Mathematical Appendix

Here we present details of mathematical derivations of the results presented in the text. The broad ideas and intuitions are discussed there; therefore here we focus on the technical aspects.

A. Subgame where neither party uses an agent

As explained in the text, in this case party L chooses l to maximize

$$U_L = \frac{f(l,l)}{f(l,l)+f(r,r)}\; V - l\; N$$

taking r as given. The first-order condition is

$$\frac{f(r,r)}{[f(l,l)+f(r,r)]^2}\; [\, f_c(l,l)+f_s(l,l)\,]\; V = N$$

or

$$\frac{f(l,l)\; f(r,r)}{[f(l,l)+f(r,r)]^2}\; \frac{f_c(l,l)+f_s(l,l)}{f(l,l)}\; V = N$$

In symmetric equilibrium this becomes

$$\frac{1}{4}\; \frac{f_c(l,l)+f_s(l,l)}{f(l,l)}\; V = N$$

Using the no-agent Cobb-Douglas form of f in (3), then multiplying both sides by l and using Euler's Theorem gives

$$\frac{1}{4}\theta_p\; V = l\, N = I_L$$

Similarly for party R. Then, with the victory probabilities of $\frac{1}{2}$ each in the symmetric equilibrium, the parties' objective function values are

$$U_n = \tfrac{1}{2}\, V - \tfrac{1}{4}\, \theta\, V = \tfrac{1}{2}\, \left[1 - \tfrac{1}{2}\, \theta\right]\, V\,, \tag{A.1}$$

where the subscript n on the utility indicates that neither party is using an agent.

B. Subgame where both parties use agents

Recall that we have a two-stage game: at the first stage the party leaders who choose the budgets and bonuses (I_L, B_L), (I_R, B_R), and at the second stage the agents choose the allocations (l_c, l_s), (r_c, r_s). We look for the symmetric subgame perfect equilibrium.

The L agent maximizes A_L defined in (5), subject to the budget constraint

$$l_c\, N_c + l_s\, N_s = I_L$$

We are assuming that the party keeps the agent's budget down to a level where he cannot steal directly, or gets no utility from such cash stealing. Then the first-order conditions are

$$\begin{aligned} \frac{f(r_c, r_s)}{[f(l_c, l_s) + f(r_c, r_s)]^2}\; f_c(l_c, l_s)\; B_L + \beta\; N_c &= \lambda\; N_c \\ \frac{f(r_c, r_s)}{[f(l_c, l_s) + f(r_c, r_s)]^2}\; f_s(l_c, l_s)\; B_L &= \lambda\; N_s \end{aligned}$$

where λ is the Lagrange multiplier.

Divide the first of these equations by N_c, the second by N_s, and subtract to eliminate λ:

$$\frac{f(r_c, r_s)}{[f(l_c, l_s) + f(r_c, r_s)]^2}\; \left[\; \frac{f_c(l_c, l_s)}{N_c} - \frac{f_s(l_c, l_s)}{N_s} \;\right]\; B_L + \beta = 0 \tag{B.1}$$

Therefore

$$\frac{f_c(l_c, l_s)}{N_c} - \frac{f_s(l_c, l_s)}{N_s} < 0, \quad \text{or} \quad \frac{f_c(l_c, l_s)}{f_s(l_c, l_s)} < \frac{N_c}{N_s} \tag{B.2}$$

To get further results, write (B.1) as

$$\frac{f(l_c,l_s)\ f(r_c,r_s)}{[f(l_c,l_s)+f(r_c,r_s)]^2}\ \left[\ \frac{l_c\ f_c(l_c,l_s)}{f(l_c,l_s)}\ \frac{1}{l_c\,N_c} - \frac{l_s\ f_s(l_c,l_s)}{f(l_c,l_s)}\ \frac{1}{l_s\,N_s}\ \right]\ B_L + \beta = 0$$

