Mathematical Appendix

Here we present details of mathematical derivations of the results presented in the text. The broad ideas and intuitions are discussed there; therefore here we focus on the technical aspects.

A. Subgame where neither party uses an agent

As explained in the text, in this case party $L$ chooses $l$ to maximize

$$U_L = \frac{f(l, l)}{f(l, l) + f(r, r)} V - l N$$

taking $r$ as given. The first-order condition is

$$\frac{f(r, r)}{[f(l, l) + f(r, r)]^2} \left[ f_c(l, l) + f_s(l, l) \right] V = N$$

or

$$\frac{f(l, l) f(r, r)}{[f(l, l) + f(r, r)]^2} \frac{f_c(l, l) + f_s(l, l)}{f(l, l)} V = N$$

In symmetric equilibrium this becomes

$$\frac{1}{4} \frac{f_c(l, l) + f_s(l, l)}{f(l, l)} V = N$$

Using the no-agent Cobb-Douglas form of $f$ in (3), then multiplying both sides by $l$ and using Euler’s Theorem gives

$$\frac{1}{4} \theta_p V = l N = I_L$$

Similarly for party $R$. Then, with the victory probabilities of $\frac{1}{2}$ each in the symmetric
equilibrium, the parties’ objective function values are

\[ U_n = \frac{1}{2} V - \frac{1}{4} \theta V = \frac{1}{2} \left[ 1 - \frac{1}{2} \theta \right] V, \]  

(A.1)

where the subscript \( n \) on the utility indicates that neither party is using an agent.

**B. Subgame where both parties use agents**

Recall that we have a two-stage game: at the first stage the party leaders who choose the budgets and bonuses \((I_L, B_L), (I_R, B_R)\), and at the second stage the agents choose the allocations \((l_c, l_s), (r_c, r_s)\). We look for the symmetric subgame perfect equilibrium.

The \( L \) agent maximizes \( A_L \) defined in (5), subject to the budget constraint

\[ l_c N_c + l_s N_s = I_L \]

We are assuming that the party keeps the agent’s budget down to a level where he cannot steal directly, or gets no utility from such cash stealing. Then the first-order conditions are

\[
\frac{f(r_c, r_s)}{[f(l_c, l_s) + f(r_c, r_s)]^2} f_c(l_c, l_s) B_L + \beta N_c = \lambda N_c
\]

\[
\frac{f(r_c, r_s)}{[f(l_c, l_s) + f(r_c, r_s)]^2} f_s(l_c, l_s) B_L = \lambda N_s
\]

where \( \lambda \) is the Lagrange multiplier.

Divide the first of these equations by \( N_c \), the second by \( N_s \), and subtract to eliminate \( \lambda \):

\[
\frac{f(r_c, r_s)}{[f(l_c, l_s) + f(r_c, r_s)]^2} \left[ \frac{f_c(l_c, l_s)}{N_c} - \frac{f_s(l_c, l_s)}{N_s} \right] B_L + \beta = 0 \quad (B.1)
\]

Therefore

\[
\frac{f_c(l_c, l_s)}{N_c} - \frac{f_s(l_c, l_s)}{N_s} < 0, \quad \text{or} \quad \frac{f_c(l_c, l_s)}{f_s(l_c, l_s)} < \frac{N_c}{N_s} \quad (B.2)
\]
To get further results, write (B.1) as

\[
\frac{f(l_c, l_s) f(r_c, r_s)}{[f(l_c, l_s) + f(r_c, r_s)]^2} \left[ \frac{l_c f_c(l_c, l_s)}{f(l_c, l_s)} \frac{1}{l_c N_c} - \frac{l_s f_s(l_c, l_s)}{f(l_c, l_s)} \frac{1}{l_s N_s} \right] B_L + \beta = 0
\]

Using the Cobb-Douglas form (2), this becomes

\[
\pi_L \pi_R \theta_a \left[ \left\{ \frac{\alpha}{l_c N_c} - \frac{1 - \alpha}{l_s N_s} \right\} \frac{1}{l_c N_c} \right] B_L + \beta = 0
\]

Define \( z_l = \frac{l_c N_c}{I_L} \), that is, the fraction of the budget spent on core supporters. Then the conditions simplifies to

\[
\frac{z_l - \alpha}{z_l (1 - z_l)} = \frac{\beta}{\theta a} \frac{1}{\pi_L \pi_R} \frac{I_L}{B_L} \quad \text{(B.3)}
\]

A similar equation governs the \( R \) agent’s allocation.

