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$$\nabla P_{\perp} + \boldsymbol{B}\nabla_{\parallel}\left(\frac{P_{\Delta}}{B}\right) + \frac{P_{\Delta}}{B}\nabla_{\perp}B = \frac{\sigma}{c}\boldsymbol{J}\times\boldsymbol{B} \tag{96}$$

$$\sigma \equiv 1 - 4\pi\frac{P_{\Delta}}{B^2} = 1 + 4\pi\frac{(P_{\perp} - P_{\parallel})}{B^2} \tag{97}$$

From eq. (97),

$$\frac{B^2}{4\pi}\nabla_{\perp}\sigma = -B\nabla_{\perp}\left(\frac{P_{\Delta}}{B}\right) + \frac{P_{\Delta}}{B}\nabla_{\perp}B \tag{a1}$$

Put (a1) into (96) using 2nd term in RHS of (a1), it could be

$$\nabla P_{\perp} + \boldsymbol{B}\nabla_{\parallel}\left(\frac{P_{\Delta}}{B}\right) + \frac{B^2}{4\pi}\nabla_{\perp}\sigma + B\nabla_{\perp}\left(\frac{P_{\Delta}}{B}\right) = \frac{\sigma}{c}\boldsymbol{J}\times\boldsymbol{B} \tag{a2}$$

$$\nabla P_{\perp} + B\nabla\left(\frac{P_{\Delta}}{B}\right) = \frac{\sigma}{c}\boldsymbol{J}\times\boldsymbol{B} - \frac{B^2}{4\pi}\nabla_{\perp}\sigma \tag{a3}$$

$$\nabla P_{\perp} + \nabla P_{\Delta} + BP_{\Delta}\nabla\left(\frac{1}{B}\right) = \frac{\sigma}{4\pi}(\nabla\times\boldsymbol{B})\times\boldsymbol{B} - \frac{B^2}{4\pi}\nabla_{\perp}\sigma \tag{a4}$$

Considering $P_{\Delta} = P_{\parallel} - P_{\perp}$, and $\nabla_{\perp}\sigma = \boldsymbol{b}\times(\nabla\sigma\times\boldsymbol{b})$, eq. (a4) can be described as

$$\nabla P_{\parallel} - \frac{P_{\Delta}}{B}\nabla B = \frac{1}{4\pi}\sigma(\nabla\times\boldsymbol{B})\times\boldsymbol{B} + \frac{1}{4\pi}(\nabla\sigma\times\boldsymbol{B})\times\boldsymbol{B} \tag{a5}$$

With $\sigma(\nabla\times\boldsymbol{B}) + (\nabla\sigma\times\boldsymbol{B}) = \nabla\times\sigma\boldsymbol{B}$, (a5) becomes

$$\nabla P_{\parallel} - \frac{P_{\Delta}}{B}\nabla B = \frac{1}{4\pi}(\nabla\times\sigma\boldsymbol{B})\times\boldsymbol{B} \tag{a6}$$

Therefore

$$\frac{1}{c}\boldsymbol{K}\times\boldsymbol{B} = \nabla P_{\parallel} - \frac{P_{\Delta}}{B}\nabla B \tag{98}$$

where

$$\boldsymbol{K} \equiv \frac{c}{4\pi}\nabla\times\sigma\boldsymbol{B} \tag{99}$$

“Northrop-Whiteman current"(Northrop and Whiteman, 1964)