OPTIONAL ADDITIONAL READING ON CONSUMER SURPLUS

The idea of consumer surplus as a measure of the benefit consumers derive from a good can be justified more rigorously if the utility function is *quasi-linear.* This concept appeared a few times in other contexts: Problem Set 1 Question 5, and as an incidental part of discussion of the relative magnitudes of compensating and equivalent variations on October 4 and Problem Set 3 Question 1, so some of you may recognize it and make a connection. The specific application of this handout, namely for measuring the total and marginal benefits from consuming a good, is optional for our course, but useful to read, understand, and keep on file because in reading or doing research on policy design you will often need to figure out these issues. (Note that this particular application of quasi-linear utility is optional, but the ability to solve problems like those mentioned above is a required component of the course.)

Let us focus on a particular good, whose quantity is denoted by x and price by p. Aggregate all the other goods in the economy into the second good y, and measure this in units of the number of dollars spent on that aggregate, so its price is 1. The consumer's budget constraint is

$$p\,x + y = I\,.$$

The quasi-linear utility function takes the form

$$u = y + F(x) \tag{1}$$

where $F(x)$ is an increasing and concave function ($F'(x) > 0$, $F''(x) < 0$). It is called quasi-linear because it is linear in one of the goods (here y) but non-linear in the other (here x). Literally, the prefix quasi means "having some resemblance to."

Substituting from the budget constraint, we can express utility as a function of x alone:

$$u = I - p\,x + F(x)$$

The consumer chooses x to maximize utility. As the budget constraint has already been used to substitute out for y, there are no further constraints. This is now an ordinary calculus problem – no Lagrange necessary. The first-order condition is

$$du/dx = -p + F'(x) = 0$$

or

$$p = F'(x) \tag{2}$$

The second-order condition is

$$d^2u/dx^2 = F''(x) < 0$$

which is satisfied because of the assumption made above about $F(x)$.

Equation (2) is the consumer's *inverse demand function,* expressing the price as a function of the quantity. It can be interpreted as follows. The part of the utility that comes from x, namely $F(x)$, can be called the benefit the consumer derives from consuming quantity x. Its derivative, $F'(x)$, is then the *marginal benefit* of x. Thus the inverse demand function can be thought of as the marginal benefit curve.

Suppose the cost of production of x is $C(x)$. As usual, this means the opportunity cost – how much of other things has to be given up. In our case the other things are y. Suppose that, if no x is produced and consumed and all of the economy's resources are devoted to producing y, the quantity (again measured in dollar units) available will be Y. When x is being produced and costing $C(x)$, the amount of y available is then

$$y = Y - C(x) \tag{3}$$

This is a *resource constraint* that will confront the economy as a whole, like the aggregate transformation frontier (production possibility frontier, or "guns-butter tradeoff frontier") of ECO 100.

Suppose that, instead of leaving things to a market, a benevolent dictator decides how much of x and y should be produced and consumed. Benevolence means maximizing the consumer's utility. (Such dictators are rare or even totally absent in reality, but this is just a conceptual exercise, setting up an ideal against which to measure the actual outcomes.) We can express utility in terms of x by substituting from the resource constraint (3) into the utility function (1):

$$u = Y - C(x) + F(x)$$

The first-order condition for the maximization of this is

$$du/dx = -C'(x) + F'(x) = 0$$

or

$$F'(x) = C'(x)$$

The left hand side was interpreted as the marginal benefit, so the condition just says that marginal benefit should equal marginal cost. This was how the condition for optimal output was intuitively stated and geometrically analyzed in the overheads handout for today, p. 2.

And now we can see how a competitive equilibrium can achieve the optimal output: consumer optimization leads to $p = F'(x)$ (the demand curve), firms' profit maximization leads to $p = C'(x)$ (the supply curve). The two together yield the optimality condition $F'(x) = C'(x)$ (intersection of the demand and supply curves).

What if the utility function is not of the quasi-linear form? Note that for the quasi-linear demand function that can be found by inverting equation (2), x depends only on p and not on I; this *zero income effect on the good* x is the important property of demands that come from maximization of quasi-linear utility. Therefore, if the good in question is a relatively small part of the economy, so that the aggregate income changes have only a small effect on the demand for this good, the analysis and calculations based on quasi-linearity serve as sufficiently good approximations.

Otherwise we need somewhat more sophisticated concepts. Basically, the total benefit of a good x can be measured by how much of income the consumer is willing to give up in order to have the ability to consume the quantity x instead of doing without it altogether, or how much income the consumer has to be given instead of this quantity to leave him just as well off. These are just the compensating and equivalent variation concepts we saw earlier in the course. Similar concepts can be developed for marginal benefit, and they relate to the pure substitution effects of price changes. But rigorous analysis of this needs more math than our course allows, so we will leave it for you to pick up if and when you need it for specific applications in more advanced courses or in research. A textbook treatment of this (at the ECO 310 level) is in Walter Nicholson, *Microeconomic Theory: Basic Principles and Extensions,* ninth edition 2005. pp. 145–9.