CONSUMER BEHAVIOR UNDER UNCERTAINTY – PART 1

CONCEPTS

Uncertainty is everywhere –

Future job prospects, income, career prospects, family, responsibilities ...
Return to savings invested in stocks, bonds, housing, own ventures ...
Good or bad outcomes re. weather, health, longevity, accidents, robberies ...

Consumers can take actions that affect the risks – education, occupation choices, lifestyle choices of diet and exercise, care in driving and home security, portfolio choices, insurance, gambling, ...

Firms are also affected by risks – future prices of outputs and inputs, technological changes that may make equipment obsolete, cost of capital, ...

Markets and related institutions enable consumers and firms to control their risks within certain limits (constraints)

Insurance markets and mutual help groups (family, friends, etc.) pool independent (or imperfectly correlated) risks

Financial markets attempt to achieve efficient allocation of risk

Usual themes – individual choice and overall equilibrium. Here focus on first.

TECHNICAL CONCEPTS (P-R pp. 154-7)

Alternative situations or “scenarios” – 1, 2, ... n

Probabilities – P_1 , P_2 , ... P_n : objective - repeatable and measurable as frequencies in long run
subjective - perceptions or beliefs about one-off events

Random variable X – its outcomes or “realizations” X_1 , X_2 , ... X_n

Expected value: $E[X] = P_1 X_1 + P_2 X_2 + ... + P_n X_n$

Variance: $V[X] = P_1 \{ X_1 - E[X] \}^2 + P_2 \{ X_2 - E[X] \}^2 + ... + P_n \{ X_n - E[X] \}^2 = E[X^2] - \{E[X] \}^2$

Standard deviation: $SD[X] = \sqrt{V[X]}$

Example – Florida orange grove owner’s income

Scenarios: 1. Normal year; 2. Hurricane in FL; 3. Frost in BR; 4. Both

Probabilities: $P_1 = 0.89$, $P_2 = 0.05$, $P_3 = 0.05$, $P_4 = 0.01$ (positive correlation: $P_4 > P_2 * P_3$)

Incomes: $I_1 = 100{,}000$, $I_2 = 10{,}000$, $I_3 = 200{,}000$, $I_4 = 30{,}000$

$E[I] = 0.89 * 100{,}000 + 0.05 * 10{,}000 + 0.05 * 200{,}000 + 0.01 * 30{,}000 = 99{,}800$

$V[I] = 0.89 * (100{,}000 - 99{,}800)^2 + 0.05 * (10{,}000 - 99{,}800)^2$
$+ 0.05 * (200{,}000 - 99{,}800)^2 + 0.01 * (30{,}000 - 99{,}800)^2$
$= 0.89 * (200)^2 + 0.05 * (- 89{,}800)^2 + 0.05 * (100{,}200)^2 + 0.01 * (- 69{,}800)^2$
$= 953{,}960{,}000$

$SD[I] = 30{,}886$

Continuous random variables –

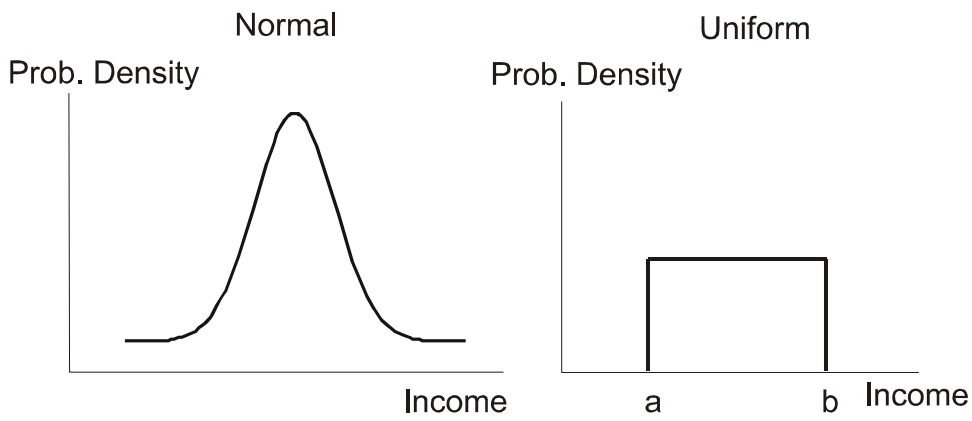
Label the scenarios by the continuous outcome itself (e.g. income)
Then show probability density function

Formulas for calculating means, variances etc. in prob. & stats. books

e.g. for uniform distribution from a to b,

$E[\,I\,] = (a+b)/2$

$V[\,I\,] = (b-a)^2 / 12$



CONSUMERS' PREFERENCES ABOUT RISK (P-R pp. 159-63)

Consumers usually dislike risk in consumption quantities or in income, wealth etc.

Example: prefer sure income of \$100,000 to 50:50 chance of \$150,000 or \$50,000

How to conceptualize and measure this?

Answer from von Neumann, Morgenstern, Milnor, ... Use utility.

