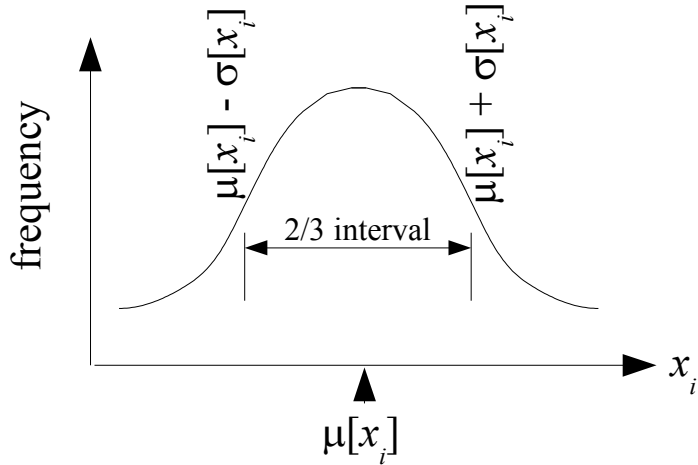


# Error Analysis Cheat Sheet for High School Physics

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## Random Errors



## Definitions

$x_i$  – variable

$x_i(j_i)$  – particular measurement of  $x_i$

$\mu[x_i]$  – mean/center of distribution of  $x_i(j_i)$ 's

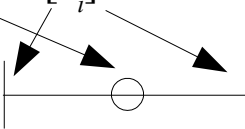
$\sigma[x_i]$  – 2/3 width “standard deviation”

$$\bar{x}_i \equiv \sum_{j_i=1}^N \frac{x_i(j_i)}{N} \approx \mu[x_i] \equiv \lim_{N \rightarrow \infty} \sum_{j_i=1}^N \frac{x_i(j_i)}{N}$$

$$s^2[x_i] \equiv \sum_{j_i=1}^N \frac{(x_i(j_i) - \bar{x}_i)^2}{N-1} \approx \sigma^2[x_i] \equiv \lim_{N \rightarrow \infty} \sum_{j_i=1}^N \frac{(x_i(j_i) - \mu[x_i])^2}{N}$$

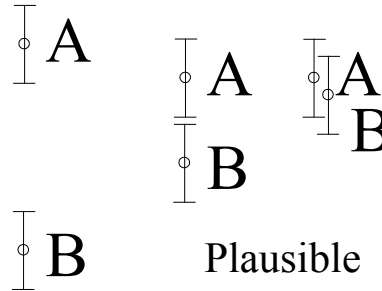
**Always** report  $x_i(j_i) \pm s[x_i]$ . Never report

$\mu[x_i] \pm s[x_i]$ .



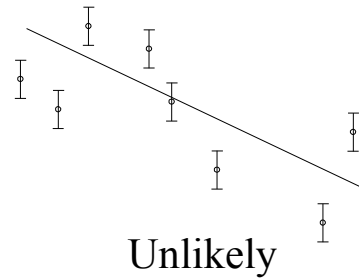
## Interpretation

Could two measurements have come from the same mean?

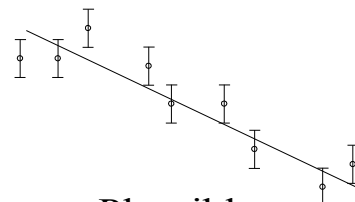


Unlikely

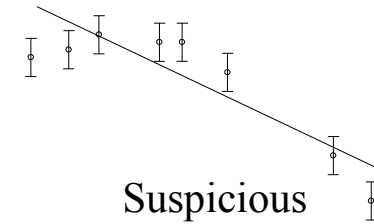
Could a set of measurements have come from a single line?



Unlikely



Plausible



Suspicious

If  $y = mx + b$  reasonable, wiggle line while still intercepting most error bars  
 (1) Maximum rotation of line about any one point gives  $m \pm s[m]$   
 (2) Maximum parallel shift gives  $b \pm s[b]$

## Quadrature

Experiment: For each  $i$ th variable  $x_i$ ,  $i = 1$  to  $M$ , measure a value  $x_i(j_i)$ .

Calculate  $y(k)$ .

$$y(k) = f(x_1(j_1), x_2(j_2), \dots, x_M(j_M))$$

Report  $y_k \pm s[y]$ .

$$s^2[y] = \sum_{i=1}^M \left[ \frac{f(x_i(j_i) + s[x_i]) - f(x_i(j_i) - s[x_i])}{2} \right]^2$$

Assuming  $x_i$  and  $x_j$  uncorrelated and  $f$  looks flat with respect to  $x_i$  near  $x_i(j_i)$ .

SDOM is one application of quadrature formula

$$y = \bar{x}_i \equiv \sum_{j_i=1}^N \frac{x_i(j_i)}{N} \quad s[\bar{x}_i] = \frac{s[x_i]}{\sqrt{N}}$$