Using the Cobb-Douglas form (2), this becomes

$$\pi_L\ \pi_R\ \theta_a\ \left[\ \frac{\alpha}{l_c\ N_c} - \frac{1-\alpha}{l_s\ N_s}\ \right]\ B_L + \beta = 0$$

Define $z_l = l_c\ N_c/I_L$, that is, the fraction of the budget spent on core supporters. Then the conditions simplifies to

$$\frac{z_l - \alpha}{z_l\ (1-z_l)} = \frac{\beta}{\theta_a}\ \frac{1}{\pi_L\ \pi_R}\ \frac{I_L}{B_L} \tag{B.3}$$

A similar equation governs the R agent's allocation.

Calculating (B.2) for the Cobb-Douglas case, we see that

$$\frac{\alpha\ l_s}{(1-\alpha)\ l_c} < \frac{N_c}{N_s}, \quad \text{or} \quad \frac{\alpha}{1-\alpha} < \frac{l_c\,N_c}{l_s\,N_s} = \frac{z_l}{1-z_l}, \quad \text{so} \quad z_l > \alpha\,.$$

This is also consistent with (B.3).

Consider small changes around equilibrium. The logarithmic differential of the left hand side (omitting l subscripts because a similar equation is valid with r subscripts also) is

$$\begin{aligned}
\left[\ \frac{1}{z-\alpha} - \frac{1}{z} + \frac{1}{1-z}\ \right]\ dz &= \frac{z(1-z) - (z-\alpha)(1-z) + z(z-\alpha)}{z(1-z)(z-\alpha)}\ dz \\
&= \frac{z - z^2 - z + z^2 + \alpha - \alpha\,z + z^2 - \alpha\,z}{z(1-z)(z-\alpha)}\ dz \\
&= \frac{z^2 - 2\,\alpha\,z + \alpha}{z(1-z)(z-\alpha)}\ dz \\
&= \frac{(z-\alpha)^2 + \alpha(1-\alpha)}{z(1-z)(z-\alpha)}\ dz \\
&= \frac{(z-\alpha)^2 + \alpha(1-\alpha)}{(z-\alpha)^2}\ \frac{z-\alpha}{z(1-z)}\ dz
\end{aligned}$$

Define

$$\Omega = \frac{(z-\alpha)^2}{(z-\alpha)^2 + \alpha(1-\alpha)} \tag{B.4}$$

Using this and (B.3), we have

$$\left[\frac{1}{z-\alpha} - \frac{1}{z} + \frac{1}{1-z} \right] dz = \frac{1}{\Omega} \, \frac{\beta}{\theta} \, \frac{1}{\pi_L \, \pi_R} \, \frac{I}{B} \, dz \tag{B.5}$$

If $z = \alpha$ (the party leaders' ideal), $\Omega = 0$, and as z increases to 1, Ω increases to $(1-\alpha)$. We can then regard the magnitude of Ω in this range as an indicator of the magnitude of the agency problem. Of course Ω is endogenous and determined by the party leaders' choices of I and B. This will emerge as a part of the solution below.

The logarithmic differential of $\pi_L \, \pi_R$ is

$$\begin{aligned} \frac{d(\pi_L \, \pi_R)}{\pi_L \, \pi_R} = \frac{d\pi_L}{\pi_L} + \frac{d\pi_R}{\pi_R} \quad &= \quad \frac{d\pi_L}{\pi_L} - \frac{d\pi_L}{1-\pi_L} \\ &= \quad \frac{1 - 2\,\pi_L}{\pi_L\,(1-\pi_L)} \, d\pi_L \end{aligned} \tag{B.6}$$

which vanishes at a symmetric equilibrium where $\pi_L = \frac{1}{2}$.

This property simplifies the algebra of the first-stage calculation. In principle, the first-stage choices (I_L, B_L), (I_R, B_R) of the leaders of both parties will affect the second-stage choices (l_c, l_s), (r_c, r_s) of both agents. The party leaders' first stage choices will look ahead to this in the subgame perfect equilibrium. But as (B.3) shows, the R-party leaders' choice affects z_l only via π_R (and of course $\pi_L = 1 - \pi_R$). But (B.6) shows that this effect fortunately vanishes at the symmetric equilibrium.

