The distribution was as follows:

<table>
<thead>
<tr>
<th>Range</th>
<th>Number</th>
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</thead>
<tbody>
<tr>
<td>100+</td>
<td>1</td>
</tr>
<tr>
<td>90–99</td>
<td>36</td>
</tr>
<tr>
<td>80–89</td>
<td>24</td>
</tr>
<tr>
<td>&lt; 79</td>
<td>8</td>
</tr>
</tbody>
</table>

**Question 1:** (Multiple choice, total 20 points, 4 each)

1. (a)
2. (d)
3. (d)
4. (e)
5. (d)
6. (b)

**Question 2:** (Total 40 points)

(a) (8 points) The profits of firm 1, expressed as a function of the quantities of both, are given by

\[
\Pi_1 = (18 - 2Q_1 - Q_2) Q_1 - 3 Q_1 = 15 Q_1 - Q_2 Q_1 - 2 (Q_1)^2
\]

To choose \( Q_1 \) to maximize this for a given \( Q_2 \), the condition is

\[
\frac{\partial \Pi_1}{\partial Q_1} = 15 - Q_2 - 4 Q_1 = 0
\]

Solving this for \( Q_1 \) in terms of (as a function of) \( Q_2 \) gives firm 1’s reaction function:

\[
Q_1 = \frac{15}{4} - \frac{1}{4} Q_2
\]

That of firm 2 is by similar calculation

\[
Q_2 = \frac{15}{4} - \frac{1}{4} Q_1
\]
(b) (7 points) To find the Cournot (quantity-setting Nash) equilibrium, we must solve these as a pair of simultaneous equations in $Q_1$ and $Q_2$. Substituting from the second into the first yields

$$Q_1 = \frac{15}{4} - \frac{1}{4} \left[ \frac{15}{4} - \frac{1}{4} Q_1 \right]$$

Therefore

$$\left[ 1 - \frac{1}{16} \right] Q_1 = \frac{15}{4} - \frac{15}{16}$$

or

$$\frac{15}{16} Q_1 = \frac{60 - 15}{16} = \frac{45}{16}, \quad \text{or} \quad Q_1 = 3$$

Similarly $Q_2 = 3$.

(Notes: [1] I am just showing you one way to solve the equations; any other logically correct method can also be used. [2] $Q_1 = Q_2 = 3$ does not mean that each firm literally sells just three pizzas. I have not told you the units of the quantities; they could be hundreds.)

Then $P_1 = P_2 = 18 - 2 \times 3 - 3 = 9$, and $\Pi_1 = \Pi_2 = (9 - 3) \times 3 = 18$.

(c) (5 points) Add the two inverse demand functions together:

$$P_1 + P_2 = 36 - 3 Q_1 - 3 Q_2 \quad \text{or} \quad \frac{1}{3} (P_1 + P_2) = 12 - Q_1 - Q_2$$

Subtract this from the inverse demand equation for firm 1:

$$\frac{2}{3} P_1 - \frac{1}{3} P_2 = 6 - Q_1$$

so

$$Q_1 = 6 - \frac{2}{3} P_1 + \frac{1}{3} P_2$$

Similarly

$$Q_2 = 6 + \frac{1}{3} P_1 - \frac{2}{3} P_2$$

Again there are many different ways to do this.

(d) (8 points) The profits of firm 1, expressed as a function of the prices of the two firms, are given by

$$\Pi_1 = (P_1 - 3) Q_1 = (P_1 - 3) \left( 6 - \frac{2}{3} P_1 + \frac{1}{3} P_2 \right)$$

To choose $P_1$ to maximize this for a given $P_2$, the condition is

$$\frac{\partial \Pi_1}{\partial P_1} = 1 * \left( 6 - \frac{2}{3} P_1 + \frac{1}{3} P_2 \right) + (P_1 - 3) * \left( -\frac{2}{3} \right)$$

$$= 8 - \frac{4}{3} P_1 + \frac{1}{3} P_2 = 0$$

or

$$P_1 = 6 + \frac{1}{4} P_2$$
This is firm 1’s reaction function. That of firm 2 is by similar calculation

\[ P_2 = 6 + \frac{1}{4} P_1 \]

(e) (7 points) To find the Bertrand (price-setting Nash) equilibrium, solve these jointly. Substitute from the second into the first to get

\[ P_1 = 6 + \frac{1}{4} \left[ 6 + \frac{1}{4} P_1 \right] = \frac{15}{2} + \frac{1}{16} P_1 \]

Therefore

\[ \frac{15}{16} P_1 = \frac{15}{2}, \quad \text{or} \quad P_1 = 8 \]

Similarly \( P_2 = 8 \). Then \( Q_1 = Q_2 = 6 - \frac{2}{3} 8 + \frac{1}{3} 8 = 6 - \frac{8}{3} = \frac{10}{3} = 3.33 \). Profits are \( \Pi_1 = \Pi_2 = (8 - 3) * \frac{10}{3} = \frac{50}{3} = 16.67 \).

