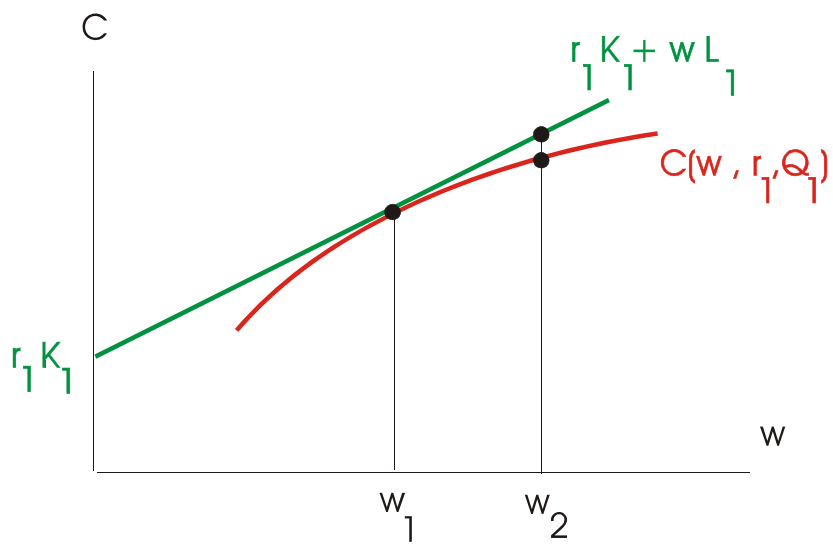
PRODUCTION AND FIRMS – PART 3

COST DEPENDS ON INPUT PRICES (Not directly in P-R)

Fix the quantity Q_1 of output to be produced
Suppose at input prices r_1 and w_1 , the cost-minimizing choices of inputs are K_1 and L_1
cost $C_1 = r_1 K_1 + w_1 L_1$
Now the wage rate changes to w_2 ; no change in Q_1 , r_1 . What happens to cost?
Could go on using K_1 and L_1 ; cost would be $r_1 K_1 + w_2 L_1$ (like Laspeyres index)
But if there is some substitution possibility,
can lower cost by changing input mix
Figure shows optimal cost as w is changed
Everywhere below line $r_1 K_1 + w L_1$
Coincides at one point $w = w_1$
Therefore line is tangent to curve at this point: Slope of line = slope of curve
$L_1 = \partial C/\partial w$ at $w = w_1$



Overall result of doing this for all possible Q, w, r :
Firm's cost function C(Q, w, r)
Firm's "conditional (for given output level Q) derived demand function for labor"
can be found simply by differentiating its cost function with respect to the price of labor w
Similarly for capital. P-R do this differently on pp. 516-8.

COBB-DOUGLAS EXAMPLE (P-R pp. 256-9)

Firm chooses L, K to minimize cost w L + r K subject to $A K^{\alpha} L^{\beta} = Q$
We did this by using tangency condition. For reinforcement let us now use Lagrange's Method

Lagrangian $\mathscr{L} = w L + r K - \lambda [A K^{\alpha} L^{\beta} - Q]$; λ is the Lagrange multiplier to be determined

Conditions $\partial\mathscr{L} / \partial L = w - \lambda A \beta K^{\alpha} L^{\beta-1} = 0$, $\partial\mathscr{L} / \partial K = r - \lambda A \alpha K^{\alpha-1} L^{\beta} = 0$
Write these as $w = \lambda \beta Q / L$ $\quad$ $r = \lambda \alpha Q / K$
$L = \lambda \beta Q / w$ $\quad$ $K = \lambda \alpha Q / r$
Therefore

$$Q = A [\lambda \alpha Q / r]^{\alpha} \quad [\lambda \beta Q / w]^{\beta} = \text{constant} * [\lambda Q]^{\alpha+\beta} \; w^{-\beta} \; r^{-\alpha}$$

Solve this for λ, then substitute into the expressions for L and K, and finally get the cost
Algebra is messy, but result:

$$L = const \left(\frac{r}{w}\right)^{\alpha/(\alpha+\beta)} Q^{1/(\alpha+\beta)}, \quad K = const \left(\frac{w}{r}\right)^{\beta/(\alpha+\beta)} Q^{1/(\alpha+\beta)}, \quad C = const \; w^{\beta/(\alpha+\beta)} \; r^{\alpha/(\alpha+\beta)} \; Q^{1/(\alpha+\beta)}$$

where all the constants are different and messy expressions involving α, β .

