ECO 300 – Fall 2005
Microeconomic Theory
Problem Set 7 – Answer Key

The distribution was as follows:

Range	Number
100+	1
90–99	36
80–89	24
< 79	8

Question 1: (Multiple choice, total 20 points, 4 each)

1. (a)
2. (d)
3. (d)
4. (e)
5. (d)
6. (b)

Question 2: (Total 40 points)

(a) (8 points) The profits of firm 1, expressed as a function of the quantities of both, are given by

$$\begin{aligned} \Pi_1 &= (18 - 2\,Q_1 - Q_2)\; Q_1 - 3\; Q_1 \\ &= 15\; Q_1 - Q_2\; Q_1 - 2\;(Q_1)^2 \end{aligned}$$

To choose Q_1 to maximize this for a given Q_2, the condition is

$$\frac{\partial \Pi_1}{\partial Q_1} = 15 - Q_2 - 4\; Q_1 = 0$$

Solving this for Q_1 in terms of (as a function of) Q_2 gives firm 1's reaction function:

$$Q_1 = \tfrac{15}{4} - \tfrac{1}{4}\; Q_2$$

That of firm 2 is by similar calculation

$$Q_2 = \tfrac{15}{4} - \tfrac{1}{4}\; Q_1$$

(b) (7 points) To find the Cournot (quantity-setting Nash) equilibrium, we must solve these as a pair of simultaneous equations in Q_1 and Q_2. Substituting from the second into the first yields

$$\begin{aligned} Q_1 &= \tfrac{15}{4} - \tfrac{1}{4} \left[\tfrac{15}{4} - \tfrac{1}{4}\, Q_1 \right] \\ &= \tfrac{15}{4} - \tfrac{15}{16} + \tfrac{1}{16}\, Q_1 \end{aligned}$$

Therefore

$$\left[1 - \tfrac{1}{16} \right] Q_1 = \tfrac{15}{4} - \tfrac{15}{16}$$

or

$$\tfrac{15}{16}\, Q_1 = \tfrac{60-15}{16} = \tfrac{45}{16}, \qquad \text{or} \qquad Q_1 = 3$$

Similarly $Q_2 = 3$.

(Notes: [1] I am just showing you one way to solve the equations; any other logically correct method can also be used. [2] $Q_1 = Q_2 = 3$ does not mean that each firm literally sells just three pizzas. I have not told you the units of the quantities; they could be hundreds.)

Then $P_1 = P_2 = 18 - 2 * 3 - 3 = 9$, and $\Pi_1 = \Pi_2 = (9 - 3) * 3 = 18$.

(c) (5 points) Add the two inverse demand functions together:

$$P_1 + P_2 = 36 - 3\, Q_1 - 3\, Q_2 \qquad \text{or} \qquad \tfrac{1}{3}\,(P_1 + P_2) = 12 - Q_1 - Q_2$$

Subtract this from the inverse demand equation for firm 1:

$$\tfrac{2}{3}\, P_1 - \tfrac{1}{3}\, P_2 = 6 - Q_1$$

so

$$Q_1 = 6 - \tfrac{2}{3}\, P_1 + \tfrac{1}{3}\, P_2$$

Similarly

$$Q_2 = 6 + \tfrac{1}{3}\, P_1 - \tfrac{2}{3}\, P_2$$

Again there are many different ways to do this.

(d) (8 points) The profits of firm 1, expressed as a function of the prices of the two firms, are given by

$$\begin{aligned} \Pi_1 &= (P_1 - 3)\, Q_1 \\ &= (P_1 - 3) \left(6 - \tfrac{2}{3}\, P_1 + \tfrac{1}{3}\, P_2 \right) \end{aligned}$$

To choose P_1 to maximize this for a given P_2, the condition is

$$\begin{aligned} \frac{\partial \Pi_1}{\partial P_1} &= 1 * \left(6 - \tfrac{2}{3}\, P_1 + \tfrac{1}{3}\, P_2 \right) + (P_1 - 3) * \left(-\tfrac{2}{3} \right) \\ &= 8 - \tfrac{4}{3}\, P_1 + \tfrac{1}{3}\, P_2 = 0 \end{aligned}$$

or

$$P_1 = 6 + \tfrac{1}{4}\, P_2$$

This is firm 1's reaction function. That of firm 2 is by similar calculation

$$P_2 = 6 + \tfrac{1}{4}\ P_1$$

(e) (7 points) To find the Bertrand (price-setting Nash) equilibrium, solve these jointly. Substitute from the second into the first to get

$$P_1 = 6 + \tfrac{1}{4}\ \left[6 + \tfrac{1}{4}\ P_1\right] = \tfrac{15}{2} + \tfrac{1}{16}\ P_1$$

Therefore

$$\tfrac{15}{16}\ P_1 = \tfrac{15}{2}, \quad \text{or} \quad P_1 = 8$$

Similarly $P_2 = 8$. Then $Q_1 = Q_2 = 6 - \frac{2}{3}\ 8 + \frac{1}{3}\ 8 = 6 - \frac{8}{3} = \frac{10}{3} = 3.33$. Profits are $\Pi_1 = \Pi_2 = (8-3)\ *\ \frac{10}{3} = \frac{50}{3} = 16.67$.