(f) (5 points) The Cournot equilibrium has higher prices \( (P_1 = P_2 = 9) \) than the Bertrand equilibrium \( (P_1 = P_2 = 8) \). The intuition is as follows. Consider firm 1 contemplating cutting its price by a little. By how much will the quantity demanded of its good expand? That depends on what the other firm is holding constant. If the other firm is holding its price constant, that is the usual concept of the own elasticity of demand for firm 1. And the goods being substitutes, this would decrease the quantity firm 2 sells. But if instead firm 2 is holding its quantity constant as in Cournot, it must be lowering its price sufficiently to be able to avoid the loss of sales induced by firm 1’s price cut (remember the two goods are substitutes). But this reduction in firm 2’s price means that the quantity sold by firm 1 does not increase by as much, again because the products are substitutes. Thus firm 1’s demand is less price responsive (less price elastic) when firm 2 holds its quantity constant than when firm 2 holds its price constant. A less price elastic demand means a lower optimal mark-up of price over marginal cost. That is why firm 1’s price is lower under the assumption of Cournot competition than under the assumption of Bertrand competition.

Note: Many people need to review the formula for the elasticity. Demand is said to be elastic if the numerical (absolute) value of the elasticity is less than 1. So just because two elasticities differ in size does not mean that one is elastic and the other is inelastic. Another point is that the products are not perfect substitutes. Therefore in the Bertrand case it is false to claim that elasticities are infinite or that profits are zero.

**Question 3:** (Total 40 points)

(a) (5 points) Adding the two demand functions,

\[ Q_1 + Q_2 = 72 - 3 P_1 - 3 P_2 \]

or

\[ \frac{1}{3} (Q_1 + Q_2) = 24 - P_1 - P_2 \]

Subtract this from the demand function for firm 1:

\[ Q_1 - \frac{1}{3} (Q_1 + Q_2) = (36 - 24) - 2 P_1 + P_1 - P_2 + P_2 \]
Therefore

\[ P_1 = 12 - \frac{2}{3} Q_1 + \frac{1}{3} Q_2 \]

Similarly for firm 2.

(b) (8 points) Firm 1’s profits as a function of the two quantities are

\[ \Pi_1 = \left( 12 - \frac{2}{3} Q_1 + \frac{1}{3} Q_2 \right) Q_1 - 2 Q_1 \]

\[ = \left( 10 + \frac{1}{3} Q_2 \right) Q_1 - \frac{2}{3} (Q_1)^2 \]

To choose \( Q_1 \) to maximize this for a given \( Q_2 \), the condition is

\[ 10 + \frac{1}{3} Q_2 - \frac{4}{3} Q_1 = 0 \]

or

\[ Q_1 = \frac{30}{4} + \frac{1}{4} Q_2 \]

This is firm 1’s reaction function. Similarly you will find that firm 2’s profit maximization condition is

\[ 10 + \frac{1}{3} Q_1 - \frac{4}{3} Q_2 = 0 \]

and its reaction function is

\[ Q_2 = \frac{30}{4} + \frac{1}{4} Q_1 \]

(c) (7 points) To find the Cournot (quantity-setting Nash) equilibrium, we solve these together. Here is a different way to do this. We proceed directly from the first-order conditions, not the solved reaction functions:

\[ 10 + \frac{1}{3} Q_2 - \frac{4}{3} Q_1 = 0, \]

\[ 10 + \frac{1}{3} Q_1 - \frac{4}{3} Q_2 = 0 \]

Subtracting the second from the first,

\[ \left[ \frac{1}{3} + \frac{4}{3} \right] (Q_2 - Q_1) = 0 \]

Therefore \( Q_1 = Q_2 \). Call the common value \( Q \); then either of the equations becomes

\[ 10 + \left[ \frac{1}{3} - \frac{4}{3} \right] Q = 0 \quad \text{or} \quad 10 - Q = 0 \]

Therefore \( Q_1 = Q_2 = 10 \), prices \( P_1 = P_2 = 26/3 = 8.667 \), and profits \( \Pi_1 = \Pi_2 = 200/3 = 66.67 \).

(d) (5 points) The Cournot equilibrium has higher prices \( (P_1 = P_2 = 8.67) \) than the Bertrand equilibrium \( (P_1 = P_2 = 8) \). The intuition is as follows. Consider firm 1 contemplating cutting its price by a little. By how much will the quantity demanded of its good expand? That depends on what the other firm is holding constant. If the other firm is holding its price constant, that is the usual concept of the own elasticity of demand for firm 1. And the goods being complements, this would increase the quantity firm 2 sells. But if instead firm 2 is holding its quantity constant as in Cournot, it must be raising its price.
sufficiently so as offset that the increase in quantity caused by firm 1’s price reduction. But
this increase in firm 2’s price means that the quantity sold by firm 1 does not increase by
as much, again because the products are complement. Thus firm 1’s demand is less price
responsive (less price elastic) when firm 2 holds its quantity constant than when firm 2 holds
its price constant. A less price elastic demand means a higher optimal mark-up of price
over marginal cost. That is why firm 1’s price is higher under the assumption of Cournot
competition than under the assumption of Bertrand competition.

(e) (5 points) A firm’s demand is less own-price elastic (quantity is less responsive to its
own price change) when the quantity of another good is held fixed than when the price of
another good is held fixed, regardless of whether the two goods are substitutes or comple-
ments!

Additional information: An implication of this is that quantity restrictions like quotas
imposed anywhere in the economy reduce the price elasticities of demands of other goods,
and are therefore conducive to the increase of market power elsewhere in the economy.

Note: A lot of people invent statements that are false or trivial, and this will often result
in reduced credit, even if you also have the true statements somewhere else in your answer,
because it shows a lack of true understanding.