Quantitative Finance

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What is Quantitative Finance?

- Application of the mathematics of uncertainty to problems concerning financial markets.
- Most important : we don't (know how to) make specific predictions . Everything is about randomness, probabilities, likelihoods, odds, ...
- Use of *stochastic models* to quantify uncertainty in prices and other economic variables.
- Often departs from classical economics by modeling at a *phenomenological* level.
- Tools derived from probability theory, differential equations, functional analysis, among others.
- Financial Mathematics is also a common name.

Outline

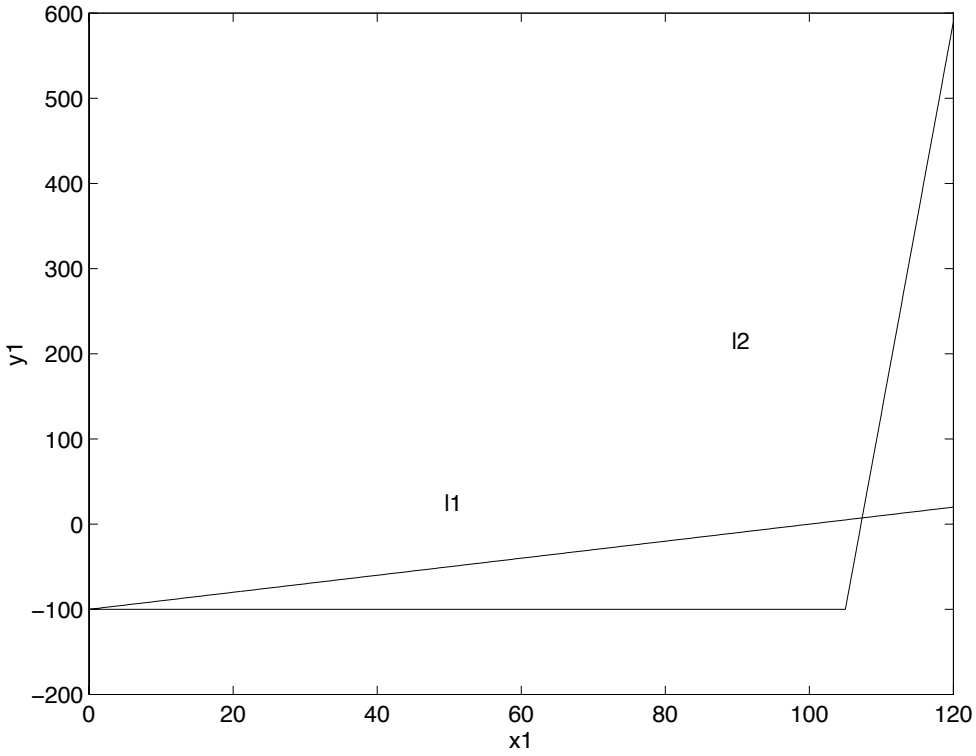
- Options Pricing & Hedging
  - Simple binomial tree model
  - Continuous-time Black-Scholes theory
- Credit Derivatives and complex structured products.
- 2007-?? Financial Crisis (Mortgages) . Role of models & quants.
- Reform, change, regulation, regret, or business as usual ?

Options + Derivative Securities

- European Call Option on a Stock: Contract giving the holder the right, but not the obligation to buy one share on expiration date T for the strike price $\$K$.
- Investor is betting that the stock price will exceed $\$K$ by date T.
- Large profits if correct, but he/she loses everything if wrong.
- Motivation is speculative .

Example

- Investing in stocks vs. investing in options.
- $K = \$105$, $T = 6$ months, today's stock price $= \$100$.
- With \$1000, can buy 10 shares or 461 call options.



Other Examples

- European Put Option on a Stock: Contract giving the holder the right, but not the obligation to sell one share on expiration date T for the strike price $\$K$.
- The put option is an insurance against a decline in the value of the firm's stock. Its price is like an insurance premium .
- Credit Default Swaps are insurance against default risk.
- Adjustable Rate Mortgages (ARMs) are derivatives that meddle with mortgage payments – dangerous if misunderstood.
- American options allow holder to exercise *any time up till* the expiration date. Given to employees as compensation and incentive.
- Asian options, lookback options, barrier options, passport options, swaptions, floortions, captions, ...

The Wild Side

- Derivatives can be utilised to smooth out (custom) exposure to financial risks. (Insurance).
- The can be (mis-)used as tools of wild speculation . (Gambling).
- Notable disasters: Barings Bank (1995), Orange County ('90s), LTCM (1998), ongoing Financial Crisis .
- BUT - derivatives will always be around and we need quantitatively trained people at the regulators' offices, ratings agencies, the Fed as well as the banks to prevent crises arising from their mismanagement.

Historical Development of the Models

- Start with a model that quantifies randomness (uncertainty about stock prices in the future).
- A start is Brownian motion : created by Bachelier in his 1900 PhD thesis modeling the Paris stock market.
- Rediscovered by Einstein in 1905. Used mainly for models in Physics over the next 60 years. Mathematical developments due to Wiener (1923), Levy (1930s), Itô (1940s), ...
- Promoted as a model for stock prices (again) by Samuelson (1960s).