Therefore the comparative statics of the agent's choice at the symmetric equilibrium (again omitting l subscripts) are given by the effects only of the budget and bonus set by that party's leaders:

$$\frac{1}{\Omega} \, \frac{\beta}{\theta_a} \, \frac{1}{\pi_L \, \pi_R} \, \frac{I_L}{B_L} \, dz_l = \frac{dI_L}{I_L} - \frac{dB_L}{B_L} \,, \tag{B.7}$$

and similarly for dz_r.

Now consider the first-stage symmetric equilibrium of the party leaders' choices. Start with

$$
\begin{aligned}
\frac{\pi_L}{1-\pi_L} &= \frac{f(l_c, l_s)}{f(r_c, r_s)} = \frac{A_a \; l_c^{\theta_a \, \alpha} \; l_s^{\theta_a (1-\alpha)}}{A_a \; r_c^{\theta_a \, \alpha} \; r_s^{\theta_a (1-\alpha)}} \\
&= \frac{l_c^{\theta_a \, \alpha} \; l_s^{\theta_a (1-\alpha)}}{r_c^{\theta_a \, \alpha} \; r_s^{\theta_a (1-\alpha)}} \quad \text{observe how } A_a \text{ cancels} \\
&= \frac{z_l^{\theta_a \, \alpha} \; (1-z_l)^{\theta_a (1-\alpha)} \; {I_L}^{\theta_a}}{N_c^{\theta_a \, \alpha} \; N_s^{\theta_a (1-\alpha)}} \; \frac{1}{r_c^{\theta_a \, \alpha} \; r_s^{\theta_a (1-\alpha)}} \qquad \text{(B.8)}
\end{aligned}
$$

Party L's leaders choose their (I_L, B_L) taking the other party leaders' choice of (I_R, B_R) and therefore the R-party agent's choice of (r_c, r_s) as given, because those have zero first-order effect on π_L as seen above. Logarithmic differentiation gives

$$
\frac{d\pi_L}{\pi_L} + \frac{d\pi_L}{1-\pi_L} = \theta_a \, \alpha \; \frac{dz_l}{z_l} - \theta_a \, (1-\alpha) \; \frac{dz_l}{1-z_l} + \theta_a \; \frac{dI_L}{I_L}
$$

or

$$
\begin{aligned}
\frac{d\pi_L}{\pi_L \, \pi_R} &= \theta_a \; \left[\frac{\alpha}{z_l} - \frac{1-\alpha}{1-z_l} \right] \; dz_l + \theta_a \; \frac{dI_L}{I_L} \\
&= -\,\theta_a \; \frac{z_l - \alpha}{z_l \, (1-z_l)} \; dz_l + \theta_a \; \frac{dI_L}{I_L} \qquad \text{(B.9)} \\
&= -\,\theta_a \; \frac{\beta}{\theta_a} \; \frac{1}{\pi_L \; \pi_R} \; \frac{I_L}{B_L} + \theta_a \; \frac{dI_L}{I_L} \qquad \text{using (B.3)} \\
&= -\,\theta_a \, \Omega_L \; \left[\frac{dI_L}{I_L} - \frac{dB_L}{B_L} \right] + \theta_a \; \frac{dI_L}{I_L} \qquad \text{using (B.7) for party } L \\
&= \theta_a \; \left[\, (1-\Omega_L) \; \frac{dI_L}{I_L} + \Omega_L \; \frac{dB_L}{B_L} \, \right] \qquad \text{(B.10)}
\end{aligned}
$$

The line (B.9) in this calculation illustrates another aspect of the agency distortion: an increase in z_l when it is already above α reduces π_l and therefore goes against the party leaders' interest. But there is also the beneficial direct effect of an increase in I_L. When everything is added together, the final result (B.10) shows that the net effect of a larger budget is beneficial for the victory probability.