(f) (5 points) The Cournot equilibrium has higher prices ($P_1 = P_2 = 9$) than the Bertrand equilibrium ($P_1 = P_2 = 8$). The intuition is as follows. Consider firm 1 contemplating cutting its price by a little. By how much will the quantity demanded of its good expand? That depends on what the other firm is holding constant. If the other firm is holding its price constant, that is the usual concept of the own elasticity of demand for firm 1. And the goods being substitutes, this would decrease the quantity firm 2 sells. But if instead firm 2 is holding its quantity constant as in Cournot, it must be lowering its price sufficiently to be able to avoid the loss of sales induced by firm 1's price cut (remember the two goods are substitutes). But this reduction in firm 2's price means that the quantity sold by firm 1 does not increase by as much, again because the products are substitutes. Thus firm 1's demand is less price responsive (less price elastic) when firm 2 holds its quantity constant than when firm 2 holds its price constant. A less price elastic demand means a lower optimal mark-up of price over marginal cost. That is why firm 1's price is lower under the assumption of Cournot competition than under the assumption of Bertrand competition.

Note: Many people need to review the formula for the elasticity. Demand is said to be elastic if the numerical (absolute) value of the elasticity is less than 1. So just because two elasticities differ in size does not mean that one is elastic and the other is inelastic. Another point is that the products are not *perfect* substitutes. Therefore in the Bertrand case it is false to claim that elasticities are infinite or that profits are zero.

Question 3: (Total 40 points)

(a) (5 points) Adding the two demand functions,

$$Q_1 + Q_2 = 72 - 3\ P_1 - 3\ P_2$$

or

$$\tfrac{1}{3}\ (Q_1 + Q_2) = 24 - P_1 - P_2$$

Subtract this from the demand function for firm 1:

$$Q_1 - \tfrac{1}{3}\ (Q_1 + Q_2) = (36 - 24) - 2\ P_1 + P_1 - P_2 + P_2$$

Therefore

$$P_1 = 12 - \tfrac{2}{3}\, Q_1 + \tfrac{1}{3}\, Q_2$$

Similarly for firm 2.

(b) (8 points) Firm 1's profits as a function of the two quantities are

$$\begin{aligned} \Pi_1 &= \left(12 - \tfrac{2}{3}\, Q_1 + \tfrac{1}{3}\, Q_2\right)\, Q_1 - 2\, Q_1 \\ &= \left(10 + \tfrac{1}{3}\, Q_2\right)\, Q_1 - \tfrac{2}{3}\, (Q_1)^2 \end{aligned}$$

To choose Q_1 to maximize this for a given Q_2, the condition is

$$10 + \tfrac{1}{3}\, Q_2 - \tfrac{4}{3}\, Q_1 = 0$$

or

$$Q_1 = \tfrac{30}{4} + \tfrac{1}{4}\, Q_2$$

This is firm 1's reaction function. Similarly you will find that firm 2's profit maximization condition is

$$10 + \tfrac{1}{3}\, Q_1 - \tfrac{4}{3}\, Q_2 = 0$$

and its reaction function is

$$Q_2 = \tfrac{30}{4} + \tfrac{1}{4}\, Q_1$$

(c) (7 points) To find the Cournot (quantity-setting Nash) equilibrium, we solve these together. Here is a different way to do this. We proceed directly from the first-order conditions, not the solved reaction functions:

$$\begin{aligned} 10 + \tfrac{1}{3}\, Q_2 - \tfrac{4}{3}\, Q_1 &= 0, \\ 10 + \tfrac{1}{3}\, Q_1 - \tfrac{4}{3}\, Q_2 &= 0 \end{aligned}$$

Subtracting the second from the first,

$$\left[\tfrac{1}{3} + \tfrac{4}{3}\right]\, (Q_2 - Q_1) = 0$$

Therefore $Q_1 = Q_2$. Call the common value Q; then either of the equations becomes

$$10 + \left[\tfrac{1}{3} - \tfrac{4}{3}\right]\, Q = 0 \qquad \text{or} \qquad 10 - Q = 0$$

Therefore $Q_1 = Q_2 = 10$, prices $P_1 = P_2 = 26/3 = 8.667$, and profits $\Pi_1 = \Pi_2 = 200/3 = 66.67$.

(d) (5 points) The Cournot equilibrium has higher prices ($P_1 = P_2 = 8.67$) than the Bertrand equilibrium ($P_1 = P_2 = 8$). The intuition is as follows. Consider firm 1 contemplating cutting its price by a little. By how much will the quantity demanded of its good expand? That depends on what the other firm is holding constant. If the other firm is holding its price constant, that is the usual concept of the own elasticity of demand for firm 1. And the goods being complements, this would increase the quantity firm 2 sells. But if instead firm 2 is holding its quantity constant as in Cournot, it must be raising its price sufficiently so as offset that the increase in quantity caused by firm 1's price reduction. But this increase in firm 2's price means that the quantity sold by firm 1 does not increase by as much, again because the products are complement. Thus firm 1's demand is less price responsive (less price elastic) when firm 2 holds its quantity constant than when firm 2 holds its price constant. A less price elastic demand means a higher optimal mark-up of price over marginal cost. That is why firm 1's price is higher under the assumption of Cournot competition than under the assumption of Bertrand competition.

(e) (5 points) A firm's demand is less own-price elastic (quantity is less responsive to its own price change) when the quantity of another good is held fixed than when the price of another good is held fixed, regardless of whether the two goods are substitutes or complements!

Additional information: An implication of this is that quantity restrictions like quotas imposed anywhere in the economy reduce the price elasticities of demands of other goods, and are therefore conducive to the increase of market power elsewhere in the economy.

Note: A lot of people invent statements that are false or trivial, and this will often result in reduced credit, even if you also have the true statements somewhere else in your answer, because it shows a lack of true understanding.