Calculating (B.2) for the Cobb-Douglas case, we see that

\[
\frac{\alpha l_s}{(1 - \alpha) l_c} < \frac{N_c}{N_s}, \quad \text{or} \quad \frac{\alpha}{1 - \alpha} < \frac{l_c N_c}{l_s N_s} = \frac{z_l}{1 - z_l}, \quad \text{so} \quad z_l > \alpha.
\]

This is also consistent with (B.3).

Consider small changes around equilibrium. The logarithmic differential of the left hand side (omitting \( l \) subscripts because a similar equation is valid with \( r \) subscripts also) is

\[
\left[ \frac{1}{z - \alpha} - \frac{1}{z} + \frac{1}{1 - z} \right] dz = \frac{z(1 - z) - (z - \alpha)(1 - z) + z(z - \alpha)}{z(1 - z)(z - \alpha)} dz = \frac{z - z^2 - z + z^2 + \alpha z + z^2 - \alpha z}{z(1 - z)(z - \alpha)} dz = \frac{z^2 - 2 \alpha z + \alpha}{z(1 - z)(z - \alpha)} dz = (z - \alpha)^2 + \alpha(1 - \alpha) \frac{z - \alpha}{z(1 - z)} dz
\]
Define
\[ \Omega = \frac{(z - \alpha)^2}{(z - \alpha)^2 + \alpha(1 - \alpha)} \]  \hspace{1cm} (B.4)

Using this and (B.3), we have
\[ \left[ \frac{1}{z - \alpha} - \frac{1}{z} + \frac{1}{1 - z} \right] \, dz = \frac{1}{\Omega} \frac{\beta}{\pi_L \pi_R} \frac{1}{\theta} \frac{I}{B} \, dz \]  \hspace{1cm} (B.5)

If \( z = \alpha \) (the party leaders’ ideal), \( \Omega = 0 \), and as \( z \) increases to 1, \( \Omega \) increases to \((1 - \alpha)\). We can then regard the magnitude of \( \Omega \) in this range as an indicator of the magnitude of the agency problem. Of course \( \Omega \) is endogenous and determined by the party leaders’ choices of \( I \) and \( B \). This will emerge as a part of the solution below.

The logarithmic differential of \( \pi_L \pi_R \) is
\[ \frac{d(\pi_L \pi_R)}{\pi_L \pi_R} = \frac{d\pi_L}{\pi_L} + \frac{d\pi_R}{\pi_R} = \frac{d\pi_L}{\pi_L} - \frac{d\pi_L}{1 - \pi_L} = \frac{1}{\pi_L (1 - \pi_L)} \, d\pi_L \]  \hspace{1cm} (B.6)

which vanishes at a symmetric equilibrium where \( \pi_L = \frac{1}{2} \).

This property simplifies the algebra of the first-stage calculation. In principle, the first-stage choices \( (I_L, B_L), (I_R, B_R) \) of the leaders of both parties will affect the second-stage choices \( (l_c, l_s), (r_c, r_s) \) of both agents. The party leaders’ first stage choices will look ahead to this in the subgame perfect equilibrium. But as (B.3) shows, the \( R \)-party leaders’ choice affects \( z_l \) only via \( \pi_R \) (and of course \( \pi_L = 1 - \pi_R \)). But (B.6) shows that this effect fortunately vanishes at the symmetric equilibrium.

Therefore the comparative statics of the agent’s choice at the symmetric equilibrium (again omitting \( l \) subscripts) are given by the effects only of the budget and bonus set by that party’s leaders:
\[ \frac{1}{\Omega} \frac{\beta}{\theta} \frac{1}{\pi_L \pi_R} \frac{I_L}{B_L} \, d\pi_L = \frac{dI_L}{I_L} - \frac{dB_L}{B_L} , \]  \hspace{1cm} (B.7)

and similarly for \( d\pi_R \).
Now consider the first-stage symmetric equilibrium of the party leaders’ choices. Start with

\[
\frac{\pi_L}{1 - \pi_L} = \frac{f(l_c, l_s)}{f(r_c, r_s)} = \frac{A_a \theta_a \alpha l_s^{\theta_a(1-\alpha)}}{A_a \theta_a \alpha r_s^{\theta_a(1-\alpha)}}
= \frac{r_c^{\theta_a(1-\alpha)}}{r_s^{\theta_a(1-\alpha)}}
\]

observe how \( A_a \) cancels

\[
\frac{z_l^{\theta_a \alpha}}{N_l^{\theta_a \alpha}} \frac{(1 - z_l) I_l^{\theta_a}}{1 - z_l} = \frac{1}{r_c^{\theta_a \alpha}}
\]