Now we can calculate the effects of variations in (I_L, B_L) around the symmetric equilibrium on the objective function (4) of L-party leaders.

$$
\begin{aligned}
dU_L &= (V - B_L)\; d\pi_L - \pi_L\; dB_L - dI_L \\
&= (V - B_L)\; \pi_L\, \pi_R\; \theta_a \left[(1 - \Omega_L)\; \frac{dI_L}{I_L} + \Omega_L\; \frac{dB_L}{B_L} \right] - \pi_L\; dB_L - dI_L \\
&= [\, (V - B_L)\; \pi_L\, \pi_R\; \theta_a\, (1 - \Omega_L) - I_L\,]\; \frac{dI_L}{I_L} + [\, (V - B_L)\; \pi_L\, \pi_R\; \theta_a\, \Omega_L - \pi_L\; B_L\,]\; \frac{dB_L}{B_L}
\end{aligned}
$$

Therefore the first-order conditions for the optimum choice of (I_L, B_L) are

$$
\begin{aligned}
(V - B_L)\; \pi_L\, \pi_R\; \theta_a\, (1 - \Omega_L) &= I_L \\
(V - B_L)\; \pi_L\, \pi_R\; \theta_a\, \Omega_L &= \pi_L\; B_L
\end{aligned}
$$

or, using $\pi_L = \pi_R = \frac{1}{2}$, and dropping subscripts since the same condition holds for both parties,

$$(V - B)\; \theta_a\, (1 - \Omega) = 4\; I \tag{B.11}$$

$$(V - B)\; \theta_a\, \Omega = 2\; B \tag{B.12}$$

Divide these to write

$$\frac{\Omega}{1 - \Omega} = \frac{1}{2}\; \frac{B}{I} \tag{B.13}$$

or

$$\frac{(z - \alpha)^2}{\alpha\, (1 - \alpha)} = \frac{1}{2}\; \frac{B}{I} \tag{B.14}$$

We know from (B.3) and (B.7) that z is an increasing function of I/B, and $z > \alpha$; therefore the left hand side of (B.14) increases as I/B increases. The right hand side decreases as I/B increases, and spans the whole range from ∞ to 0. Therefore this equation yields a unique solution for I/B. Then z and Ω can be calculated.

Next, (B.12) gives

$$B = \frac{\theta_a\,\Omega}{2+\theta_a\,\Omega}\;V \tag{B.15}$$

This completes the solution. Note that $B < V$, and the ratio B/V is higher when θ_a is higher (the agent has higher marginal productivity) and when Ω is higher (when the agency problem is more severe).

Finally, using (B.13), we get the size of each party's budget assigned to its agent transfers to the electorate:

$$I = \frac{1}{2}\,\frac{1-\Omega}{\Omega}\;B = \tfrac{1}{2}\;\tfrac{\theta_a\,(1-\Omega)}{2+\theta_a\,\Omega}\;V\,.$$

Therefore each party's utility in equilibrium is

$$U_b = \tfrac{1}{2}\,(V-B) - I = \tfrac{1}{2}\;\left[1 - \frac{\theta_a}{2+\theta_a\,\Omega}\right]\;V\,, \tag{B.16}$$

where the subscript b on the utility indicates that both parties are using agents.

Now we can compare utilities in the equilibria of the subgames where neither party is using an agent and where both are using agents. From (**??**) and (B.16), we have

$$U_b - U_n = \frac{\theta_a\,\theta_p\,\Omega - 2\,(\theta_a - \theta_p)}{4\,(2+\theta_a\,\Omega)}\;V\,.$$

In the limiting case where $\theta_a = \theta_p$, this is positive. If the equilibrium of the full game is one where both parties use agents, it cannot be a prisoner's dilemma. But if θ_a is sufficiently greater than θ_p, such a dilemma is possible. In the text we discuss this in the context of numerical results and historical applications.

