

## CONSUMER BEHAVIOR UNDER UNCERTAINTY – PART 2

### SOME PROBLEMS WITH EXPECTED UTILITY APPROACH (P-R pp. 180-3)

#### 1. Anchoring, reference point, or status quo effect –

People care about not just final consumption or wealth, but relative to reference point  
Losses count more than gains, even for small amounts

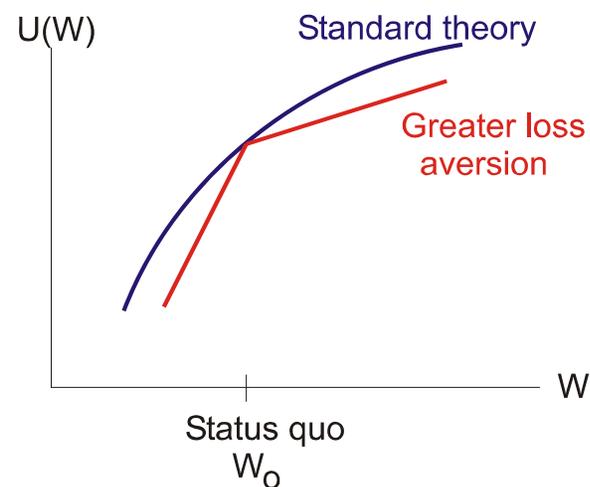
Utility function has kink at initial point

Kink would be minor modification

of standard theory; dependence on  
initial point is a more drastic departure

Related issue – framing, creation of an  
artificial status quo or reference point

Whether something is expressed  
as gain or loss affects attitude to risk



#### 2. Errors in estimation and calculation of probabilities

Overestimation of some risks (dramatic,  
highly publicized, large-loss catastrophes)

Underestimation of others (mostly routine, non-news events)

Systematic biases – Overconfidence, Stubbornness in the face of contrary evidence

Errors in conditioning or updating probabilities, especially failure to apply Bayes' rule correctly

Evidence of all these in responses to questionnaire

### 3. Allais Paradox:

Consider your responses to Questions 4 and 5 in the questionnaire

If you are an expected utility maximizer with utility function  $U$ ,

then your expected utilities from the various random prospects are

A:  $0.33 U(2500) + 0.66 U(2400) + 0.01 U(0)$

B:  $U(2400)$

C:  $0.33 U(2500) + 0.67 U(0)$

D:  $0.34 U(2400) + 0.66 U(0)$

You prefer A to B if and only if  $0.33 U(2500) + 0.66 U(2400) + 0.01 U(0) > U(2400)$ ,  
or  $0.33 U(2500) + 0.01 U(0) > 0.34 U(2400)$

You prefer C to D if and only if  $0.33 U(2500) + 0.67 U(0) > 0.34 U(2400) + 0.66 U(0)$   
or  $0.33 U(2500) + 0.01 U(0) > 0.34 U(2400)$

Therefore you should prefer A to B if and only if you prefer C to D

Choice such as B in Question 4 and C in Question 5 is just plain  
inconsistent with expected utility maximization

regardless of attitude to risk – whether  $U$  is concave or convex

But many people make just such choice – as 20 out of 34 responders in this class!

Some having seen this calculation may change their mind; others won't

Nothing irrational about such behavior, it can have underlying complete, transitive preferences

But cannot be expressed as expected utility maximization

## ALTERNATIVES TO EXPECTED UTILITY

Prospect Theory of Kahneman and Tversky – Decision process consists of two phases:

(1) Editing phase that organizes, reformulates, and simplifies options

This is where status quo and framing effects originate

(2) Evaluation phase. This uses not only a utility scale for prospects,  $U(x)$ , but also a weighting scale for probabilities,  $\pi(p)$

Overall objective function: if there are  $n$  “edited prospects”

