

## OLIGOPOLY – PART 2

## PRISONER'S DILEMMA

Example from P-R p. 455; also 476-7, 481-2  
Price-setting (Bertrand) duopoly

Demand functions

$$X_1 = 12 - 2P_1 + P_2, X_2 = 12 - 2P_2 + P_1$$

Fixed cost 20, marginal cost 0 for each firm

Try just two price choices, 6 and 4 for each

Table shows profits of the two

		Firm 2	
		4	6
Firm 1	4	12 12	20 4
	6	4 20	16 16

Game theory jargon – players Row, Column

Players' objectives – payoffs

In each cell, Row's payoff shown first

Nash equilibrium – If both choose price 4,

neither can gain by deviating unilaterally

Stronger argument – 4 is “dominant strategy”

for each: each does better choosing 4 than choosing 6, no matter what the other does

But both would do better if **both** choose 6

Payoffs 16 each instead of 12 each

This is the prisoner's dilemma

Dilemma in oligopoly – collusion (cartel)

is better for both than competition

But each is tempted to cheat (and would only lose by conforming if the other cheats)

Question for firms – Can they overcome this temptation and maintain cartel / high prices?

Need ways to detect and punish cheaters

Question for society – How to deter cartels and ensure competition (P closer to MC)?

But both need understanding of the game and its possible equilibria

## REPEATED INTERACTION (P-R pp. 484-8)

Above game played each period. From one period to the next, payoffs grow at rate  $g$ , and money values are discounted at the interest rate  $r$

Also, probability  $p$  that for some outside reason, interaction will discontinue

Tacit collusion can be sustained by various strategies that make it costly to cheat.

Simplest example here: “Grim trigger strategies” – both start by playing the collusive actions, but if anyone ever cheats, the collusion collapses in all future periods

and the game is thereafter played to its competitive or dilemma equilibrium

Cheater gains  $20 - 16 = 4$  in one period, but loses (opportunity sense)  $16 - 12 = 4$  ever after

The future loss has expected present discounted value

$$4 \frac{(1-p)(1+g)}{1+r} + 4 \left[ \frac{(1-p)(1+g)}{1+r} \right]^2 + 4 \left[ \frac{(1-p)(1+g)}{1+r} \right]^3 + \dots = 4 \frac{k}{1-k}$$

where  $k = (1-p)(1+g)/(1+r)$ . Example: If  $p = 0.2$ ,  $g = 0.05$ ,  $r = 0.1$ , then  $k = 0.76$ ,  $\text{RHS} = 12.1 > 4$

But if  $p = 0.4$ ,  $g = -0.02$ ,  $r = 0.2$ , then  $k = 0.49$ ,  $\text{RHS} = 3.84 < 4$

So collusion more likely if [1] interest rate  $r$  at which future money flows are discounted is low,

[2] interaction is likely to persist (low  $p$ ), [3] industry is growing or stable (high  $g$ )

All these are conditions ensuring that the future is important relative to the present

Implicit in this – length of the period. Interpretation – how long it takes to detect cheating

Therefore speed and accuracy of cheating important for successful cartelization

In practice, firms' demands and costs are asymmetric. Price wars are more costly for large firms, so small firms may hope to get away with cheating

P-R pp. 457-67 for other practical issues – signaling collusive price and allocating market shares as demand and cost conditions change, deterring or responding to outside competition, ...

## ENTRY DETERRENCE (P-R pp. 491-502, specific example from p. 497-8)

Existing firm (incumbent) trying to deter potential entrant by threat –

Says “I will charge low price (start a price war) if you enter”

Problem – threat may not be credible; potential entrant may believe that if it comes in and puts the matter to the test, the incumbent will back off and accommodate the new firm

Game “tree” shown on right. Consists of “nodes” and “branches”

Each node shows the player who takes action there; emerging branches are available actions

At final or terminal nodes, show resulting “payoffs” if that path is followed in play of game

Asterisks show the optimal action of each player at each node

With these numbers, if Entrant chooses In,  
Incumbent’s optimal strategy is High Price  
Therefore threat of price war is not credible

Equilibrium of game found by rollback:

Start at last decision nodes and work back

Find optimal choice at each node,

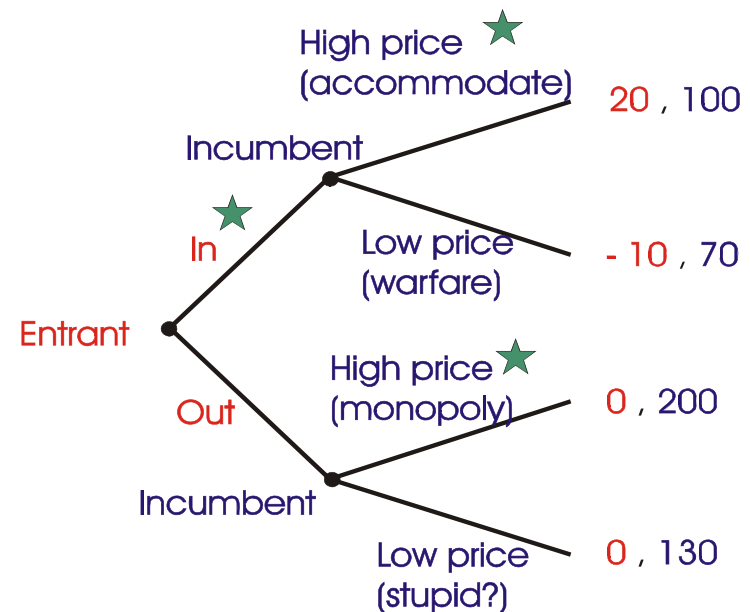
looking ahead and folding in

all optimal choices at later nodes

Resulting path of play - continuous sequence  
of asterisks from the initial to a final node

Incumbent would like to make threat credible

One method – install (sink) capacity ahead of time, enough to meet demand at the low price



The extra capacity costs 50

If high price is charged, quantity is low, capacity goes unused, but its cost is sunk

Profits lower than if capacity was not sunk

If low price is charged, capacity is used. Profits no different than non-sunk case

where the same capacity would have to be acquired later to meet demand at the low price

Bigger game tree with new initial node

for incumbent's capacity choice

Lower half same as old tree

In upper half, if entrant In,

Low price becomes optimal  
for incumbent – threat of  
price war becomes credible

In rollback equilibrium play

Incumbent installs capacity

Entrant stays Out

Incumbent prices High

Excess capacity stays idle

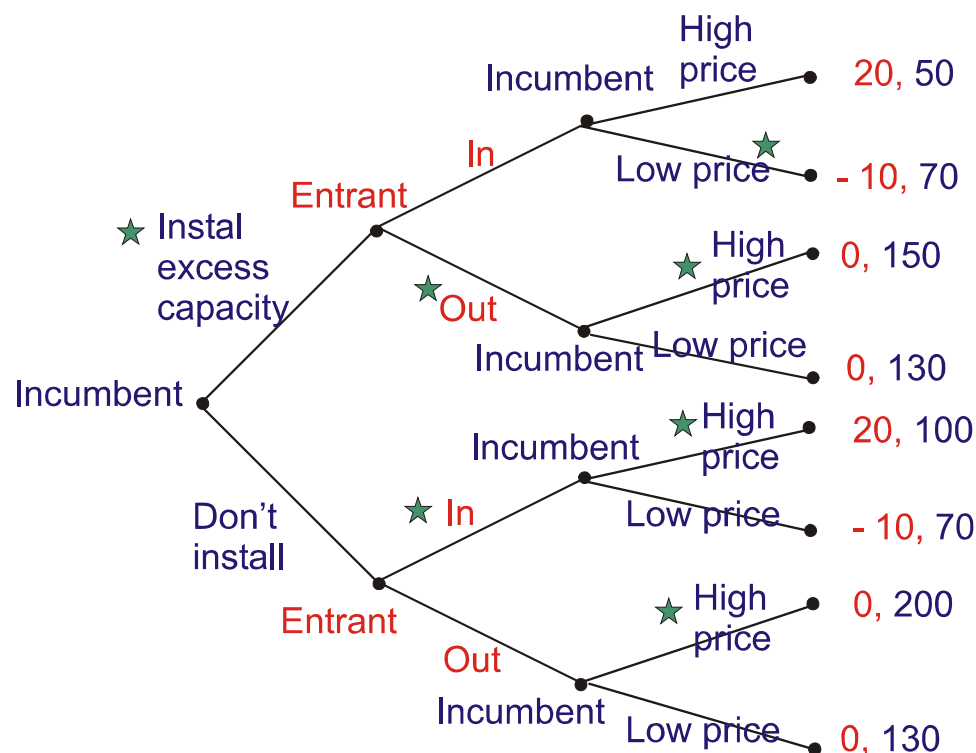
Incumbent's profit 150

(200 monopoly - 50 capacity cost)

Not as good as 200, but better than

the 130 that comes without the

capacity strategy that makes threat credible



Other paths to credibility – reputation in repeated play, irrationality, ...

## STACKELBERG LEADERSHIP (P-R pp. 447-8, 452)

Quantity-setting as in Cournot, but firm 1 moves first, firm 2 moves second

Numbers as in November 17 Oligopoly handout; pp. 3-4

Firm 2's reaction curve is  $Q_2 = 75 - 0.75 Q_1$ ; this will be its choice at its second move

Firm 1 will recognize this, so at its first move will choose  $Q_1$  to maximize its profit

$$\Pi_1 = (200 - Q_1 - Q_2) Q_1 - 100 Q_1 = (200 - Q_1 - 75 + 0.75 Q_1) Q_1 - 100 Q_1 = 25 Q_1 - 0.25 (Q_1)^2$$

To maximize this,  $d\Pi_1 / dQ_1 = 25 - 0.5 Q_1 = 0$ , or  $Q_1 = 50$

Have not drawn game tree, but this is rollback equilibrium of the sequential-move game