C. Subgame where only party L has an agent

Here we have a two-stage game. At the first stage, party L chooses the budget I_L and bonus B_L for its agent while party R chooses its uniform per capita transfer amount r. In the second stage, L's agent chooses the targeted transfers l_c and l_s. As usual this is solved by backward induction, starting with the second-stage decision problem given (I_L, B_L) and r.

The agent wants to maximize A_L subject to the given budget I_L. This is the same problem as in Appendix C, and leads to the same condition (B.3), which I rewrite as

$$\pi_L \ (1 - \pi_L) \ \frac{z_l - \alpha}{z_l \, (1 - z_l)} = \frac{\beta}{\theta_a} \ \frac{I_L}{B_L} \, , \tag{C.1}$$

where $z_l = l_c \, N_c / I_L$ is the fraction of the budget the agent allocates to the core supporters.

Also, the same calculation that led to (B.8), but now remembering $r_c = r_s = r$, yields

$$\begin{aligned} \frac{\pi_L}{1 - \pi_L} &= \frac{f(l_c, l_s)}{f(r_c, r_s)} = \frac{A_a \, l_c^{\theta_a \, \alpha} \ l_s^{\theta_a (1-\alpha)}}{A_p \, r^{\theta_p}} \\ &= \frac{A_a}{A_p} \ \frac{z_l^{\theta_a \, \alpha} \ (1 - z_l)^{\theta_a (1-\alpha)} \ I_L^{\theta_a}}{N_c^{\theta_a \, \alpha} \ N_s^{\theta_a (1-\alpha)}} \ \frac{1}{r^{\theta_p}} \end{aligned} \tag{C.2}$$

These two equations define z_l and π_L as functions of (I_L, B_L) and r.

Consider how z_l and π_L change as (I_L, B_L) and r change. Logarithmic differentiation of (C.1) yields

$$\frac{d\pi_L}{\pi_L} - \frac{d\pi_L}{1 - \pi_L} + \left[\frac{1}{z_l - \alpha} - \frac{1}{z_l} + \frac{1}{1 - z_l} \right] \ dz_l = \frac{dI_L}{I_L} - \frac{dB_L}{B_L} \, ,$$

or, using (B.5), which remains valid because the L agent's optimality conditions thus far are the same,

$$\frac{1-2\,\pi_L}{\pi_L\,(1-\pi_L)}\;d\pi_l+\frac{1}{\Omega}\,\frac{\beta}{\theta_a}\,\frac{1}{\pi_L\,(1-\pi_L)}\,\frac{I_L}{B_L}\,dz_l=\frac{dI_L}{I_L}-\frac{dB_L}{B_L}\,.$$

This simplifies to

$$(1-2\,\pi_L)\;d\pi_l+\frac{1}{\Omega}\,\frac{\beta}{\theta_a}\,\frac{I_L}{B_L}\,dz_l=\pi_L\,\pi_R\;\left[\,\frac{dI_L}{I_L}-\frac{dB_L}{B_L}\,\right]\,. \tag{C.3}$$

Next, logarithmic differentiation of (C.2) yields

$$\frac{d\pi_L}{\pi_L}+\frac{d\pi_L}{1-\pi_L}=\theta_a\;\frac{dI_L}{I_L}+\theta_a\;\left[\,\alpha\,\frac{dz_l}{z_l}-(1-\alpha)\,\frac{dz_l}{1-z_l}\,\right]-\theta_p\,\frac{dr}{r}\,,$$

or

$$\frac{1}{\pi_L\,(1-\pi_L)}\;d\pi_L=\theta_a\;\frac{dI_L}{I_L}-\theta_a\,\frac{z_l-\alpha}{z_l\,(1-z_l)}\,dz_l-\theta_p\,\frac{dr}{r}\,,$$

or, using (C.1),

$$\frac{1}{\pi_L\,(1-\pi_L)}\;d\pi_L=\theta_a\;\frac{dI_L}{I_L}-\frac{\beta}{\pi_L\,(1-\pi_L)}\,\frac{I_L}{B_L}\,dz_l-\theta_p\,\frac{dr}{r}\,.$$

