PRODUCTION AND FIRMS – PART I

WHAT IS A FIRM?

Traditional or textbook view: Firm essentially defined by its technology of production:

Outputs = F(Inputs); Buys or hires inputs in markets; produces and sells outputs in markets

Owner chooses quantities to (1) minimize cost: MRTS = price ratio for any pair of inputs

(2) maximize profits: MC = p (MC = MR if monopoly power)

The two together imply: price for each input = output price * marginal physical product

(Or MR * marginal physical product in case of a monpolist firm)

New view: Studies internal organization of a firm

Firms' internal operations are based on commands and hierarchies;

they are "islands of central planning in a sea of markets" (Coase, Nobel Prize 1991)

Choice of market versus hierarchy depends on

(1) Is the transaction occasional or recurring?

(2) Is there any product-specific investment creating opportunities for "hold-up" ?

(3) Is quality of product, effort etc. observable? verifiable as evidence in a court?

Recurring transactions requiring product-specific investment and unobservable dimensions are difficult to handle efficiently in an arm's length market

They create "principal-agent" relationships, requiring design of contracts, incentives

Shareholders are principals and top management are their agents

Middle managers in turn are agents to top management, ...

Some issues of principal-agent situations and incentive design later in this course

Old view is still useful for characterizing firm's relationship with rest of economy –
supply functions for outputs and "derived" demand functions for inputs
Will proceed with that for a while

TIME ASPECTS OF PRODUCTION

1. STOCKS VERSUS FLOWS
Production is a flow – quantity per period (e.g. month or quarter or year)
Costs and profits should also be flows, $ per unit of time
Some inputs are also flows – they are used up when used
raw materials, labor services
Other inputs are stocks – they remain (perhaps somewhat worn) at end of period
land, machinery, equipment ...
Relevant price for such durable inputs is not the whole purchase price
but the cost of using the input for one period
= Interest on purchase price (opportunity cost of tying up the money for one period)
+ Depreciation (loss in value due to use or passage of time)
(Rarely in this context, value may rise through time, in which case – Capital gains)

2. SLOW ADJUSTMENT
Not possible to adjust inputs optimally at every instant – Time for installation and moving
Contracts with suppliers or workers cannot be changed until next negotiation date ...
This creates "sunk costs" – short run costs that cannot be avoided even if producing zero
Longer time span $\Rightarrow$ fewer sunk costs. In eventual long run, no sunk costs (free entry/exit)
Marshallian convention: capital is sunk input, labor is variable input
Meant as pedagogic device to keep concepts distinct; not meant to be realistic

ONE INPUT (P-R pp. 190-9)

