ECO 300 – MICROECONOMIC THEORY
Fall Term 2005
PROBLEM SET 8 – ANSWER KEY

The distribution of scores was as follows:

100	1
90-99	15
80-89	9
70-79	11
< 70	3

Martin thanks all those (over half the class) who took freebies and saved him work. But do check the answer key carefully; some related questions may appear on the exam.

QUESTION 1: (Total 35 points)

1. (b)
2. (d)
3. (d)
4. (b)
5. (d)
6. (a)
7. (c)

QUESTION 2: (Total 25 points)

Many answers were weak. The incentive compatibility conditions are not very well understood. Many students, even good ones, didn't get it as soon as more than two types were involved. Review the concepts and methods carefully.

(a) (12 points) First note that if the aim is to be recognized as one's true type, no one is going to acquire education beyond what is needed for this purpose. In particular, type A will get exactly E_A, type B will get exactly E_B, and type C will get 0.

Then there are six incentive-compatibility conditions (2 points each):

For type A not to mimic type B: $150 – 25\ E_A > 100 – 25\ E_B$,
or $E_A – E_B < (150-100)/25 = 2$
For type A not to mimic type C: $150 – 25\ E_A > 20$,
or $E_A < (150-20)/25 = 5.2$

For type B not to mimic type A: $100 – 40\ E_B > 150 – 40\ E_A$,
or $E_A – E_B > (150-100)/40 = 1.25$.
For type B not to mimic type C: $100 – 40\ E_B > 20$,
or $E_B < (100-20)/40 = 2$

For type C not to mimic type A: $20 > 150 - 50\ E_A$,

or $E_A > (150-20)/50 = 2.6$

For type C not to mimic type B: $20 > 100 - 50\ E_B$,

or $E_B > (100-20)/50 = 1.6$

(b) 3 points for each of the following deductions:

If $E_A - E_B < 2$ and $E_B < 2$, then adding these together, $E_A < 4$. That automatically ensures $E_A < 5.2$

If $E_A - E_B > 1.25$ and $E_B > 1.6$, then adding these together, $E_A > 2.85$. That automatically ensures $E_A < 2.6$

2 points for the inference: If the requirements to be accepted as type B are such that type C does not want to mimic type B, and the requirements to be accepted as type A are such that type B does not want to mimic type A, then the requirements to be accepted as type A are automatically strict enough that type C does not want to mimic type A. Similarly, we have a kind of "transitivity" in the conditions that rule out better types mimicking worse types.

Additional information: This transitivity of incentive compatibility conditions is formally known as the "single crossing property" and named after its Nobel laureate discoverers, Mirrlees and Spence.

(c) Math 2 points, comment 3 points. The bounds on E_A emerged in part (b) above: $2.85 < E_A < 4$. These are more stringent than the ones when type B did not exist, namely $2.6 < E_A < 5.2$. General inference: If a type closer to you exists, you have to signal more strongly to separate yourself from that type, but the upper limit on your signal also narrows, to reduce your temptation to mimic that close type, too.

QUESTION 3: (Total 25 points)

(a) (5 points) If you bid $110 million for the company, your bid will surely succeed. The expected value of the company in the hands of the current management is (10+110)/2 = 60 million dollars. You expect to turn this to $105 million. Therefore your expected profit is – $5 million.

(b) (8 points) If you bid $50 million, for the company, your bid will succeed if and only if the true value in the hands of the current management is below this number. Since the true value is uniformly distributed between 10 and 110, the probability of it being less than 50 is (50-10)/(110-10) = 0.4 .

If your bid succeeds, you will get a company whose value is uniformly distributed over the range from 10 to 50 million. The expected value of this is (10+50)/2 = 30. You expect to turn this into 1.75 * 30 = 52.5, making an expected profit of 52.5 – 50 = 2.5 .

But at the point in time where you make your bid of $50 million, you reckon to get this only with probability 0.4. Therefore your expected profit at bid time is 0.4 * 2.5 = 1 million dollars.

A common error was failing to distinguish between probabilities and expected values calculated at this original point (unconditional probabilities and expected values) and those after the bid has succeeded (conditional – on success of the bid – probabilities and expected values).

(c) (12 points) If you bid $X million, for the company, your bid will succeed if and only if the true value in the hands of the current management is below this number. Since the true value is uniformly distributed between 10 and 110, the probability of it being less than X is (X – 10)/(110 – 10) = (X – 10)/100 .

If your bid succeeds, you will get a company whose value is uniformly distributed over the range from 10 to X million. The expected value of this is (10+X)/2 . You expect to turn this into 1.75 * (10+X)/2 , making an expected profit of

1.75 * (10+X)/2 – X = (17.5 – 0.25 X) / 2 = (70 – X) / 8

But at the point in time where you make your bid, you reckon to get this only with probability (X – 10)/100 . Therefore your expected profit at bid time is

(X – 10) (70 – X) / 800 = (– 700 + 80 X – X^2) / 800

To choose X to maximize this, we set the derivative equal to zero: (80 – 2 X) / 800 = 0

Therefore X = 40 is your optimal bid. The expected profit at the time when you make the bid is

(40 – 10) * (70 – 40) / 800 = 30* 30 / 800 = 900 / 800 = 1.125

(million dollars).

QUESTION 4 (total 15 points)

Reminder of the grading scheme: You should expect about 12 points for a logically correct answer where the game does not have any special imagination or interest. The final 3 points are for such special qualities, as judged by Martin. You can get less than 12 points for logical errors about incentive compatibility or other aspects of the game.

This produced many nice answers. Some students referred even to counter-signalling. The only general suggestion: You should point out more carefully the role of differential costs of signalling to different types. In other words, relate your stories better to the formal language of game theory and information economics.