In this example, upside utility gain in risky situation must be worth less than downside utility loss:

$U(150) - U(100) < U(100) - U(50)$, $2\,U(100) > U(150) + U(50)$, $U(100) > ½\,[\,U(150) + U(50)\,]$

More generally, with scenarios etc. as on previous page, consumer evaluates,

not expected income $E[X]$, but "expected utility"

$$E[U(X)] = P_1\,U(X_1) + P_2\,U(X_2) + \ldots + P_n\,U(X_n)$$

Among alternatives with different risks and returns, chooses one with highest expected utility

Example – Wealth W is random variable, measured in $ millions

Utility function is U(W) = ln (W) ("natural logarithm" with base e; could also use base 10)

Risky situation:

W = 2 or 8 with probabilities 0.5 each

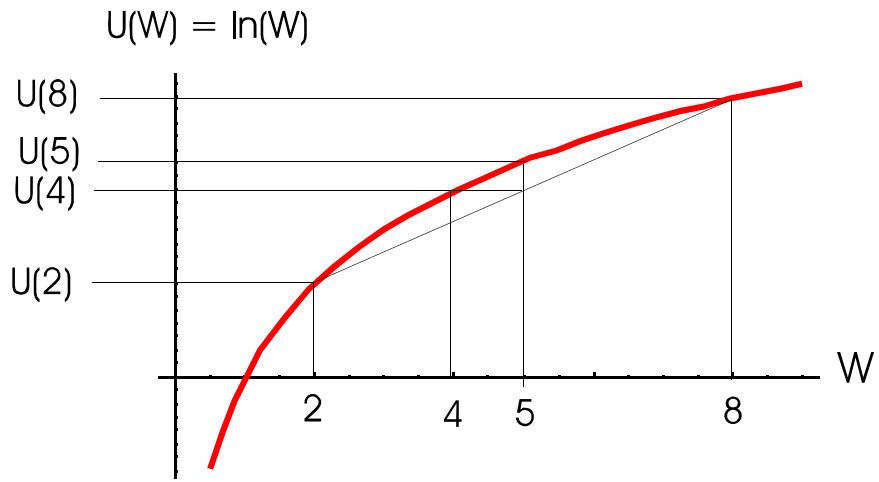
Expected utility = 0.5 ln (2) + 0.5 ln (8)

= 0.5 ln(2 * 8) = 0.5 ln(16)

= 0.5 ln(4 * 4) = 0.5 * 2 ln(4) = ln(4)

same as utility from having

W = $4 million for sure



But E[W] = 0.5 * 2 + 0.5 * 8 = 5

So this consumer is willing to give

up $1 million to remove the risk

Like an "insurance premium"

Others may have different attitudes to risk. Some may even love risk.

Want a way to quantify risk-aversion or risk preference

Basic idea – risk-aversion arises because utility function is concave

that is, marginal utility is diminishing

So how fast marginal utility decreases as wealth increases

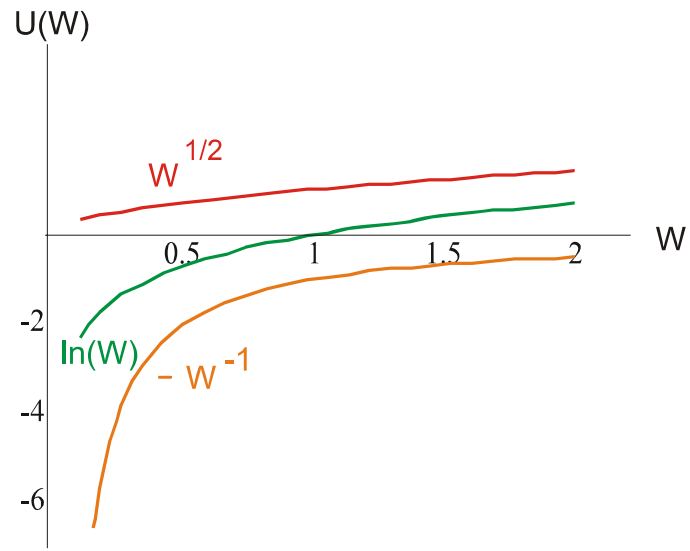
should provide a quantitative measure of risk-aversion

But for this to make sense, differences in the size of utility numbers must matter

– when used in the context of expected utility comparisons for characterizing behavior under risk, utility is cardinal.

Consider $U(W) = W^k$. Marginal utility $MU = U'(W) = k\,W^{k-1}$
Its elasticity as W increases is

$$\frac{W}{MU}\frac{dMU}{dW} = \frac{W}{k\,W^{k-1}}\,k(k-1)W^{k-2} = k-1$$



For risk aversion, this should be negative, that is, $k < 1$.
(A risk-loving consumer would have $k > 1$)
Define $\rho = 1 - k$ as the measure of risk-aversion
Technical term for ρ:
coefficient of relative risk-aversion

$\rho = 1$ ($k = 0$) is special case, where $U(W) = \ln(W)$
Can even have $\rho > 1$ ($k < 0$); then
utility function should be $U(W) = -\,W^k$
to keep it an increasing function of W
(OK to do this; the zero of utility is still arbitrary)

Example: Consumer with $100,000 faces risk of gaining or losing $50,000 with prob. ½ each