Elasticity of L, K and C with respect to Q is $1/(\alpha+\beta)$
So, for example, If $\alpha+\beta > 1$, then inputs and costs increase slower than Q
Increasing returns to scale, decreasing AC curve. Exercise: check that MC < AC
If good at math, use full formulas in P-R to check that $L = \partial C/\partial w$, $K = \partial C/\partial r$
L, K depend only on relative prices. As (w/r) increases, L falls and K increases: substitution

EMPIRICAL EXAMPLES OF COST FUNCTIONS

AGGREGATE U.S. MANUFACTURING (E. Berndt, Practice of Econometrics, 1991, pp. 469-76)
Four inputs: capital, labor, energy, and non-energy raw materials

$$\ln(C) = \ln(\alpha_0) + \sum_{i=1}^{4} \alpha_i \ln(P_i) + \frac{1}{2} \sum_{i=1}^{4}\sum_{j=1}^{4} \gamma_{ij} \ln(P_i) \ln(P_j) + \alpha_Y \ln(Y) + \frac{1}{2} \gamma_{YY} [\ln(Y)]^2 + \sum_{i=1}^{4} \gamma_{iY} \ln(P_i) \ln(Y)$$

The parameters α_0 , ... γ_{iY} are to be estimated by regression analysis.
Economic theory places restrictions, e.g. $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 1$, because
if all input prices go up in proportion, total cost must rise by the same factor
Then interpretation: input demand functions, how each depends on own and other prices
returns to scale, etc. Berndt reports elasticities of demand:

Elasticity of demand for	With respect to the price of			
	Capital	Labor	Energy	Materials
Capital	- 0.34	0.29	- 0.16	0.21
Labor	0.05	- 0.45	0.03	0.37
Energy	- 0.17	0.20	- 0.53	0.51
Materials	0.02	0.18	0.04	- 0.24

So capital and energy are complements, and other pairs of inputs are substitutes
Unfortunately Berndt does not report on returns to scale

CREDIT UNIONS (Stephen Moeller, Princeton senior thesis 1999)

These are saving and borrowing cooperatives at workplaces, community associations etc.
Must choose measure of quantity Q. Moeller discusses alternatives:

numbers of loans outstanding, or loans granted in a year, or savings accounts,
dollar volume of loans outstanding ... He chooses the first of these.

$$\ln(C) = a + b_1 \ln(Q) + b_2 [\ln(Q)]^2 + \sum_i c_i \ln(W_i) + \sum_j d_j F_j$$

where W_i are input prices, and F_j are other structural variables. His estimates are

b_1 = 0.6537, with standard error 0.0231
b_2 = 0.0204, with standard error 0.0015

How to interpret this? AC = C/Q, so ln(AC) = ln(C) - ln(Q)

$$\ln(AC) = a - (1 - b_1) \ln(Q) + b_2 [\ln(Q)]^2 + \sum_i c_i \ln(W_i) + \sum_j d_j F_j$$

So as ln(Q) increases starting from very small values (near ln(Q) = 0 or Q = 1),

AC decreases at first (negative coefficient on linear term) and
AC eventually increases (positive coefficient on quadratic term

Economic intuition: [1] Large Q enables better risk-pooling, but

[2] Eventually Q becomes so large that moral hazard cannot be controlled

Where is AC minimized? Choosing Q is equivalent to choosing ln(Q), so

take derivative of right hand side with respect to ln(Q) and set it equal to zero

$-(1-b_1) + 2\, b_2 \ln(Q) = 0$, or $\ln(Q) = (1-b_1)/(2\, b_2) = 8.48$, or Q = 4764

In reality, 85% of U.S. credit unions were to the left of this

Median Q was 705; at this point AC was 7.8% higher than the minimum achievable