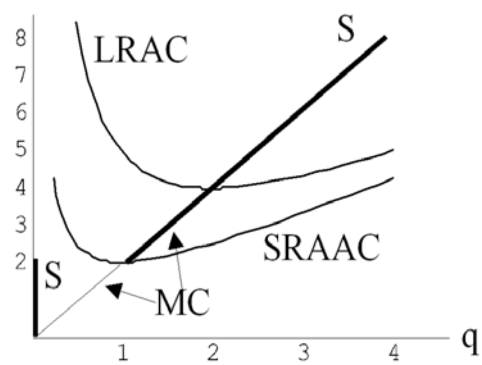
$SRAAC'(q) = -1/q^2 + 1$, $SRAAC''(q) = 2/q^3 > 0$. So $SRAAC(q)$ is minimized at $q = 1$ and the minimum is 2.

Since the MC curve passes through the bottom points of all the AC curves, you can also find the minima of the average cost curves by setting MC equal to each in turn. Bonus of 1 point for doing this (for the extra economic perception).

The firm's short run supply curve coincides with its marginal cost curve so long as the price does not fall below the minimum $SRAAC$. Thus $p = 2\,q$ when $p \geq 2$. Or

$$q = \begin{cases} 0 & \text{if } p \leq 2 \\ p/2 & \text{if } p \geq 2 \end{cases}$$

Observe that if $p = 2$, the firm is indifferent between producing 0 and 1 (it merely loses its sunk cost, and only just recovers its avoidable cost if it produces 1). No points taken off this time for the finickiness of allowing for $p = 2$ in both cases.



(d) (4 points) In the long run, with free entry and exit of fishing firms, the industry's supply curve is a horizontal line at the level of the minimum $LRAC$, that is, at $p = 4$. To be more pedantic, it consists of all points with $p = 4$ and $q = 2$, 4, 6, 8, 10, ... No points off for not being pedantic.

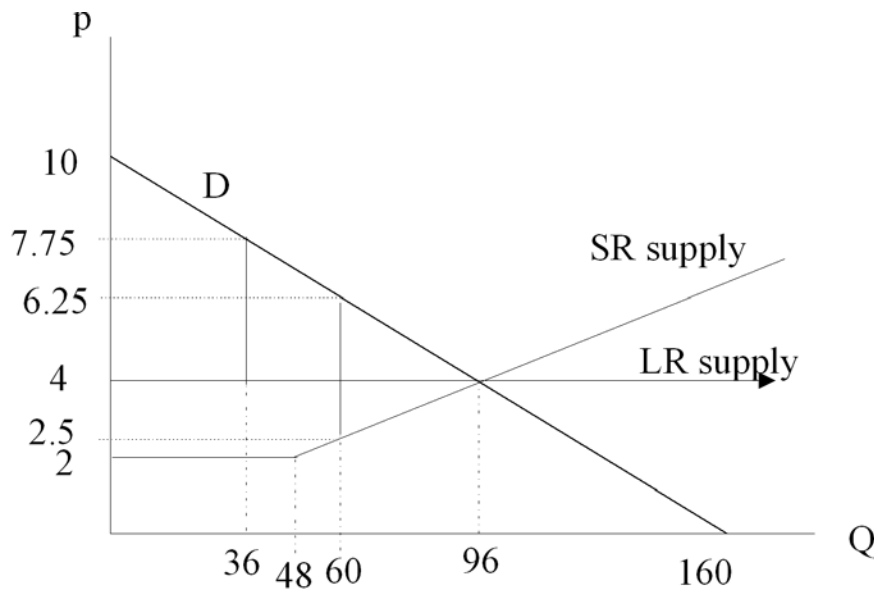
(e) (9 points) (From the supply curve, $p = 4$. Then from the demand curve, $Q = 16(10 - 4) = 96$. Each firm produces $q = 2$ at the bottom point of its $LRAC$, so there are 48 firms. Price equals the $LRAC$ at this point, so each makes zero profit. The aggregate consumer surplus is the area to the left of the market demand curve, so it equals $\frac{1}{2}\,(10-4)\,96 = 288$.

I show a composite figure for all the equilibria and the dead-weight loss calculations, you can draw separate ones. Also, I find the equilibrium with tax by keeping the demand and supply curves unchanged, but simply looking for a quantity where the price on the demand curve exceeds the price on the supply curve by the amount of the tax. It is perfectly legitimate instead to shift the demand curve vertically downward by the amount of the tax, or the supply curve upward by the amount of the tax (but not both).