Calculate what amount (insurance premium) he/she is willing to give up to avoid this risk

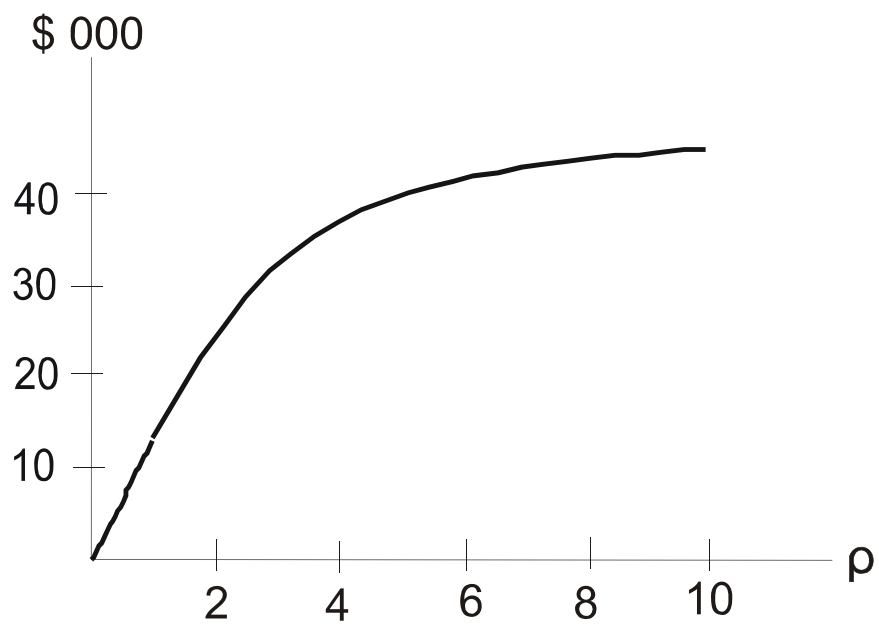
Solve for X from U(100000-X) = 0.5 * U(150000) + 0.5 * U(50000)

where the function U has the formula derived on the previous page

Do this for various possible ρ , to calibrate our intuition about ρ

Results shown in table and graph below:

ρ	Ins. Prem.
0.1	1,300
0.2	2,700
0.5	6,700
0.7	9,400
1.0	13,400
2.0	25,000
5.0	40,800
10.0	46,400

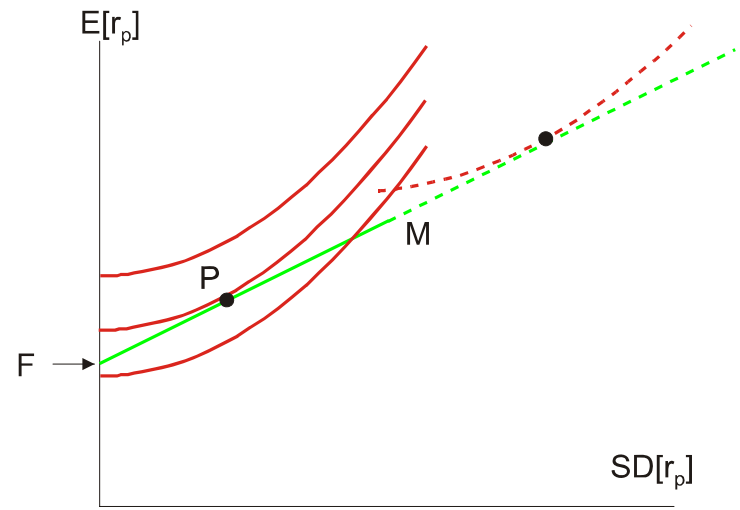


This is exactly like the job choice Question 3 in the questionnaire

So we can figure out the risk aversion coefficient implied by your answer

MEAN-VARIANCE APPROACH (P-R p. 163)

Consumer likes higher E[W] and dislikes higher V[W], so consider
objective function to be maximized: E[W] - α V[W]
where α is a measure of risk-aversion
Indifference curve: E[W] - α V[W] = constant, or
E[W] = constant + α V[W] = constant + α { SD[W] }^{2}
Indifference curves are parabolas
Higher α makes steeper parabola
If we can find budget line, we can do maximization



Application to financial markets: (P-R pp.174-9)
Two assets:
1. Riskless government bond, R_f = rate of return
2. Risky asset (stock market - broad mutual fund)
random rate of return r_m
$E[r_m] = R_m$, $SD[r_m] = \sigma_m$, $V[r_m] = (\sigma_m)^2$
If you invest fraction b of your wealth in stocks and
(1-b) in the government bond, the random rate of return r_p on your portfolio has
$E[r_p] = b R_m + (1-b) R_f = R_f + b (R_m - R_f)$, $V[r_p] = (b \sigma_m)^2$, $SD[r_p] = b \sigma_m$

$$E[r_p] = R_f + \frac{R_m - R_f}{\sigma_m} SD[r_p]$$ This is the budget line.

When maximizing, we may find b constrained to be between 0 and 1
But b can exceed 1 if it is possible to buy stocks on margin (or leveraged investment)