ECO 300 – MICROECONOMIC THEORY
Fall Term 2005
PROBLEM SET 3
Due 12.30 p.m. on Thursday October 13

QUESTION 1: (total 35 points)

You have $20 per week to spend, and two possible uses for this money: telephoning friends back home, and drinking coffee. Each hour of phoning costs $2, and each cup of coffee costs $1. Your utility function is U(X,Y) = XY, where X is the hours of phoning you do, and Y the number of cups of coffee you drink. (X and Y are continuous variables. Interpret fractions as averages over several weeks.)

(a) What are your optimal choices? What is the resulting utility level? (5 points)

(b) Now suppose the price of telephone calls drops to $1 per hour. What are your optimal choices? What is the resulting utility level? (5 points)

(c) Calculate the compensating variation for the price decrease, defined as the reduction in income at the new prices that will leave you with the same level of utility as you had before the price drop. (10 points)

(d) Calculate the equivalent variation for the price decrease, defined as the increase in income that will give you the same utility at the old prices as your $20 budget enables you to enjoy at the new prices. (10 points)

(e) Compare the compensating and equivalent variations to each other. Which is bigger? What is the economic intuition for this? (3 points)

(f) In class we did similar calculations of compensating and equivalent variations when the initial prices of X and Y were $1 each, and then the price of X increased to $2. Compare the two variations of that exercise with the two variations of this problem, and comment on your finding. (2 points)

QUESTION 2: (total 35 points)

The isolated, rich, but volcanic island of San Serife is scheduled to disappear in an eruption at the end of 2006. Therefore the residents' time horizon is limited to the two years, 2005 and 2006. Each of them earns $200,000 in 2005, and will earn $220,000 in 2006.

The residents have formed a non-profit cooperative to facilitate borrowing and lending among themselves. This pays 10% interest to savers, and charges borrowers 10% interest. If a sum is borrowed in 2005, the principal must be repaid in 2006, together with the year's interest. And lending in 2005 gets the principal back and earns interest in 2006. No *new* borrowing or lending can occur in 2006.

(a) Consider a saver who saves S thousand dollars in 2005, thus consuming C(2005) = 200 – S thousand dollars. Find an expression for his/her consumption in 2006, written C(2006) and also measured in thousands of dollars, using the income in 2006 plus the principal and interest earned on the savings. (5 points)

(b) Consider a borrower who borrows B thousand dollars in 2005, thus consuming C(2005) = 200 + B thousand dollars. Find an expression for his/her consumption in 2006, written C(2006) and also measured in thousands of dollars, using the income in 2006 but having to pay back the principal and interest on the borrowing. (5 points)

(c) Show that, for the saver as well as the borrower, the two consumption amounts are linked by the "budget constraint"

$$C(2005) + \frac{C(2006)}{1.1} = 200 + \frac{220}{1.1} = 400$$

Interpret this equation in the terminology of economics. (5 points)

(d) Now consider two residents. Ina Rush has the utility function

$$U(Ina) = [C(2005)]^{0.6}\,[C(2006)]^{0.4}$$

Vera Patient has the utility function

$$U(Vera) = [C(2005)]^{0.3}\,[C(2006)]^{0.7}$$

Find their optimal saving or borrowing plans. Use without proving afresh the "Cobb-Douglas" results from the precepts for Week 2 or Pindyck-Rubinfeld pp. 148-9. (10 points for each)

QUESTION 3: (30 points)

Joe Luckless has incurred a debt and must pay S each day to service it (to pay the interest and to pay back the principal gradually over time). By working, he can earn w per hour in addition.

(a) If he works L hours, write down an equation for the amount Y he will be able to consume after servicing the debt. (2 points)

(b) Joe's utility function is U = (20 – L) Y. Substitute your expression for Y to express U as a function of L. (Your expression will also involve the algebraic constants or "parameters" S and w). (8 points)

(c) Find an algebraic expression for his optimal choice of hours of work L, with L on the left hand side and S and w on the right hand side. What economic name will you give the resulting expression? (10 points)

(d) If Joe's debt is magically forgiven, while w remains constant, will L increase or decrease? (5 points)

(e) If w increases holding S constant, will L increase or decrease? Give an economic interpretation for your answers. (5 points)