This simplifies to

$$d\pi_L+\beta\,\frac{I_L}{B_L}\,dz_l=\pi_L\,\pi_R\;\left[\,\theta_a\;\frac{dI_L}{I_L}-\theta_p\frac{dr}{r}\,\right] \tag{C.4}$$

The two comparative statics equations (C.3) and (C.4) can be solved for dz_l and $d\pi_L$ to get

$$dz_l \;=\; \frac{1}{\Delta}\,\frac{\pi_L\,\pi_R}{\beta}\,\frac{B_L}{I_L}\;\left\{\,[1+\theta_a\,(2\pi_L-1)]\,\frac{dI_L}{I_L}-\frac{dB_L}{B_L}-\theta_p\,(2\pi_L-1)\,\frac{dr}{r}\,\right\} \tag{C.5}$$

$$d\pi_L \;=\; \frac{1}{\Delta}\,\pi_L\,\pi_R\;\left\{\,\frac{1-\Omega}{\Omega}\,\frac{dI_L}{I_L}+\frac{dB_L}{B_L}-\frac{\theta_p/\theta_a}{\Omega}\,\frac{dr}{r}\,\right\} \tag{C.6}$$

where (C.4):

$$\Delta = \frac{1}{\theta\,\Omega} + 2\,\pi_L - 1\,. \tag{C.7}$$

If $\pi_L > \frac{1}{2}$, which in turn ensures $\Delta > 0$, all comparative static effects have the intuitive signs. (1) An increase in I_L increases z_L, the fraction the agent spends on core supporters: the more relaxed budget enables him to indulge more in his preference. (2) An increase in B_L decreases z_l: the incentive works to align the agent's choice more closely with the party leaders' preferred level $z_l = \alpha$. (3) An increase in r decreases z_L: greater pressure of competition from the other party's transfers forces the agent to reduce his spending to indulge his own preference for a larger core club. (4) An increase in I_L increases π_L: worsening of the agent's moral hazard (higher z_l) is not so severe as the reduce the party's probability of victory. (5) An increase in B_L increases π_L and an increase in r reduces π_L: these are obvious.

The property $\pi_L > \frac{1}{2}$ is intuitively appealing: an important reason to employ the agent is to use his ability to make transfers with better targeting and higher productivity, which should increase the probability of winning. But the general theory does not allow us to prove this definitively. We will examine the issue using numerical solutions.

The comparative static results for stage 2 are needed for analyzing the stage 1 Nash game between the party leaders. The L leaders choose (I_L, B_L) for given r to maximize

$$U_L = \pi_L\,(V - B_L) - I_L\,,$$

and the R leaders choose r for given (I_L, B_L) to maximize

$$U_R = (1 - \pi_L)\,V - r\,N\,.$$

We can use the comparative statics results of (C.6) to find the parties' calculation of effects of changes in their strategies (I_L, B_L) and r respectively, taking into account the L agent's response at the second stage. We have total differentials of the objective functions:

$$\begin{aligned} dU_L &= (V - B_L)\ d\pi_L - \pi_L\ dB_L - dI_L \\ &= (V - B_L)\ \frac{\pi_L\, \pi_R}{\Delta}\ \left\{ \frac{1-\Omega}{\Omega}\, \frac{dI_L}{I_L} + \frac{dB_L}{B_L} \right\} - \pi_L\ dB_L - dI_L \end{aligned}$$

and

$$\begin{aligned} dU_R &= -V\ d\pi_L - N\, dr \\ &= V\ \frac{\pi_L\, \pi_R}{\Delta\, \Omega}\, \frac{\theta_p}{\theta_a}\ \frac{dr}{r} - N\, dr \end{aligned}$$

Note the absence of dr in the expression for dU_L and of (dI_L, dB_L) in the expression for dU_R, reflecting the Nash noncooperative assumption where each party takes the other's strategy as given.

