

ECO 300 – MICROECONOMIC THEORY
Fall Term 2005
PROBLEM SET 6 – ANSWER KEY

The distribution of scores was as follows:

100 +	11
90-99	30
80-89	11
70-79	6
< 70	2

And 19 people took freebies.

QUESTION 1: (Total 10 points)

The key to answering this is a basic lesson you learned in ECO 100: who initially hands over the money to the government for a tax and who ends up bearing the burden of the tax after all the prices have adjusted to a new equilibrium are two different things. You have to work out the new equilibrium. There are three ways to do this:

VERBAL

Since the labor market is competitive, that is, both employers and employees take the wage as given, therefore shifting an equal tax amount from the employee to the employer will have no effect on the amount of labor employed and on the wage kept by the employee after taxes. The equilibrium amount of labor employed is determined by the total amount of tax paid by both employees and employers. This is represented by the difference between the wage paid by the employer and the wage received by the employee. As long as the total tax doesn't change, the same amount of labor is employed and the wages paid by the employer and received by the employee (after tax) will not change. Hence, employees would be no better or worse off if the employers paid the full amount of the social security tax.

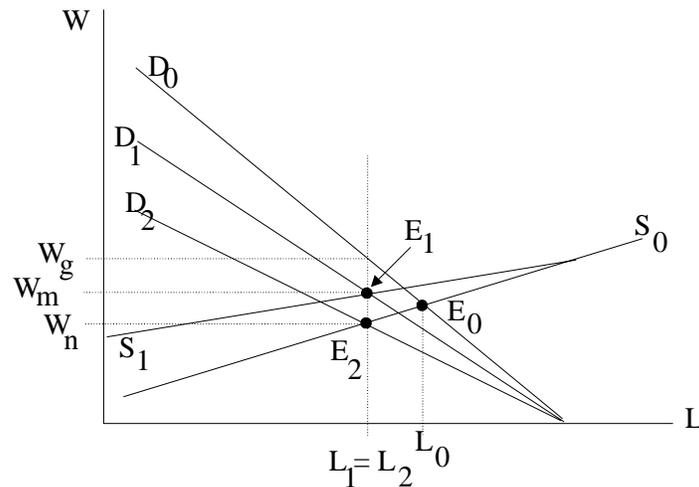
However, for the total tax not to change, it is important that in all cases the percentages are expressed relative to the pre-tax or gross-of-tax wage paid by the firms. And that is indeed the case in the way the IRS treats social security contributions.

DIAGRAMMATIC

The figure is on the next page. The demand curve for labor D_0 shows on the vertical axis the gross wage the employers are willing to pay as a function of the quantity of labor hired shown on the horizontal axis. The supply curve of labor S_0 shows on the vertical axis the net wage for which the workers are willing to sell the services of the quantity of labor on the horizontal axis. In the absence of any tax, the equilibrium would be E_0 , and a quantity L_0 of labor services would be hired and supplied.

In the original scenario of the tax, the employers pay 6.2% of the gross wage to the government instead of to the worker; therefore their demand curve shifts downward vertically by this percentage to the new position D_1 . (Note that the shift is not vertically parallel; it is a given percentage of the vertical height of the original curve.) Then the workers hand over a similar sum

to the government. Therefore their supply curve shifts vertically upward by the same amount to the new position S_1 . (Note that for each L , the vertical difference between S_1 and S_0 equals the vertical difference between D_1 and D_0 .) The new equilibrium is at E_1 , and the quantity of labor services transacted is L_1 . For this much labor, the employers are willing to pay a total of W_g , of which they hand over W_m to the workers and $(W_g - W_m) = 0.062 W_g$ to the government. Of the W_m in their paycheck, the workers must pay $(W_m - W_n) = 0.062 W_g$ to the government, and keep W_n . Thus $(W_g - W_n) = (W_g - W_m) + (W_m - W_n) = 0.062 W_g + 0.062 W_g = 0.124 W_g$.