Party \( L \)’s leaders choose their \((I_L, B_L)\) taking the other party leaders’ choice of \((I_R, B_R)\) and therefore the \( R \)-party agent’s choice of \((r_c, r_s)\) as given, because those have zero first-order effect on \( \pi_L \) as seen above. Logarithmic differentiation gives

\[
\frac{d\pi_L}{\pi_L} + \frac{d\pi_L}{1 - \pi_L} = \theta_a \alpha \frac{dz_l}{z_l} - \theta_a (1 - \alpha) \frac{dz_l}{1 - z_l} + \theta_a \frac{dI_L}{I_L}
\]

or

\[
\frac{d\pi_L}{\pi_L \pi_R} = \theta_a \left[ \frac{\alpha}{z_l} - \frac{1 - \alpha}{1 - z_l} \right] dz_l + \theta_a \frac{dI_L}{I_L}
= - \theta_a \frac{z_l - \alpha}{z_l (1 - z_l)} dz_l + \theta_a \frac{dI_L}{I_L}
= - \theta_a \frac{\beta}{\theta_a} \frac{1}{\pi_L \pi_R} B_L + \theta_a \frac{dI_L}{I_L}
= - \theta_a \Omega_L \left[ \frac{dI_L}{I_L} - \frac{dB_L}{B_L} \right] + \theta_a \frac{dI_L}{I_L}
= \theta_a \left[ (1 - \Omega_L) \frac{dI_L}{I_L} + \Omega_L \frac{dB_L}{B_L} \right]
\]

The line (B.9) in this calculation illustrates another aspect of the agency distortion: an increase in \( z_l \) when it is already above \( \alpha \) reduces \( \pi_L \) and therefore goes against the party leaders’ interest. But there is also the beneficial direct effect of an increase in \( I_L \). When everything is added together, the final result (B.10) shows that the net effect of a larger budget is beneficial for the victory probability.
Now we can calculate the effects of variations in \((I_L, B_L)\) around the symmetric equilibrium on the objective function (4) of L-party leaders.

\[
dU_L = (V - B_L) \left( \pi_L \pi_R \Omega_a \left( \frac{dI_L}{I_L} + \frac{dB_L}{B_L} \right) \right) - \pi_L dB_L - dI_L
\]

\[
dU_L = [(V - B_L) \pi_L \pi_R \Omega_a (1 - \Omega_L) - I_L] \left( \frac{dI_L}{I_L} + [(V - B_L) \pi_L \pi_R \Omega_a \Omega_L - \pi_L B_L] \right) \frac{dB_L}{B_L}
\]

Therefore the first-order conditions for the optimum choice of \((I_L, B_L)\) are

\[
(V - B_L) \pi_L \pi_R \Omega_a (1 - \Omega_L) = I_L
\]

\[
(V - B_L) \pi_L \pi_R \Omega = \pi_L B_L
\]

or, using \(\pi_L = \pi_R = \frac{1}{2}\), and dropping subscripts since the same condition holds for both parties,

\[
(V - B) \theta_a (1 - \Omega) = 4 I \quad \text{(B.11)}
\]

\[
(V - B) \theta_a \Omega = 2 B \quad \text{(B.12)}
\]

Divide these to write

\[
\frac{\Omega}{1 - \Omega} = \frac{1}{2} \frac{B}{I} \quad \text{(B.13)}
\]

or

\[
\frac{(z - \alpha)^2}{\alpha (1 - \alpha)} = \frac{1}{2} \frac{B}{I} \quad \text{(B.14)}
\]

We know from (B.3) and (B.7) that \(z\) is an increasing function of \(I/B\), and \(z > \alpha\); therefore the left hand side of (B.14) increases as \(I/B\) increases. The right hand side decreases as \(I/B\) increases, and spans the whole range from \(\infty\) to 0. Therefore this equation yields a unique solution for \(I/B\). Then \(z\) and \(\Omega\) can be calculated.
Next, (B.12) gives
\[ B = \frac{\theta_a \Omega}{2 + \theta_a \Omega} V \] \hspace{1cm} (B.15)
This completes the solution. Note that \( B < V \), and the ratio \( B/V \) is higher when \( \theta_a \) is higher (the agent has higher marginal productivity) and when \( \Omega \) is higher (when the agency problem is more severe).