Maximize  $\pi(p_1) U(x_1) + \pi(p_2) U(x_2) + \dots + \pi(p_n) U(x_n)$

Typically,  $\pi(p) > p$  for small  $p$ ,  $\pi(p) < p$  for  $p$  close to 1, and  $\pi(p_1) + \pi(p_2) + \dots + \pi(p_n) < 1$

Minimax regret – Consider the outcomes (wealth, or utility):

		Scenario	
		1	2
Action	1	a	b
	2	c	d

Suppose  $a > c$ , and  $d > b$ , so action 1 is better in scenario 1 and action 2 in scenario 2

If you do 1 and scenario 2 materializes, you get  $b$  instead of  $d$ ; define “regret” of action 1 as  $(d-b)$

Similarly regret of action 2 is  $(a-c)$ . Rule - Choose the action with the smaller regret

Merit - Don't need probabilities. Defect - May be guarding against highly unlikely events.

Each of these has its own difficulties – such a person is subject to a “Dutch book”, will accept a succession of bets in a cycle that will make expected loss

## INSURANCE (brief treatment in P-R pp. 166-8)

Consider a consumer with the initial wealth of  $W_0 = \$10$  million

Faces 10% risk that half of this would be wiped out

(big house in Stanford destroyed by earthquake or one in Palm Beach by hurricane)

Can buy insurance at price  $p$  per dollar of coverage.

If buys  $x$  dollars, pays  $px$  right now, and gets back  $x$  if suffers loss

So faces risky prospect: 10% probability final wealth  $W_1 = 10 - 5 + x - px = 5 + (1-p)x$

90% probability final wealth  $W_2 = 10 - px$

Utility function is  $U(W) = \ln(W)$

Expected utility is  $EU = 0.1 * \ln[W_1] + 0.9 * \ln[W_2] = 0.1 * \ln[5 + (1-p)x] + 0.9 * \ln[10 - px]$

Chooses  $x$  to maximize this.

Two methods:

MAT 103 method: Use the “chain rule” for differentiation of a function of a function

$$\frac{dEU}{dx} = 0.1 \frac{1}{W_1} \frac{dW_1}{dx} + 0.9 \frac{1}{W_2} \frac{dW_2}{dx} = 0.1 \frac{(1-p)}{5 + (1-p)x} + 0.9 \frac{-p}{10 - px}$$

Set this equal to zero, and simplify:

$$0.1 (1-p) (10 - px) - 0.9 p [5 + (1-p)x] = 0$$

or

$$x = \frac{1 - 5.5 p}{p (1 - p)}$$

This is the demand curve for insurance.  
See figure. Observe

$p = 0.1$  is “actuarially fair insurance”, and  
at this price,  $x = 5$  : full coverage demanded

Typically,  $p > 0.1$  because of  
insurance company’s admin. costs  
and market power

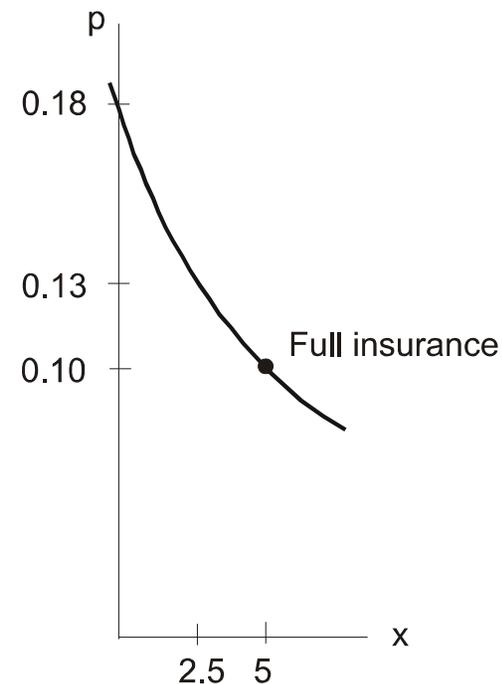
So consumer chooses partial coverage

Other reason for partial coverage - moral hazard

Mathematically, could contemplate  $p < 0.1$ , but this is economically irrelevant:  
insurance co. offering this would lose money;  
moral hazard would be overwhelming