Now party L's first-order conditions can be found by setting the coefficients of dI_L and dB_L separately equal to zero in the expression for dU_L:

$$(V - B_L)\ \frac{\pi_L\, \pi_R}{\Delta}\ \frac{1-\Omega}{\Omega}\ \frac{1}{I_L} - 1 \;=\; 0\,, \tag{C.8}$$

$$(V - B_L)\ \frac{\pi_L\, \pi_R}{\Delta}\ \frac{1}{B_L} - \pi_L \;=\; 0\,. \tag{C.9}$$

The R party's first-order condition is found by setting the coefficient of dr equal to zero in the expression for dU_R:

$$V\ \frac{\pi_L\, \pi_R}{\Delta\, \Omega}\ \frac{\theta_p}{\theta_a}\ \frac{1}{r} - N = 0\,. \tag{C.10}$$

The complete solution for the two stages together – for all five endogenous variables I_L, B_L, r, z_l and π_L – is then implicitly defined by the five equations (C.1), (C.2), (C.8), (C.9) and (C.10). No general inferences can be drawn from the algebra, so we resort to numerical solution.

D. Deriving θ from a Contest success function

From the text recall that Skaperdas shows in his Theorem 2 that the only form satisfying certain desirable axioms is that when players 1 and 2 expend scalar efforts x_1 and x_2 respectively, the probability of winning for the first player should take the form

$$\pi_1 = \frac{x_1^\theta}{x_1^\theta + x_2^\theta}\,,$$

and of course $\pi_2 = 1 - \pi_1$ is the probability that player 2 wins.* The parameter θ captures the marginal (incremental) returns to expending effort.

This is more easily understood by considering the odds ratio

$$\frac{\pi_1}{\pi_2} = \left(\frac{x_1}{x_2}\right)^\theta.$$

Taking logarithms of both sides and differentiating,

$$\frac{d\ \ln(\pi_1/\pi_2)}{d\ \ln(x_1/x_2)} = \theta\,.$$

Thus θ is the elasticity of the odds ratio with respect to the effort ratio: increasing x_1 by 1% relative to x_2 will shift the odds ratio by θ% in player 1's favor. Second-order conditions of maximization impose limits on θ; for our purpose $\theta \leq 1$ will suffice.

Numerical Appendix

The two tables below provide more information about some of the equilibria that figure (1) depicts. The tables contain all of the endogenous outcomes of the model, the values of θ_a and V, and the four possible payoffs for each party. Table (1) contains the endogenous outcomes of the model for both the case when only one party employs an agent

$*$. Skaperdas 1996.

and when both parties employ an agent. Table (2) contains the payoffs for the parties for all of the subgames in the model.