Finally, using (B.13), we get the size of each party’s budget assigned to its agent transfers to the electorate:
\[ I = \frac{1}{2} \frac{1 - \Omega}{\Omega} B = \frac{1}{2} \frac{\theta_a (1 - \Omega)}{2 + \theta_a \Omega} V. \]
Therefore each party’s utility in equilibrium is
\[ U_b = \frac{1}{2} (V - B) - I = \frac{1}{2} \left[ 1 - \frac{\theta_a}{2 + \theta_a \Omega} \right] V, \] \hspace{1cm} (B.16)
where the subscript \( b \) on the utility indicates that both parties are using agents.

Now we can compare utilities in the equilibria of the subgames where neither party is using an agent and where both are using agents. From (??) and (B.16), we have
\[ U_b - U_n = \frac{\theta_a \theta_p \Omega - 2 (\theta_a - \theta_p)}{4 (2 + \theta_a \Omega)} V. \]
In the limiting case where \( \theta_a = \theta_p \), this is positive. If the equilibrium of the full game is one where both parties use agents, it cannot be a prisoner’s dilemma. But if \( \theta_a \) is sufficiently greater than \( \theta_p \), such a dilemma is possible. In the text we discuss this in the context of numerical results and historical applications.
C. Subgame where only party $L$ has an agent

Here we have a two-stage game. At the first stage, party $L$ chooses the budget $I_L$ and bonus $B_L$ for its agent while party $R$ chooses its uniform per capita transfer amount $r$. In the second stage, $L$’s agent chooses the targeted transfers $l_c$ and $l_s$. As usual this is solved by backward induction, starting with the second-stage decision problem given $(I_L, B_L)$ and $r$.

The agent wants to maximize $A_L$ subject to the given budget $I_L$. This is the same problem as in Appendix C, and leads to the same condition (B.3), which I rewrite as

$$
\pi_L \left(1 - \pi_L\right) \frac{z_l - \alpha}{z_l (1 - z_l)} = \frac{\beta}{\theta_a \frac{I_L}{B_L}},
$$

(C.1)

where $z_l = l_c N_c / I_L$ is the fraction of the budget the agent allocates to the core supporters.

Also, the same calculation that led to (B.8), but now remembering $r_c = r_s = r$, yields

$$
\frac{\pi_L}{1 - \pi_L} = \frac{f(l_c, l_s)}{f(r_c, r_s)} = \frac{A_a \theta \alpha l_s^{\theta \alpha(1 - \alpha)}}{A_p r^{\theta \rho}} = \frac{A_a}{A_p} \frac{z_l^{\theta \alpha (1 - z_l)^{\theta \alpha(1 - \alpha)}} I_L^{\theta \alpha}}{N_c^{\theta \alpha} N_s^{\theta \alpha(1 - \alpha)}} \frac{1}{r^{\theta \rho}},
$$

(C.2)

These two equations define $z_l$ and $\pi_L$ as functions of $(I_L, B_L)$ and $r$.

Consider how $z_l$ and $\pi_L$ change as $(I_L, B_L)$ and $r$ change. Logarithmic differentiation of (C.1) yields

$$
\frac{d\pi_L}{\pi_L} - \frac{d\pi_L}{1 - \pi_L} + \left[ \frac{1}{z_l - \alpha} - \frac{1}{z_l} + \frac{1}{1 - z_l} \right] dz_l = \frac{dI_L}{I_L} - \frac{dB_L}{B_L},
$$

or, using (B.5), which remains valid because the L agent’s optimality conditions thus far
are the same,

\[
\frac{1 - 2\pi_L}{\pi_L (1 - \pi_L)} d\pi_I + \frac{1 - \pi_I}{\pi_I (1 - \pi_I)} I_L \frac{1}{B_L} d\pi_I = \frac{dI_L}{I_L} - \frac{dB_L}{B_L}.
\]