When  $p = 0.18$ ,  $x = 0$

If insurance is sufficiently “unfairly priced”  
even risk-averse consumer will prefer to bear the whole risk



## ALTERNATIVE INTERPRETATION OF INSURANCE – MARKETS FOR RISK (not in P-R)

When you pay say  $p = 0.12$  for a dollar of fire insurance on your house  
you are “betting” 0.12; you “win the bet” and get  $\$1 - 0.12 = 0.88$  if the fire occurs,  
you “lose the bet” and lose 0.12 if the fire does not occur.

More generally, consider a market in “contingent claims”  
held before anyone knows which of the events 1 and 2 occurs

The objects traded in this market are “betting slips”

You buy and sell these ahead of time. Then, after the uncertainty is resolved  
you cash in the slips corresponding to the event that occurs;  
the other kind are worthless after the fact

A scenario-1 betting slip will pay \$1 if scenario 1 occurs, 0 otherwise

A scenario-2 betting slip will pay \$1 if scenario 2 occurs, 0 otherwise

The market determines the prices say  $p(1)$ ,  $p(2)$  of these slips;

the demand and supply depend on probabilities and participants’ risk attitudes

If you hold one of each kind of slip, you will get \$1 for sure; so prices  $p(1) + p(2) = 1$

If time to resolution of uncertainty is distant,  $\text{sum} = 1/(1 + \text{interest rate})$

In real world there are zillions of different scenarios, and zillions of assets  
can be created by bundling and rebundling the “betting slips” for them

Financial markets trade in objects of exactly this kind, usually bundled in particular ways

Equity stocks give you different dividends and capital gains (or losses)

in different scenarios corresponding to how well or poorly the firm does

Bonds give interest and capital gains or losses as interest rates change

Options, derivatives are ways of rebundling risks to suit specific classes of investors

Other examples:

[1] Contingent claims markets on weather scenarios (temperature on particular day / season)

These are used by energy producers and utilities to hedge their costs

Website <http://www.evomarkets.com/weather/>

[2] Political betting markets on elections - <http://www.biz.uiowa.edu/iem/>

[3] Mortgage-backed securities and collateralized mortgage obligations

Website <http://www.investopedia.com/terms/m/mbs.asp>

Bad news – to be a real expert in such securities, to be actually able to design new ones, and make seven-figure incomes, requires very high-end math:

MAT 315 real analysis, MAT 391 stochastic processes etc.

## EFFICIENT ALLOCATION OF RISK (not in P-R)

Can markets greatly reduce or eliminate risk for all participants?

Sometimes yes: [1] negative correlation enables elimination by sharing

[2] law of large numbers allows substantial reduction by pooling independent risks

But more generally, risks can only be traded –

for every hedger there is a speculator on the other side of the trade

Then the question is whether markets will achieve efficient allocation of risks

Some such tendencies can be identified –

less risk-averse person will accept the risk for a smaller price than

what a more risk-averse person is willing to pay

So mutually beneficial trade – risk ends up with whoever is most willing to bear it

But problems arise, mainly because of asymmetric information

Moral hazard – One person's actions can affect probability or magnitude of risk

But others cannot observe this, so cannot write contract to exclude such actions

Then e.g. availability of insurance may make the insured behave carelessly

To cope with this: only partial insurance is offered

Adverse selection – One person knows more about the nature of the risk than another

To cope with this: strategies of signaling and screening

Example: Potential employers want quantitative skills, dedication, perseverance, ...

You can signal these qualities by taking tough courses, sticking to them, getting good grades

Or employer can screen applicants by requiring such courses and grades on your resume

In either case - the action must be such that someone who does not have the desired characteristic would find it too costly to imitate the action