Table 1: Equilibria Outcomes for $\beta = 0.5$

V	θ_a-θ_p	$B_L 1$	$I_L 1$	$l_c 1$	$r1$	$\pi_L 1$	$B_L 2$	$I_L 2$	$l_c 2$
100	0.8	6.526	2.731	3.579	0.974	0.837	17.718	9.655	9.35
100	0.6	7.333	2.926	3.809	1.338	0.784	15.501	7.037	7.298
100	0.4	7.996	2.829	3.699	1.838	0.703	12.721	4.55	5.172
100	0.2	7.621	2.005	2.749	2.339	0.587	9.054	2.294	2.99
100	0.	3.988	0.517	0.87	2.503	0.473	3.796	0.507	0.862
80	0.8	5.569	2.405	3.094	0.836	0.825	14.174	7.724	7.48
80	0.6	6.194	2.526	3.237	1.131	0.771	12.401	5.629	5.838
80	0.4	6.655	2.377	3.072	1.52	0.69	10.177	3.64	4.137
80	0.2	6.221	1.634	2.227	1.89	0.578	7.243	1.835	2.392
80	0.	3.19	0.414	0.696	2.003	0.473	3.037	0.406	0.69
60	0.8	4.538	2.039	2.561	0.686	0.808	10.631	5.793	5.61
60	0.6	4.979	2.086	2.619	0.909	0.753	9.301	4.222	4.379
60	0.4	5.25	1.895	2.413	1.189	0.672	7.632	2.73	3.103
60	0.2	4.787	1.253	1.695	1.435	0.566	5.433	1.376	1.794
60	0.	2.393	0.31	0.522	1.502	0.473	2.278	0.304	0.517
40	0.8	3.399	1.61	1.954	0.518	0.782	7.087	3.862	3.74
40	0.6	3.658	1.585	1.934	0.666	0.725	6.2	2.815	2.919
40	0.4	3.754	1.371	1.709	0.837	0.646	5.088	1.82	2.069
40	0.2	3.307	0.86	1.152	0.972	0.548	3.622	0.918	1.196
40	0.	1.595	0.207	0.348	1.001	0.473	1.519	0.203	0.345
20	0.8	2.07	1.061	1.214	0.318	0.729	3.544	1.931	1.87
20	0.6	2.152	0.976	1.134	0.387	0.67	3.1	1.407	1.46
20	0.4	2.108	0.776	0.935	0.454	0.596	2.544	0.91	1.034
20	0.2	1.753	0.449	0.591	0.496	0.518	1.811	0.459	0.598
20	0.	0.798	0.103	0.174	0.501	0.473	0.759	0.101	0.172
4	0.8	0.646	0.359	0.358	0.095	0.551	0.709	0.386	0.374
4	0.6	0.615	0.28	0.291	0.1	0.504	0.62	0.281	0.292
4	0.4	0.54	0.188	0.21	0.102	0.464	0.509	0.182	0.207
4	0.2	0.398	0.096	0.122	0.101	0.444	0.362	0.092	0.12
4	0.	0.16	0.021	0.035	0.1	0.473	0.152	0.02	0.034

The number after the outcome variables indicates the number of agents.

Table 2: Party Utilities for $\beta = 0.5$

V	θ_a-θ_p	1 No Agent	1 Agent	No Agent	2 Agent
100	0.8	15.365	75.47	47.5	31.486
100	0.6	20.22	69.764	47.5	35.213
100	0.4	27.822	61.888	47.5	39.09
100	0.2	38.92	52.259	47.5	43.179
100	0	50.198	44.895	47.5	47.595
80	0.8	13.186	58.98	38.	25.189
80	0.6	17.174	54.393	38.	28.17
80	0.4	23.272	48.238	38.	31.272
80	0.2	31.865	41.015	38.	34.543
80	0	40.159	35.916	38.	38.076
60	0.8	10.828	42.78	28.5	18.892
60	0.6	13.911	39.344	28.5	21.128
60	0.4	18.478	34.909	28.5	23.454
60	0.2	24.614	29.989	28.5	25.907
60	0	30.119	26.937	28.5	28.557
40	0.8	8.204	27.01	19.	12.595
40	0.6	10.334	24.762	19.	14.085
40	0.4	13.339	22.03	19.	15.636
40	0.2	17.094	19.261	19.	17.272
40	0	20.079	17.958	19.	19.038
20	0.8	5.107	12.005	9.5	6.297
20	0.6	6.21	10.985	9.5	7.043
20	0.4	7.621	9.892	9.5	7.818
20	0.2	9.149	8.998	9.5	8.636
20	0	10.04	8.979	9.5	9.519
4	0.8	1.702	1.489	1.9	1.259
4	0.6	1.883	1.427	1.9	1.409
4	0.4	2.041	1.419	1.9	1.564
4	0.2	2.121	1.505	1.9	1.727
4	0	2.008	1.796	1.9	1.904

“1” indicates the payoff is when 1 party uses an agent.
“No Agent” indicates that the payoff is for the party that is not using an agent.