CONSUMER BEHAVIOR – PART 2

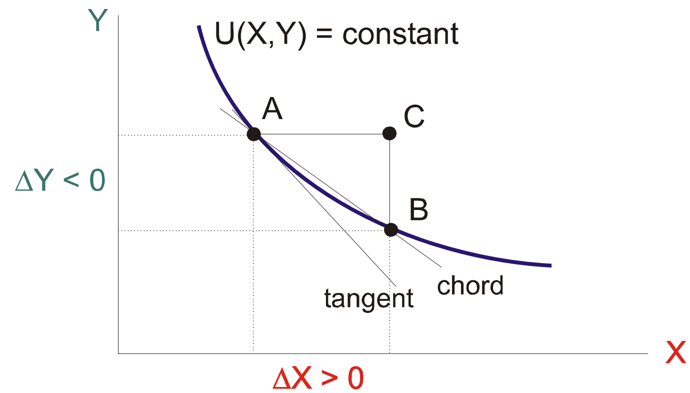
UTILITY FUNCTION (P-R pp. 76-9)

Basis for quantifying indiff. curve analysis, estimating demand functions, normative analysis
A formula that assigns a number to each basket, such that

(1) all baskets on one indifference curve are assigned the same number

(2) baskets on higher indifference curves are assigned higher numbers

Then attaining highest indifference curve on the budget line (or nonlinear budget constraint) can be done using algebra and calculus – maximize utility function subject to budget constraint



Need algebra of tangency condition MRS = price ratio
Slope of budget line is P_X / P_Y
What is expression for MRS? (P-R pp. 93, 147)
Figure shows the analysis
Utility is the same at B as at A
Move from A to B via C
Utility increases as X increases
by ΔX in going from A to C, Increase = $MU_X \ \Delta X$
Then utility decreases as Y decreases
$\Delta Y < 0$, in going from C to B, Change = $MU_Y \ \Delta Y$
Taylor's Formula approximation for total change = $MU_X \ \Delta X + MU_Y \ \Delta Y = 0$

$$MRS = -\frac{\Delta Y}{\Delta X} = \frac{MU_X}{MU_Y}$$ And by definition of derivatives, $MU_X = \partial U/\partial X$, $MU_Y = \partial U/\partial Y$

Finally, let changes become small, so MRS = slope of tangent.

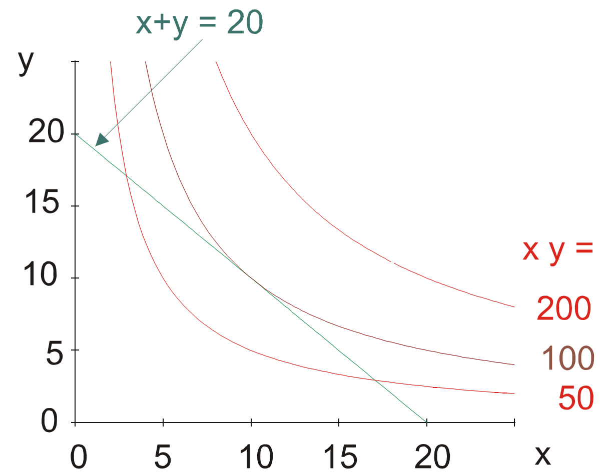
Example: indifference curves are rectangular hyperbolas, along each of which X Y = constant

So take utility function U(X,Y) = X Y itself; this satisfies requirements (1) and (2) above

Suppose $I = 20$, $P_X = 1$, $P_Y = 1$, so budget constraint X + Y = 20

Mathematically, exactly same as land enclosing problem of Sep. 20

Solution: X = Y = 10



Verify this using MRS = price ratio condition

$MRS = MU_X / MU_Y$

When U(X,Y) = X Y,

$MU_X = \partial U/\partial X = Y$, $MU_Y = \partial U/\partial Y = X$

So MRS = Y/X

Optimality condition: Y/X = 1/1, or Y = X

But X + Y = 20, so X = Y = 10

General rule: To max U(X,Y) = X Y subject to $P_X X + P_Y Y = I$

Condition $MRS = P_X / P_Y$ becomes $Y / X = P_X / P_Y$, or $P_X X = P_Y Y$

Optimal to split income equally between X, Y: $P_X X = ½ I$, $P_Y Y = ½ I$

So optimal choices (demand functions) $X = \dfrac{I}{2P_X}, Y = \dfrac{I}{2P_Y}$

Generalizations of this are basis for simple rules of thumb: “spend 30 % of income on rent” etc.