Then can solve for other magnitudes. Table shows comparison

Type of oligopoly	$Q_1$	$Q_2$	P	$\pi_1$	$\pi_2$
Cournot	20	60	120	400	2400
Stackelberg (firm 1 leads)	50	37.5	112.5	625	937.5

Quantity-leadership is better for firm 1 and for society than simultaneous play. Reason –

by increasing  $Q_1$  above its Cournot level, firm 1 can get firm 2 to produce less  $Q_2$

But firm 2's backing off is less than 1-for-1, so total Q increases, P decreases closer to MC

So consumers gain. Total industry profit declines, but firm 1 gains at the expense of firm 2

With price-setting, leadership is often disadvantageous when products are substitutes –

the other firm moving second can see your price and then just undercut you

## MONOPOLISTIC COMPETITION (P-R pp. 436-41)

Case where several firms sell close but not perfect substitutes

Free entry / exit yields (approximately) zero profit for each firm

Examples – many consumer products, supermarkets in large town, global auto industry?

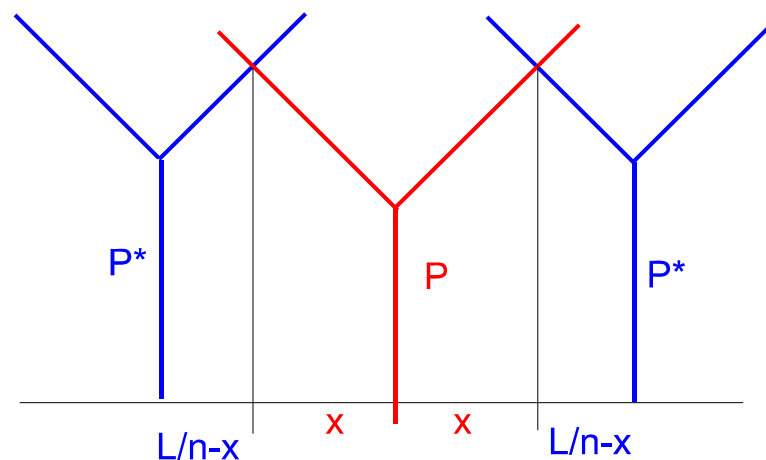
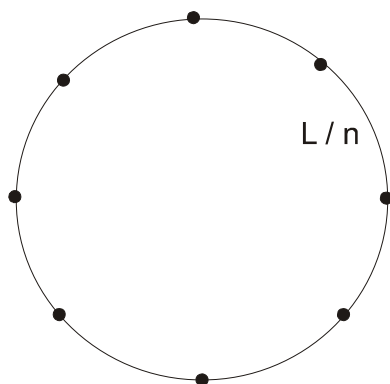
Multi-firm model of location differentiation. Consider  $n$  firms along a circle of circumference  $L$

Each firm has fixed cost  $F$  and constant marginal cost  $c$

If firm charges  $P$ , effective price at distance  $x$  is  $P + kx$

Consider symmetric equilibrium where each firm charges price  $P^*$  (more complex cases exist)

Left figure shows circle with 8 firms; right, just 3 neighbors, with prices and effective prices



If one firm deviates to charge  $P$ , it sells to distance  $x$  either side, defined by

$$P + kx = P^* + k(L/n - x), \quad \text{or} \quad x = L/(2n) + (P^* - P)/(2k)$$

So total quantity it sells is  $Q = 2x = L/n + (P^* - P)/k$

This firm's profit  $\Pi = (P - c)Q - F = (P - c) \left[ \frac{L}{n} + \frac{P^* - P}{k} \right] - F$

This firm treats others'  $P^*$  as given, and chooses  $P$  to maximize its own profit

$$\frac{\partial \Pi}{\partial P} = \left[ \frac{L}{n} + \frac{P^* - P}{k} \right] - (P - c) \frac{1}{k} = 0$$

We want a symmetric equilibrium where  $P = P^*$ , so  $(P - c) / k = L / n$ , or  $P = c + k L / n$

Then each firm sells to  $x = L/(2n)$  either side of its location, for total quantity  $Q = L/n$

Profit of each firm is  $\pi = (P - c) Q - F = k (L/n)^2 - F$

Entry / exit will reduce this to zero, so  $k (L/n)^2 = F$ , or  $n = L \sqrt{(k/F)}$

More firms if [1] larger market, [2] lower fixed cost, [3] larger disutility cost of non-ideal product

Equilibrium can be illustrated in figure on right

similar to P-R p. 438 Fig. 12.1 (b)

except that here MC is horizontal

For one firm,  $P = P^*$ ,  $Q = L/n$  is optimal

because its  $MR = MC$  there

Its maximized profit is zero

because  $AR = AC$  there

So fixed cost  $F$  must equal

profit rectangle  $(P^* - MC) (L/n)$

