ECO 300 – MICROECONOMIC THEORY
Fall Term 2005
PROBLEM SET 4
Due 12.30 p.m. on Thursday October 20

QUESTION 1: (Total 30 points; 8 for the calculation in each case, 6 for the comments.)

The small island of San Serife in the Pacific Ocean is divided into northern and southern halves by a mountain range in its middle. Rain-bearing winds blow west to east. Their precise track is uncertain; in a given year it may cover all of the island or only the northern or the southern half of it, or neither half. Specifically, you are asked to consider three alternative cases:

CASE A – Each year, the rains will either cover the whole island or miss it entirely. The probabilities of these two scenarios are ½ each.

CASE B – Each year, the rains will fall on exactly one of the two halves. The probability that the rains fall on the northern half but not the southern is ½, and the probability that the rains fall on the southern half but not the northern is also ½ .

CASE C – The rains covering the northern and the southern half are independent events, and the probability of each is ½. Thus in this case there are four scenarios: (i) both halves get rain, (ii) the north but not the south gets rain, (iii) the south but not the north gets rain, and (iv) neither half gets rain. The probability of each scenario is ¼.

There are two landowners, Nancy North and Sam South. Nancy owns the northern half of the island and Sam owns the southern half. Each half, in a year when it gets rain, will produce coconuts worth $100, and in a year without rain, will produce only $50.

To cope with the uncertainty, the two owners agree on a mutual help policy: each year they will pool their incomes together and then share the total equally. In each of the three cases A-C, calculate the mean and the standard deviation of the income of each owner.

Briefly give the economic intuition for the differences among your results for the three cases.

QUESTION 2: (Total 45 points, allocated as stated below)

Now suppose the situation is as in Case A of Question 1. Instead of looking at the consequences of full risk-sharing where each owner gets half of the total output, let us look at a partial shifting of risk between them. Suppose Nancy's utility function is

$$U(\text{Nancy}) = E[I(\text{Nancy})] - 0.004\ V[I(\text{Nancy})]$$

and Sam's utility function is

$$U(\text{Sam}) = E[I(\text{Sam})] - 0.006\ V[I(\text{Sam})]$$

where I denotes income after the agreed risk-sharing has been carried out, E denotes expected value, V denotes variance, and U denotes utility. So Sam is more risk-averse than Nancy ($0.006 > 0.004$), and we should expect a mutually beneficial transaction whereby Nancy takes on some of Sam's risk for a price.

Consider an arrangement of the following form. Sam pays Nancy $(p * x)$ in advance. After the year has passed and the coconuts have been collected, if the rains have been bad so each has only $50, then Nancy will pay Sam x, so that Nancy finally has income $(50 + p * x – x)$ and Sam has $(50 – p * x + x)$. If the rains have been good, so each has $100, then Nancy does not pay

Sam anything, so finally Nancy has (100 + p * x) and Sam has (100 – p * x). We will soon interpret p as the price of insurance and x as the quantity traded.

Calculate the algebraic expressions for the expected values and variances of Nancy's and Sam's final incomes, as functions for p and x. Hence find the algebraic expressions for their utility functions. (20 points, 10 each)

Sam chooses his optimal x taking p as given; find the equation for, and graph, his choice of x as a function of p, that is, his "demand curve for insurance". (10 points, 7 for equation, 3 for graph)

Nancy chooses her optimal x taking p as given; find the equation for, and graph, her choice of x as a function of p, that is, her "supply curve of insurance". (10 points, 7 for equation, 3 for graph)

Find the equilibrium p for which the quantity x of insurance demanded by Sam equals the quantity supplied by Nancy. (5 points)

QUESTION 3: (Total 25 points, 3 for the graphs, 2 for each cell of the table, 2 for comments)

Here we compare the consequences of two different kinds of utility functions with different kinds of attitudes toward risk: a conventional case with smoothly declining marginal utility of wealth, and a loss-aversion case with a kink at the initial point.

Suppose the initial wealth is 1 (in millions of dollars). The risk is gaining or losing R, with equal probabilities; thus the final wealth W will be either 1+R or 1-R, each with probability ½ . The two utility functions are as follows. In the conventional case

$$U(W) = \sqrt{W}$$

In the loss-aversion case

$$U(W) = \begin{cases} 1 + 0.2 * (W-1) & \text{if } W > 1 \\ W & \text{if } W < 1 \end{cases}$$

(a) Sketch graphs of these.

(b) We are to find the risk premium P that a consumer would be willing to pay to avoid this risk, that is, to enjoy (1-P) with certainty. That is, we are to solve for P from the equation

$$U(1-P) = 0.5 * U(1+R) + 0.5 * U(1-R)$$

Show your results in a table like the following, for two cases of the function U shown and for five values of R shown in the column and row headings. The entries in the cells will be your solution for P in each situation.

R	Conventional case	Loss-aversion case
0.5		
0.4		
0.3		
0.2		
0.1		

Comment on your results.