(f) (17 points) In the short run, the full industry supply curve is

$$Q = \begin{cases} 0 & \text{if } p < 2 \\ 0 \text{ or } 1 \text{ or } 2 \text{ or } \ldots 48 & \text{if p = 2} \\ 48(p/2) = 24\,p & \text{if } p > 2 \end{cases}$$

Note that if p is exactly equal to 2, since each firm is indifferent between producing 0 and 1, we could have zero or 1 or ... 48 firms each producing 1 unit, so the industry supply has



discrete points at $p = 2$ and $Q = 0, 1, 2, \ldots 48$. In practice we show this as a horizontal line at $p = 2$ and extending from $Q = 0$ to $Q = 48$. No points off for missing this bit of pedantry either.

ASSUMING as stated that all 48 firms remain active, the price received by the firms is given as a function of the market quantity by solving from the last portion of this supply curve, that is

$$p = Q \,/\, 24\,.$$

The price paid by the consumers is given as a function of Q by

$$p = 10 - Q \,/\, 16\,.$$

With the tax, the equilibrium Q is defined by

$$10 - Q/16 = 3.75 + Q/24, \quad \text{or} \quad Q\left(\frac{1}{16} + \frac{1}{24}\right) = 10 - 3.75 = 6.25,‘$$

$$\text{or} \quad Q\,\frac{5}{48} = 6.25, \quad \text{or} \quad Q = 60\,.$$

Then the consumers pay the price $p = 10 - 60/16 = 10 - 3.75 = 6.25$. Firms receive the price $60/24 = 2.5$. Since this exceeds 2, the minimum $SRAAC$, firms will not exit in the short run, thereby justifying the “for the moment” assumption made above that all 48 firms stayed active. (Note the steps of the argument. If it had turned out that all 48 firms could not cover their avoidable costs, we would have had to abandon the “for the moment” assumption and re-do the problem, assuming n active firms, find their supply curve etc, and solve for n from the condition that avoidable cost be covered. Geometrically, you would recognize that the equilibrium must then lie along the horizontal portion of the short run supply curve at $p = 2$ and $0 \leq Q \leq 48$; therefore the price the consumers pay must be that plus the tax, and so on.)

Each of the 48 firms produces $2.5/2 = 1.25$ units. So its revenue is $1.25 * 2.5 = 3.125$, and its total cost is $4 + (1.25)^2 = 5.5625$. It makes a loss of 2.4375, but that is less than its sunk cost namely 3. The total loss of the 48 firms is 117 Arials. You can also calculate it as the loss of producer surplus trapezoid

$$(4 - 2.5)\frac{96 + 60}{2} = 1.5 \times 78 = 117$$

As the price paid by consumers rises from 4 to 6.25, and the quantity they buy shrinks from 96 to 60, the aggregate loss of consumer surplus is the trapezoid

$$(6.25 - 4)\ \frac{96 + 60}{2} = 2.25 \times 78 = 175.5$$

The government collects tax revenue $3.75 \times 60 = 225$ Arials. The dead-weight loss is $117 + 175.5 - 225 = 67.5$ Arials. You could also calculate it as the area of the triangle

$$\tfrac{1}{2}\,(6.25 - 2.5)\,(96 - 60)\,.$$

(g) (9 points) Firms will want to exit in the long run. As some exit, the price rises and the remaining ones lose less, eventually breaking even again. In the new long run equilibrium, the active firms must again be producing at their minimum $LRAC$, and this must equal the price they receive. So the price received by firms equals 4. Then that paid by consumers equals $4 + 3.75 = 7.75$. At this price, the quantity demanded is $16\,(10 - 7.75) = 36$. Since each firm produces 2, there must be 18 active firms. Each makes zero profit.

The aggregate loss of consumer surplus (as compared to the old long run equilibrium) is

$$(7.75 - 4)\ \frac{96 + 36}{2} = 3.75 \times 66 = 247.5\,.$$

The government's revenue is $3.75 \times 36 = 135$. The dead-weight loss is $247.5 - 135 = 112.5$. You could also calculate it as the area of the triangle

$$\tfrac{1}{2}\,(7.75 - 4)\,(96 - 36)\,.$$

(h) (5 points) The dead-weight loss is larger in the long run than in the short run. The reason is that elasticity of supply is larger (in fact infinite) in the long run. Therefore, for a given rate of tax, the quantity decreases by more in the long run. The dead-weight loss triangle has the same height (the tax) but a greater width (the quantity reduction) in the long run.