Historical Development *(ctd.)*

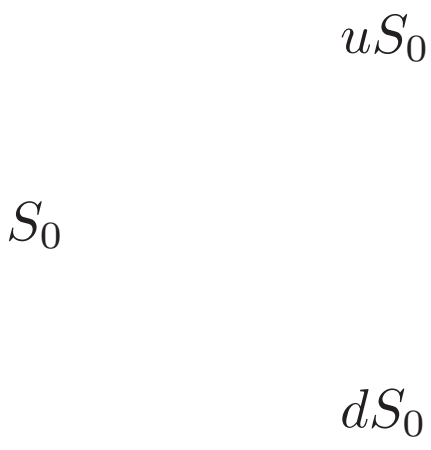
- Successful option pricing theory Black-Scholes (1973). Explicit pricing formula .
- CBOE (Chicago Board Options Exchange) opened April 26, 1973 (coincidence). Initially traded call options on just 16 stocks. (No puts till 1977).
- Mid-1970s: TI hand held calculator programmed with Black-Scholes formula.
- In 2000, calls and puts traded on > 1200 stocks.
- 1997, Merton & Scholes awarded Nobel Prize in Economics. (LTCM crisis, 1998).

Basic Idea *(ctd.)*

- Try and find a replicating strategy in the underlying stock and bank account that has the same payoff as the option, no matter where the stock ends up.
- Then we have two things that pay the same no matter what. They must be priced the same or else there is an arbitrage opportunity . *Mark to market.*
- Initially derided by classical economists as Ketchup Economics .
- By product: the replicating strategy HEDGES a short position in the option.
- Hedging is the minimization or elimination of risk. Pioneered by Napoleon's mother ?

One-Period Binomial Tree Model

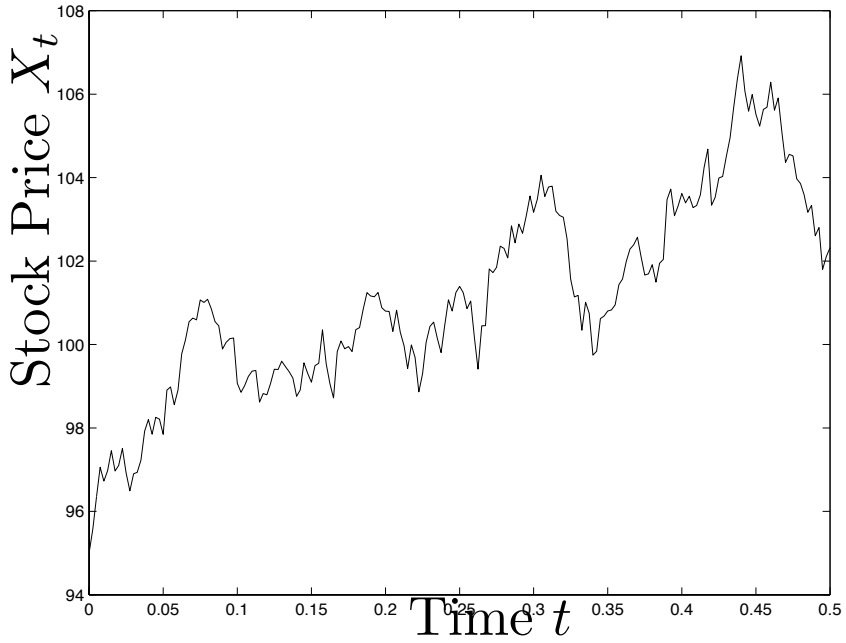
- One time period of length T years.
- Current stock price S_0 . Stock goes up to uS_0 with probability p , or down to dS_0 with probability $1 - p$.



Parameters (u, d, p) .

Samuelson Geometric Brownian Motion Model

Stock price random walk (drunken sailor) model X_t



$$\frac{dX_t}{X_t} = \mu\, dt + \sigma\, dW_t,$$

where W=Brownian motion, σ = volatility ($CONSTANT$). $\longrightarrow$ Black-Scholes formula.

Credit Risk

- Defaultable instruments, or credit-linked derivatives, are financial securities that pay their holders amounts that are contingent on the occurrence (or not) of a *default event* such as the bankruptcy of a firm, non-repayment of a loan or missing a mortgage payment.
- The market in credit-linked derivative products grew more than seven-fold in recent years, from $631.5 billion global volume in first-half of 2001, to almost $12 trillion in first half of 2005.
- Securitization and demand for *things to bet on* fueled by flat stock market, low interest rates, influx of foreign investors.

Complex Structured Products (Corporate Debt)

- CDOs (Collateralized Debt Obligations) depend on the number of defaults over a fixed time of a number (~ 300) firms. (Mortgage-Backed Securities even more complex).
- Various slices of the loss distribution are sold as tranches , and these are sensitive to the correlation between default events.
- CDO^2's collate tranches of different CDOs. There was even a CDO^3 !!! Too hard to value: dropped in 2005.
- From late 1990s, industry adopted copula models – highly artificial "correlator". These are fast, simple and wrong .
- Is the current crisis a surprise? Frontpage Wall Street Journal article 9/12/05: How a formula ignited a market that burned investors .