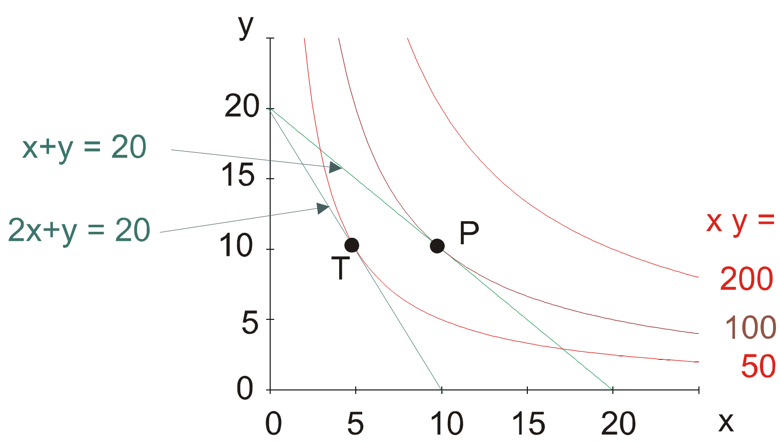
Will review and generalize in precepts (Also P-R pp. 148-9)

You can cite and use Cobb-Douglas optimization without proof in problem sets, exams, unless I explicitly ask for proof

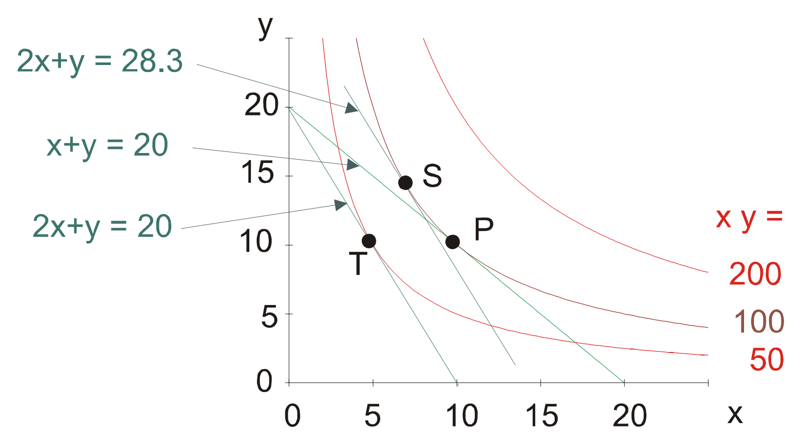
EFFECTS OF PRICE CHANGE (P-R pp. 81-3, 116-119)

In the above example, suppose P_X increases from 1 to 2, holding I = 20 and P_Y = 1 unchanged

Total effect



Breakdown into income and substitution effects



Choice in initial situation = P
Choice after price increase = T
Total effect of price change: move P to T
Money income is held constant
Leaves consumer worse off
(on lower indifference curve, lower utility)

Pure substitution effect = move P to S
Money income increased to 28.3
(consumer is given compensation
enabled to be as well off as before)
Pure income effect = move from S to T
Effect of withdrawing compensation

Practical importance: to calculate cost-of-living adjustments in wage contracts, social security ...

If money income is increased to 30, the consumer can buy the same basket of goods as before

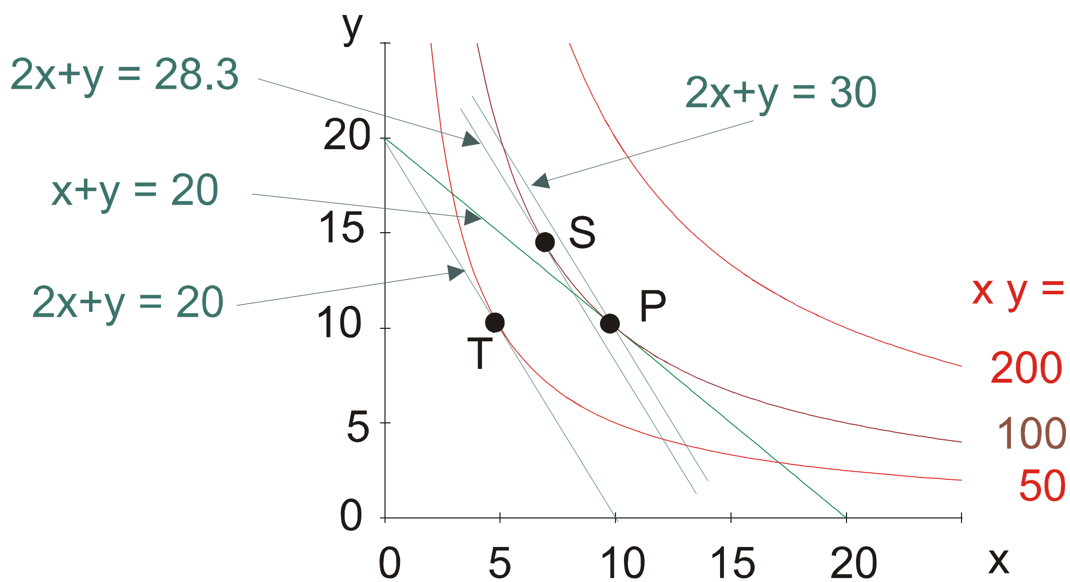
But don't need this much compensation

because at new prices the consumer can remain as well off

by substituting away from the good whose price rises

If given compensation to enable purchase of old basket

consumer will still substitute and actually become better off



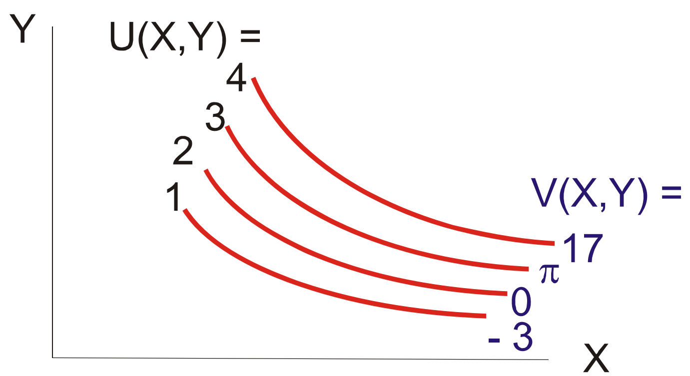
This has implications for construction and interpretation of cost of living indexes (like CPI)