This simplifies to

\[
(1 - 2\pi_L) d\pi_I + \frac{1 - \pi_I}{\pi_I (1 - \pi_I)} I_L \frac{1}{B_L} d\pi_I = \pi_L \pi_R \left[ \frac{dI_L}{I_L} - \frac{dB_L}{B_L} \right].
\]  (C.3)

Next, logarithmic differentiation of (C.2) yields

\[
\frac{d\pi_L}{\pi_L} + \frac{d\pi_I}{1 - \pi_I} = \theta_a \frac{dI_L}{I_L} + \theta_a \left[ \alpha \frac{d\pi_I}{z_I} - (1 - \alpha) \frac{d\pi_I}{1 - z_I} \right] - \theta_p \frac{dr}{r},
\]

or

\[
\frac{1}{\pi_L (1 - \pi_L)} d\pi_L = \theta_a \frac{dI_L}{I_L} - \theta_a \frac{z_I - \alpha}{z_I (1 - z_I)} d\pi_I - \theta_p \frac{dr}{r},
\]

or, using (C.1),

\[
\frac{1}{\pi_L (1 - \pi_L)} d\pi_L = \theta_a \frac{dI_L}{I_L} - \frac{\beta}{\pi_L (1 - \pi_L)} I_L \frac{1}{B_L} d\pi_I - \theta_p \frac{dr}{r}.
\]

This simplifies to

\[
d\pi_L + \beta \frac{I_L}{B_L} d\pi_I = \pi_L \pi_R \left[ \theta_a \frac{dI_L}{I_L} - \theta_p \frac{dr}{r} \right].
\]  (C.4)

The two comparative statics equations (C.3) and (C.4) can be solved for \(dz_I\) and \(d\pi_L\) to get

\[
dz_I = \frac{1}{\Delta} \frac{\pi_L \pi_R}{\beta} \frac{B_L}{I_L} \left\{ \left[ 1 + \theta_a (2\pi_L - 1) \right] \frac{dI_L}{I_L} - \frac{dB_L}{B_L} - \theta_p (2\pi_L - 1) \frac{dr}{r} \right\} \]  (C.5)

\[
d\pi_L = \frac{1}{\Delta} \pi_L \pi_R \left\{ \frac{1 - \Omega}{\Omega} \frac{dI_L}{I_L} + \frac{dB_L}{B_L} - \theta_p / \theta_a \frac{dr}{r} \right\} \]  (C.6)
where (C.4):
\[ \Delta = \frac{1}{\theta \Omega} + 2 \pi_L - 1. \] (C.7)

If \( \pi_L > \frac{1}{2} \), which in turn ensures \( \Delta > 0 \), all comparative static effects have the intuitive signs. (1) An increase in \( I_L \) increases \( z_L \), the fraction the agent spends on core supporters: the more relaxed budget enables him to indulge more in his preference. (2) An increase in \( B_L \) decreases \( z_l \): the incentive works to align the agent’s choice more closely with the party leaders’ preferred level \( z_l = \alpha \). (3) An increase in \( r \) decreases \( z_L \): greater pressure of competition from the other party’s transfers forces the agent to reduce his spending to indulge his own preference for a larger core club. (4) An increase in \( I_L \) increases \( \pi_L \): worsening of the agent’s moral hazard (higher \( z_l \)) is not so severe as the reduce the party’s probability of victory. (5) An increase in \( B_L \) increases \( \pi_L \) and an increase in \( r \) reduces \( \pi_L \): these are obvious.

The property \( \pi_L > \frac{1}{2} \) is intuitively appealing: an important reason to employ the agent is to use his ability to make transfers with better targeting and higher productivity, which should increase the probability of winning. But the general theory does not allow us to prove this definitively. We will examine the issue using numerical solutions.

The comparative static results for stage 2 are needed for analyzing the stage 1 Nash game between the party leaders. The L leaders choose \( (I_L, B_L) \) for given \( r \) to maximize
\[ U_L = \pi_L (V - B_L) - I_L, \]
and the R leaders choose \( r \) for given \( (I_L, B_L) \) to maximize
\[ U_R = (1 - \pi_L) V - r N. \]

We can use the comparative statics results of (C.6) to find the parties’ calculation of effects of changes in their strategies \( (I_L, B_L) \) and \( r \) respectively, taking into account the L agent’s
response at the second stage. We have total differentials of the objective functions:

\[ dU_L = (V - B_L) \left( \pi_L \pi_R \frac{1 - \Omega}{\Delta} \left\{ \frac{1}{\Omega} \frac{dI_L}{I_L} + \frac{dB_L}{B_L} \right\} - \pi_L dB_L - dI_L \right) \]

and

\[ dU_R = -V d\pi_L - N dr \]

\[ = V \frac{\pi_L \pi_R \theta_p}{\Delta \Omega} \frac{dr}{\theta_a} r - N dr \]

Note the absence of \( dr \) in the expression for \( dU_L \) and of \((dI_L, dB_L)\) in the expression for \( dU_R \), reflecting the Nash noncooperative assumption where each party takes the other’s strategy as given.