Just to get some basic concepts

Total product TP or Q function of
input L

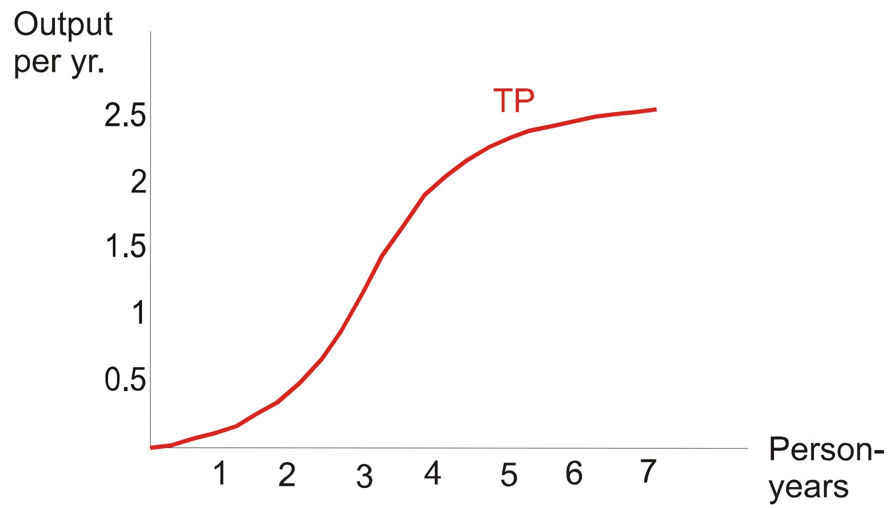
AP = Q / L,   MP = dQ/dL

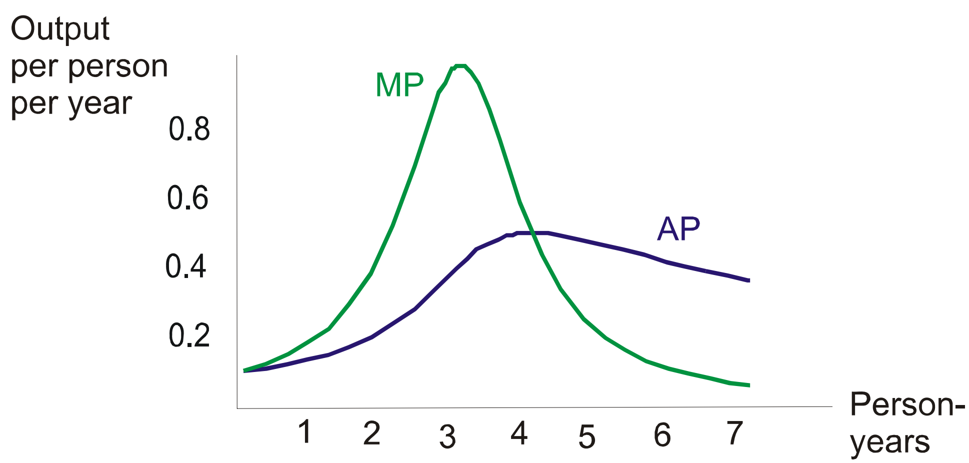
Increasing returns (to scale):
AP increases as L increases

$$MP = \frac{d(L * AP)}{dL}$$

$$= 1 * AP + L * \frac{d(AP)}{dL}$$

$$MP - AP = L * \frac{d(AP)}{dL}$$



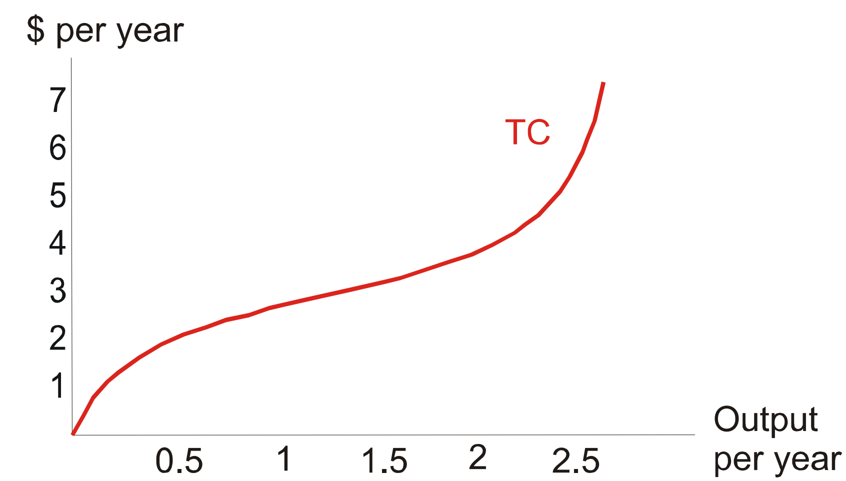


So MP > AP if and only if d(AP)/dL > 0
i.e. in the region of increasing returns
Usual general case in teaching – increasing returns first, then decreasing returns
But in various special circumstances can get other possibilities

Usual reasons for increasing returns:

(1) Fixed or "first-copy" costs. If L_0 of labor needed to enable any production at all, and then constant amount c per unit of output, we have $Q = c (L - L_0)$

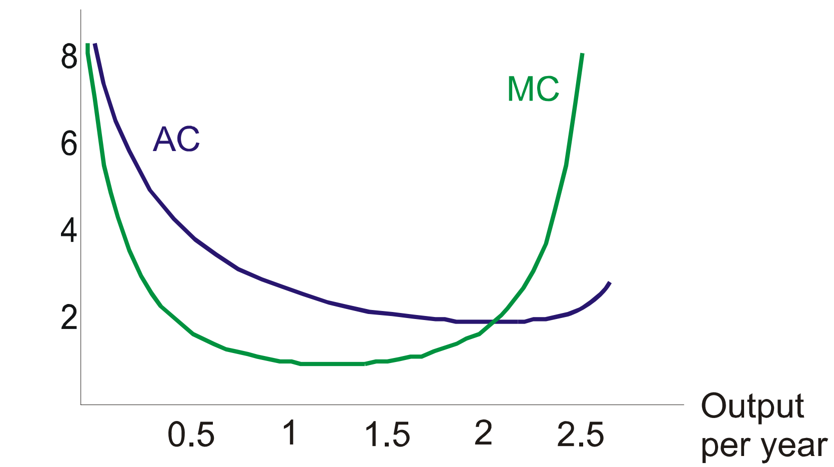
(2) "Geometry" of chemical processes:
input materials $M \propto$ (proportional to) length, Output $\propto$ area, so $Q = M^2$
Or $M \propto$ area, $Q \propto$ volume, so $Q \propto M^{3/2}$



Usual reason for decreasing returns:

Some input is fixed by its very nature, for example attention of top management
So expanding the other inputs eventually runs into limited attention span at the top

Cost: Invert total product function $Q = TP(L)$ to get $L = L(Q)$, and then $TC = w\, L(Q)$



$AC = C / Q = w L / Q = w / AP$
So increasing returns to scale if and only if AC is decreasing as Q increases