We develop this application explicitly later

ORDINAL UTILITY (P-R pp. 77-8)

Utility does not have any natural or physical units, or even a natural sense of "zero"
Can construct any scale that accurately reflects underlying preference relationships
To say whether A is preferred to B, all that matters is whether U(A) > U(B)
not the actual magnitudes of U(A) and U(B),
nor the magnitude of [U(A)-U(B)], only the sign of this difference

In other words, the "cardinal numbers" of utility are not relevant for our purpose
only the ordering matters – utility is ordinal
Figure shows example
Can represent the same indifference map
using either the left hand side function U(X,Y)
or the right hand side function V(X,Y)



In previous example, instead of $U(X,Y) = X\,Y$,
we could have chosen the functions

$$V(X,Y) = (X\,Y)^2 = X^2\,Y^2 \text{ , or}$$

$$W(X,Y) = \ln(X\,Y) = \ln(X) + \ln(Y), \ldots$$

All of these would represent the same preferences,
and maximizing any of them would yield the same demand functions

Later, when we consider choice under uncertainty, actual cardinal numbers of utilities will matter
Cardinality can also matter if we are comparing utilities of different people
for normative evaluation of economic outcomes and policies

RELATIVE PRICES (related to P-R pp. 12-13, 80)

Budget constraints $P_X X + P_Y Y = I$ and $(k P_X) X + (k P_Y) Y = k I$
allow exactly the same choices of X and Y
And utility depends only on (X,Y), not on k
Therefore equiproportionate change in all prices and in income have no effect on choices

Can also write budget constraint as $X + \frac{P_Y}{P_X} Y = \frac{I}{P_X}$ OR $\frac{P_X}{P_Y} X + Y = \frac{I}{P_Y}$

Demand functions depend only on the relative price P_X / P_Y (or P_Y / P_X)
and on the income relative to the price of a good, I / P_X (or I / P_Y)
(or more generally, I / an index of prices)

What is the economic meaning of the relative price P_X / P_Y ?
X is cloth measured in meters, Y is wheat measured in kilograms, income in Euros (€)

Then P_X is so many € per meter, P_Y so many € per kilogram. So P_X / P_Y is $\frac{Euros / M}{Euros / Kg} = \frac{Kg}{M}$

the number of kilograms of wheat you have to give up to get a meter of cloth
This is the true rate of barter or exchange of one good for another;
money is just an intermediate step
That is how transactions in microeconomics ultimately operate –
people exchange their labor, or services of other factors they own (capital, land)
for goods, via one or more intermediate steps

Income expressed in monetary units matters only because of its purchasing power
over actual goods and services, so take ratio over a price index

CHOICE AMONG DISCRETE ENTITIES (not in P-R)

Suppose your choice is from among n cars

Car k has price P_k ; has characteristics C_k ; leaves $Y_k = (I - P_k)$ to spend on other goods

Yields utility $U_k = U(C_k , Y_k)$. Should choose that k which yields the highest U_k

Empirical implementation: Consumer makes slight independent errors e_k in estimating utilities

Chooses that k which yields highest $(U_k + e_k)$

For example: Probability that car k = 1 is chosen

$= \text{Prob}(U_1 + e_1 > U_2 + e_2 ; U_1 + e_1 > U_3 + e_3 ; \ldots U_1 + e_1 > U_n + e_n)$

$= \text{Prob}(e_1 - e_2 > U_2 - U_1 ; e_1 - e_3 > U_3 - U_1 ; \ldots\ e_1 - e_n > U_n - U_1)$

$= \text{Prob}(e_1 - e_2 > U_2 - U_1) * \text{Prob}(e_1 - e_3 > U_3 - U_1) * \ldots * \text{Prob} (e_1 - e_n > U_n - U_1)$

Given the statistical hypothesis about the distribution of the estimation errors,

can find expressions for these probabilities in terms of the preference parameters, prices

and then fit to observed frequencies (market shares) to estimate the parameters

(Of course needs much more detail than this brief statement)

Typical example:

$$\text{Prob(car k = 1 is chosen)} = \frac{e^{a\,U_1}}{e^{a\,U_1} + e^{a\,U_2} + \ldots + e^{a\,U_n}}$$

where each $U_k = U(C_k , I - P_k)$ contains some parameters (algebraic constants),

and *a* is another parameter that measures how fast the probability responds to utility levels

Econometrics of such choices uses pioneering work by McFadden (Nobel prize 2000)