In the second scenario, the employers must pay 0.124 of the gross wage to the government and the rest to the workers; the workers do not pay any further payroll tax. Therefore the demand curve for labor shifts down 12.4% to the new position D_2 ; the supply curve of labor does not shift and stays at S_0 . The new equilibrium is where the two meet, at E_2 . But as we just saw, $(W_g - W_n) = 0.124 W_g$; therefore the intersection has the same quantity of labor transacted as in the first scenario: $L_2 = L_1$. Therefore the gross wages the employers pay and the net wages the workers receive are also the same, namely W_g and W_n respectively. The incidence of the payroll tax is the same no matter who initially hands over the money to the government.

ALGEBRAIC:

Mathematically, the solution and the issue can be expressed as follows: If the firm's demand curve (on which the pre-tax wage must lie) is $w = D(L)$, and the workers' supply curve is $w = S(L)$, then:

- [1] Under the system where each side pays tax equal to $0.062 D(L)$, the equilibrium is defined by $D(L) - 0.062 D(L) = S(L) + 0.062 D(L)$
- [2] When the employers pay the whole tax $0.124 D(L)$, $D(L) - 0.124 D(L) = S(L)$
- [3] When employees pay the whole tax $0.124 D(L)$, $D(L) = S(L) + 0.124 D(L)$

All three have the same solution L . Note that if the employees' tax liability was computed as a percentage of their net-of-tax wage $S(L)$, the solutions would be different.

QUESTION 2: (Total 35 points, 7 for part a and 14 each for parts b, c)

Note that slight differences in the numbers can arise because of rounding errors.

a. (7 points) We are given the equations for the total market demand for sugar in the U.S. and the supply of U.S. producers:

$$Q_D = 26.53 - .285P$$

$$Q_S = -8.70 + 1.214P.$$

The difference between the quantity demanded and supplied, $Q_D - Q_S$, is the amount of sugar imported that is restricted by the quota. If the quota is increased from 3 billion pounds to 6.5 billion pounds, then we will have $Q_D - Q_S = 6.5$ and we can solve for P:

$$(26.53 - .285P) - (-8.70 + 1.214P) = 6.5$$

$$35.23 - 1.499P = 6.5$$

$$P = 19.17 \text{ cents per pound.}$$

At a price of 19.17 cents per pound $Q_S = -8.70 + (1.214)(19.17) = 14.6$ billion pounds and $Q_D = Q_S + 6.5 = 21.1$ billion pounds.

b. (14 points. We notionally reserve 5 points for the figure and 9 for the various numerical calculations. P-R do not ask for a figure; if you answer correctly without the figure, including explaining all the steps of your algebraic and numerical arguments, you will get full credit, with 14 points for the whole algebra. But the figure is a simpler way to “show the steps of your work,” and I hope most of you will draw a figure in such a situation without being asked explicitly.)

The figure on the next page is reproduced from Pindyck-Rubinfeld Figure 9.16 p. 325, with a second horizontal line at the new US price 19.17 added. The scale is not accurate, this is deliberate in order to mark the various areas more clearly. Otherwise the strip between the prices 21.5 and 19.17 would be too narrow.

The gain in consumer surplus is area a+b+c+d. The loss to domestic producers is equal to area a.

Numerically:

$$a = (21.5 - 19.17)(14.6) + (17.4 - 14.6)(21.5 - 19.17)(.5) = 36.8$$

$$b = (17.4 - 14.6)(21.5 - 19.17)(.5) = 3.22$$

$$c = (21.5 - 19.17)(20.4 - 17.4) = 6.9$$

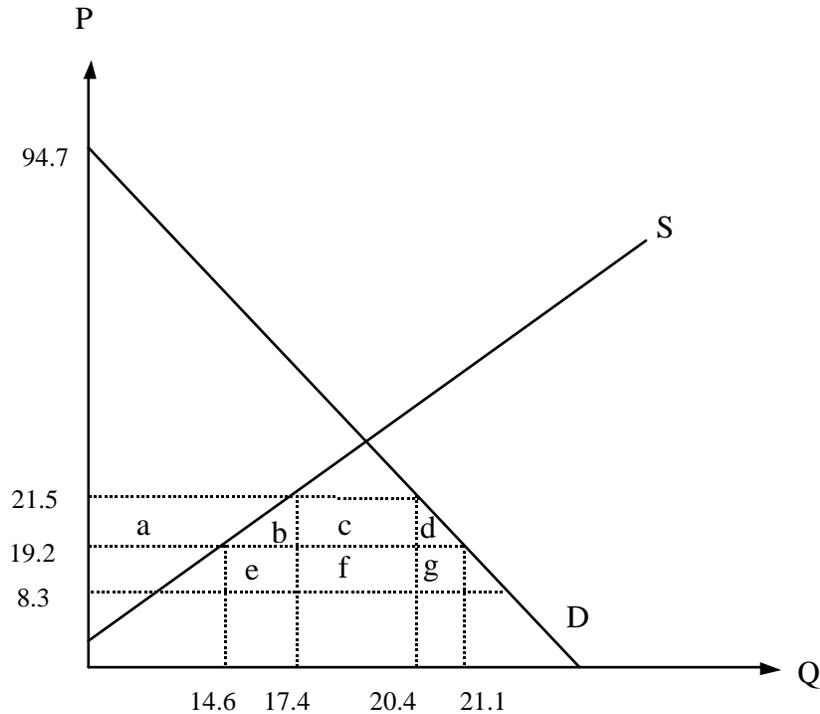
$$d = (21.5 - 19.17)(21.1 - 20.5)(.5) = 0.69.$$

These numbers are in billions of cents or tens of millions of dollars.

Thus, consumer surplus increases by \$483 million, while domestic producer surplus decreases by \$372 million.

c. (14 points. Again notionally 5 points for showing the areas in the figure and 9 for the numerical calculations.)

When the quota was 3 billion pounds, the profit earned by foreign producers is the difference between the domestic price and the world price (21.5 - 8.3) times the 3 billion units sold, for a total of 39.6, or \$396 million. When the quota is increased to 6.5 billion pounds, domestic price will fall to 19.17 cents per pound and profit earned by foreigners will be (19.17 - 8.3) * 6.5 = 70.85, or \$708.5 million. Profit earned by foreigners therefore increased by \$311 million. On the graph above, this is area (e+f+g) - (c+f) = e+g-c. The deadweight loss of the quota decreases by area b+e+d+g, which is equal to \$420 million.

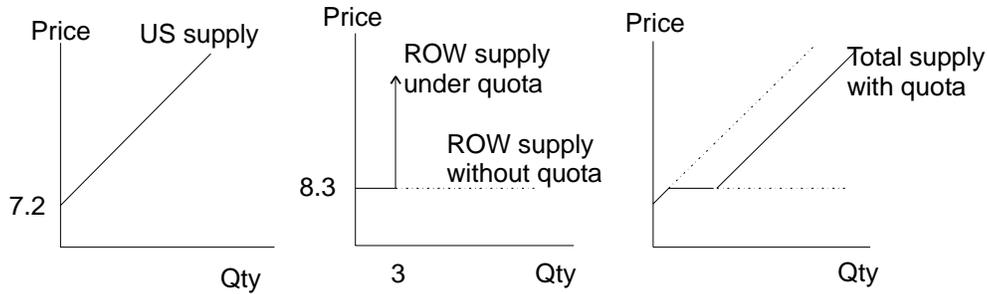


Additional material to improve your understanding of equilibrium with quotas:

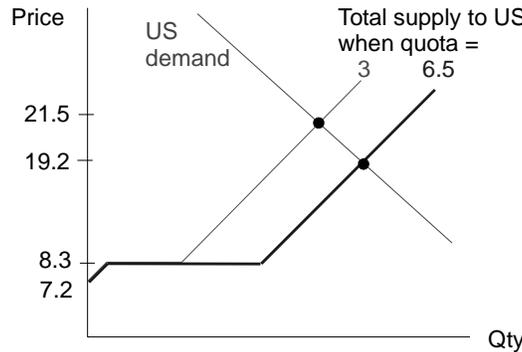
I said in class that there were two ways of finding the equilibrium under a tax. Either shift the supply and demand curves vertically by the amount of the tax each side pays and then find the intersection of the shifted curve; this is the method used in P-R and in many textbooks. Or look for a quantity such that the price paid by the demanders exceeds the price received by the suppliers by exactly the amount of the tax; this is looking for a vertical gap of just the right height between the demand and supply curves. This was the method I showed in the overheads handout of November 8, pp. 2-3.

There are similarly two methods for analyzing a quota. Either you can shift the foreign supply curve horizontally as the quota is increased. Or you can look for a US price (greater than the price that prevails in the rest of the world) for which the horizontal gap between our consumers' quantity demanded and our firms' quantity supplied equals the quota quantity that foreigners are allowed to export to our market. Here P-R use the latter method. How would you develop the former method?

The figure just below shows how the total supply curve as the horizontal sum of the US firms' supply curve and the ROW supply curve. If no quota is imposed, the ROW supply will be a horizontal line at the world price 8.3. The quota restricts foreign quantity to 3, so the ROW supply becomes L-shaped. Then the total supply is the horizontal summation shown in the right hand panel of the figure.



When the quota is increased, this curve shifts to the right staying parallel to itself. Therefore the equilibrium moves down along the demand curve. This is shown in the figure below, which again is not quite to scale:



If you are interested, you can figure out how to show and calculate various areas – US producer surplus changes, rents to foreign suppliers, etc. using this figure.

QUESTION 3: (Total 35 points)

- (a) (5 points) The expressions for the numerical values of price elasticities of demand are:

U.S.

$$E_U = -\frac{P_U}{Q_U} \frac{dQ_U}{dP_U} = -\frac{P_U}{700(160 - P_U)} (-700) = \frac{P_U}{160 - P_U}$$

Europe

$$E_E = -\frac{P_E}{Q_E} \frac{dQ_E}{dP_E} = -\frac{P_E}{300(80 - P_E)} (-300) = \frac{P_E}{80 - P_E}$$

At any common price $P < 80$, we have $80 - P < 160 - P$, so $P/(80 - P) > P/(160 - P)$; the U.S. demand is less elastic.

(Added explanation – In Europe it is much less common to have textbooks “required for purchase;” students rely more on libraries, sharing etc. In the U.S. there is still some elasticity of demand because of used copies, and some sharing, library use etc.)

- (b) (12 points, 4 each for profit expressions, first-order conditions, and subsequent calculations.) If Megatext sets price = P , its profit per book is $P - 18 - 0.1 P = 0.9 P - 18$. Therefore the expressions for profits in the two markets are

US

$$\Pi_U = 700(160 - P_U)(0.9P_U - 18)$$

Europe

$$\Pi_E = 300(80 - P_E)(0.9P_E - 18)$$

The conditions for the separate prices to maximize the separate profits are

US

$$\frac{d\Pi_U}{dP_U} = 700 [-(0.9P_U - 18) + 0.9(160 - P_U)] = 700[18 + 144 - 1.8P_U] = 0$$

$$\text{So } P_U = 162/1.8 = 90.$$

Europe

$$\frac{d\Pi_E}{dP_E} = 300 [-(0.9P_E - 18) + 0.9(80 - P_E)] = 300[18 + 72 - 1.8P_E] = 0$$

$$\text{So } P_E = 90/1.8 = 50.$$

Result:

$$\text{Quantities: } Q_U = 700(160-90) = 700 * 70 = 49,000$$

$$Q_E = 300(80-50) = 300 * 30 = 9,000$$

$$\text{Megatext's profits: } \Pi_U = 49,000(0.9 * 90 - 18) = 49,000 * 63 = 3,087,000$$

$$\Pi_E = 9,000(0.9 * 50 - 18) = 9,000 * 27 = 243,000$$

$$\text{Total } \Pi = 3,330,000$$

$$\text{Authors' (joint) royalties} = 49,000 * 0.1 * 90 + 9,000 * 0.1 * 50 = 486,000$$

(c) (12 points, 3 each for correct demand function, profit expression, first-order condition, and subsequent calculations) When Megatext must set a common price P in the two markets, the total quantity it sells is given by

$$Q = 700(160 - P) + 300(80 - P) = 136000 - 1000P = 1000(136 - P)$$

Its profit is

$$\Pi = 1000(136 - P)(0.9P - 18)$$

The condition for P to maximize this is

$$\frac{d\Pi}{dP} = 1000 [-(0.9P - 18) + 0.9(136 - P)] = 1000[18 + 122.4 - 0.9P] = 0$$

$$\text{Therefore } P = 140.4/18 = 78.$$

Result:

$$\text{Quantities: } Q_U = 700(160-78) = 700 * 82 = 57,400$$

$$Q_E = 300(80-78) = 300 * 2 = 600$$

$$\text{Megatext's profit: } \Pi = (57,400+600)(0.9 * 78 - 18) = 3,027,600$$

$$\text{Authors' (joint) royalties} = (57,400+600)(0.1 * 78) = 452,400$$

(d) (6 points, 1 point bonus for calculating the worldwide surplus change.) As a consequence of Megatext's inability to price-discriminate:

Megatext's profit goes down by $3,330,000 - 3,027,600 = 302,400$

The authors' royalties go down by $486,000 - 452,3000 = 33,600$

U.S. students pay the price \$78 instead of \$90, and buy 57,400 books instead of 49,000

So their consumer surplus goes up by the trapezoidal area

$$(90-78) * \frac{1}{2} (57,400 + 49,000) = 638,400$$

European students pay \$78 instead of \$50, and buy 600 books instead of 9,000

So their consumer surplus goes down by the trapezoidal area

$$(78-50) * \frac{1}{2} (9,000+600) = 134,400$$

$$\text{Worldwide increase in surplus} = 638,400 - (302,400 + 33,600 + 134,400) = 168,000$$

QUESTION 4: (20 points)

(a) (5 points) Perfect price discrimination profit = $40 * 50 + 150 * 50 = 2000 + 7500 = 9500$

(b) (10 points; 1 each for each single-type strategy, and 6 for the two-type)

Single-type strategies:

R only priced at 140: $40 * 100 = 4000$

R only priced at 225: $135 * 50 = 6250$

U only priced at 175: $25 * 100 = 2500$

U only priced at 300: $150 * 50 = 7500$

Two-type strategy: The self-selection and participation constraints do not change. Therefore the optimal pricing is still $X = 140$, $Y = 215$. Then

$$\text{Profit} = 40 * 50 + 65 * 50 = 2000 + 3250 = 5250$$

(c) (5 points; 1 to identify the optimal, 4 for explanation)

The optimal strategy for PITS is U-only, priced at 300.

The difference between this and the class example arises from the fact that now the mix of travelers changes in favor of the business types. Therefore PITS stands to lose more by keeping the U price at 215 to stop the business types from defecting to the choice of R. It becomes better not to provide the R service at all, thereby forgoing the profit from the tourist types, but being able to charge the business travelers their full willingness to pay (300).

(In the real world you will have noticed different airlines making different choices from this menu of strategies, and sometimes shifting from one to another, according to their view of the demand and the competition.)