Now party L’s first-order conditions can be found by setting the coefficients of \( dI_L \) and \( dB_L \) separately equal to zero in the expression for \( dU_L \):

\[ (V - B_L) \frac{\pi_L \pi_R}{\Delta} \frac{1 - \Omega}{\Omega} \frac{1}{I_L} - 1 = 0, \quad (C.8) \]

\[ (V - B_L) \frac{\pi_L \pi_R}{\Delta} \frac{1}{B_L} - \pi_L = 0. \quad (C.9) \]

The R party’s first-order condition is found by setting the coefficient of \( dr \) equal to zero in the expression for \( dU_R \):

\[ V \frac{\pi_L \pi_R}{\Delta \Omega} \frac{\theta_p}{\theta_a} \frac{1}{r} - N = 0. \quad (C.10) \]

The complete solution for the two stages together – for all five endogenous variables \( I_L, B_L, r, z_l \) and \( \pi_L \) – is then implicitly defined by the five equations (C.1), (C.2), (C.8), (C.9) and (C.10). No general inferences can be drawn from the algebra, so we resort to numerical solution.
D. Deriving $\theta$ from a Contest success function

From the text recall that Skaperdas shows in his Theorem 2 that the only form satisfying certain desirable axioms is that when players 1 and 2 expend scalar efforts $x_1$ and $x_2$ respectively, the probability of winning for the first player should take the form

$$\pi_1 = \frac{x_1^\theta}{x_1^\theta + x_2^\theta},$$

and of course $\pi_2 = 1 - \pi_1$ is the probability that player 2 wins.* The parameter $\theta$ captures the marginal (incremental) returns to expending effort.

This is more easily understood by considering the odds ratio

$$\frac{\pi_1}{\pi_2} = \left( \frac{x_1}{x_2} \right)^\theta.$$

Taking logarithms of both sides and differentiating,

$$\frac{d \ln(\pi_1/\pi_2)}{d \ln(x_1/x_2)} = \theta.$$

Thus $\theta$ is the elasticity of the odds ratio with respect to the effort ratio: increasing $x_1$ by 1% relative to $x_2$ will shift the odds ratio by $\theta\%$ in player 1’s favor. Second-order conditions of maximization impose limits on $\theta$; for our purpose $\theta \leq 1$ will suffice.

Numerical Appendix

The two tables below provide more information about some of the equilibria that figure (1) depicts. The tables contain all of the endogenous outcomes of the model, the values of $\theta_a$ and $V$, and the four possible payoffs for each party. Table (1) contains the endogenous outcomes of the model for both the case when only one party employs an agent

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* Skaperdas 1996.
and when both parties employ an agent. Table (2) contains the payoffs for the parties for all of the subgames in the model.
Table 1: Equilibria Outcomes for $\beta = 0.5$

<table>
<thead>
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<th>$V$</th>
<th>$\theta_d - \theta_p$</th>
<th>$B_L1$</th>
<th>$I_L1$</th>
<th>$l_c1$</th>
<th>$r1$</th>
<th>$\pi_L1$</th>
<th>$B_L2$</th>
<th>$I_L2$</th>
<th>$l_c2$</th>
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<td>2.731</td>
<td>3.579</td>
<td>0.974</td>
<td>0.837</td>
<td>17.718</td>
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<td>0.6</td>
<td>7.333</td>
<td>2.926</td>
<td>3.809</td>
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<td>0.784</td>
<td>15.501</td>
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<td>7.298</td>
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<td>3.699</td>
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<td>0.703</td>
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</tr>
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The number after the outcome variables indicates the number of agents.
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"1" indicates the payoff is when 1 party uses an agent.

"No Agent" indicates that the payoff is for the party that is not using an agent.