Relation between AC and MC:

$$MC - AC = Q \frac{d(AC)}{dQ}$$

TWO INPUTS (P-R pp. 199-207)

Production function Q = F(K,L)

Average products

$AP_L = Q / L = F(K,L) / L$

$AP_K = Q / K = F(K,L) / K$

Marginal products

$MP_L = \partial Q / \partial L = F_L(K,L)$

$MP_K = \partial Q / \partial K = F_K(K,L)$

Isoquant – curve in (L,K) diagram along which Q = F(K,L) is constant

Just like consumer indifference curve, but now labels on curve are objective, measurable output quantities, not an arbitrary utility scale

MRTS = numerical value of slope dK/dL along indifference curve

$$MRTS = -\left.\frac{dK}{dL}\right|_{Q=const.} = \frac{MP_L}{MP_K}$$

Cobb-Douglas Example: $Q = A K^{\alpha} L^{\beta}$

$AP_L = A K^{\alpha} L^{\beta} / L = A K^{\alpha} L^{\beta-1}$

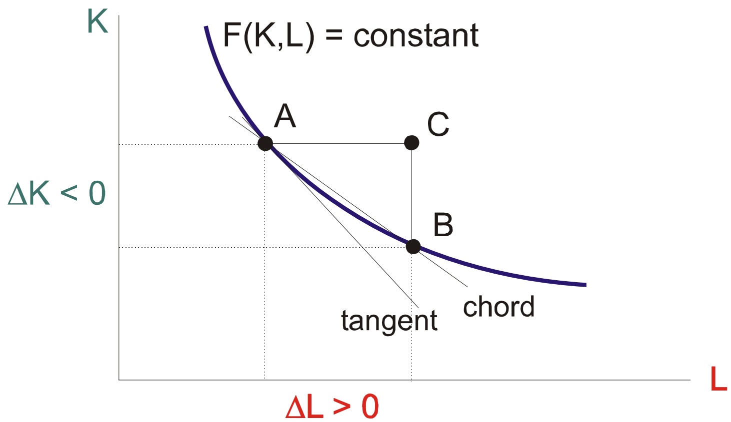
$AP_K = A K^{\alpha} L^{\beta} / K = A K^{\alpha-1} L^{\beta}$

$MP_L = A K^{\alpha} \beta L^{\beta-1} = \beta A K^{\alpha} L^{\beta-1} = \beta AP_L$

$MP_K = A \alpha K^{\alpha-1} L^{\beta} = \alpha A K^{\alpha-1} L^{\beta} = \alpha AP_K$

Isoquant: If $A K^{\alpha} L^{\beta} = Q_0$, a given constant,

$K^{\alpha} = Q_0 A^{-1} L^{-\beta}$, $K = (Q_0)^{1/\alpha} A^{-1/\alpha} L^{-\beta/\alpha}$



$$MRTS = \frac{\beta A K^{\alpha} L^{\beta-1}}{\alpha A K^{\alpha-1} L^{\beta}} = \frac{\beta K}{\alpha L}$$

RETURNS TO SCALE, AND TO EACH INPUT (P-R pp. 207-10)

When there are two inputs, must distinguish between the effects of
increasing only one of them holding the other constant, and
increasing both of them in equal proportions

When only one input changes, we usually ask what happens to its marginal return:
Diminishing marginal returns to labor: MP_L diminishes when L increases holding K constant

$$\frac{\partial}{\partial L}(MP_L) = \frac{\partial}{\partial L}\left(\frac{\partial Q}{\partial L}\right) = \frac{\partial^2 Q}{\partial L^2} < 0$$

Diminishing marginal returns to capital similar
Cobb-Douglas example: $MP_L = \beta\, A\, K^{\alpha}\, L^{\beta - 1}$;
if $\beta < 1$, this decreases as L increases, with K held constant

When both inputs change in the same proportion, we also ask about their average return:
Increasing returns to scale: If (K,L) is replaced by (sK,sL) where s > 1, then does output increases by more than the factor s? : Is F(sK,sL) / s > F(K,L), or F(sK,sL) > s F(K,L)
Decreasing returns to scale: when s > 1, is F(sK,sL) / s < F(K,L), or F(sK,sL) < s F(K,L)
Constant returns to scale: for any s > 0, F(sK,sL) = s F(K,L)

Cobb-Douglas example: $A\,(sK)^{\alpha}\,(sL)^{\beta} = s^{\alpha+\beta}\, A\, K^{\alpha}\, L^{\beta}$, this is $> s\, A\, K^{\alpha}\, L^{\beta}$ if $\alpha + \beta > 1$
Returns to scale are decreasing if $\alpha + \beta < 1$; constant if $\alpha + \beta = 1$
So can have diminishing marginal returns to each input but increasing returns to scale:
just need $\alpha < 1$, $\beta < 1$, $\alpha + \beta > 1$