WSJ I

When a credit agency downgraded General Motors Corp.'s debt in May, the auto maker's securities sank. But it wasn't just holders of GM shares and bonds who felt the pain.

Like the proverbial flap of a butterfly's wings rippling into a tornado, GM's woes caused hedge funds around the world to lose hundreds of millions of dollars in other investments on behalf of wealthy individuals, institutions like university endowments – and, via pension funds, regular folk.

All this traces back, in a sense, to a day eight years ago when a Chinese-born New York banker got to musing about love and death – specifically, how people tend to die soon after their spouses do. Therein lies a tale of how a statistician unknown outside a small coterie of finance theorists helped change the world of investing.

WSJ II

The banker, David Li, came up with a computerized financial model to weigh the likelihood that a given set of corporations would default on their bond debt in quick succession. Think of it as a produce scale that not only weighs a bag of apples but estimates the chance that they'll all be rotten in a week.

The model fueled explosive growth in a market for what are known as credit derivatives: investment vehicles that are based on corporate bonds and give their owners protection against a default. This is a market that barely existed in the mid-1990s. Now it is both so gigantic – measured in the trillions of dollars – and so murky that it has drawn expressions of concern from several market watchers. The Federal Reserve Bank of New York has asked 14 big banks to meet with it this week about practices in the surging market.

WSJ III

The model Mr. Li devised helped estimate what return investors in certain credit derivatives should demand, how much they have at risk and what strategies they should employ to minimize that risk. Big investors started using the model to make trades that entailed giant bets with little or none of their money tied up. Now, hundreds of billions of dollars ride on variations of the model every day.

The problem: The scale's calibration isn't foolproof. "The most dangerous part," Mr. Li himself says of the model, "is when people believe everything coming out of it." Investors who put too much trust in it or don't understand all its subtleties may think they've eliminated their risks when they haven't.

WSJ IV

The story of Mr. Li and the model illustrates both the promise and peril of today's increasingly sophisticated investment world. That world extends far beyond its visible tip of stocks and bonds and their reactions to earnings or economic news. In the largely invisible realm of derivatives – investment contracts structured so their value depends on the behavior of some other thing or event – credit derivatives play a significant and growing role. Endless trading in them makes markets more efficient and eases the flow of money into companies that can use it to grow, create jobs and perhaps spread prosperity.

But investors who use credit derivatives without fully appreciating the risks can cause much trouble for themselves and potentially also for others, by triggering a cascade of losses. The GM episode proved relatively minor, but some experts say it could have been worse.

Financial Crisis 2008

- The crisis erupted from mortgage-backed securities (MBS). These are pools of mortgages from different regions, tranched and rated.
- CDOs on MBS are in fact CDO^2's. Their senior tranches were given AAA ratings.
- Ratings agencies' worst-case scenario was that housing prices would go up by 0.0%. The AAA nomenclature gives CDOs the allure of treasury bonds.
- The argument of independent housing markets across the country justified the "safety" of these securities.
- The MBS book was 10 times the size of the corporate debt book, but far less *quant*-ified. They were AAA, who needed analytics ...?

Who knew/understood what?

- Traders were incentivized to take short-term risks by huge bonuses .
- Quants reported to traders and either did not speak up or were not heard.
- Management may not have known the details, but were buffered by the knowledge they were too big to fail .
- Ratings agencies were paid to rate: more deals, more business.

Present, future, changes, BAU

- The bonus culture persists. Once one firm does it, everyone has to.
- Quants report to management, not traders.
- Too big to fail firms got bigger.
- Regulations being discussed. Exchange-traded CDSs. National Institute of Finance.
- Structured products have dried up, but will return. Design has to come from quantitatively-trained people who can assess their complexity and risk.

Sideways (with P. Cotton)

Flying across the Atlantic in dense fog, Lindbergh descended several times to an altitude of just ten feet. He judged wind direction from the spray of the waves, and maneuvered his plane sideways to catch a glance of what was ahead. The Spirit of St Louis had no windshield; looking sideways was Lindbergh's only option.

On Wall Street, looking sideways at the prices of similar securities is called marking the book. Without sideways-looking models, business cannot be transacted and bonuses cannot be paid. Wall Street models enabled pilots to see that everyone else was more or less in formation ... as they all ploughed into the mountain.

Engineers cannot predict the weather, but they can design planes which fly through storms. Wall Street could have taken a scientific approach to financial product design based on forward looking simulation. Most institutions chose not to. On Wall Street, in the heyday of credit default swaps and collateralized debt obligations (the notorious CDOs), the trader bringing in tens of millions in risky bets was king, and the quants, the mathematicians, if they wanted to keep their jobs, served up models for one purpose: looking sideways to mark the books.

Looking sideways took Lindbergh to Paris, and CDOs to Sydney. But like aircraft, complex financial products ultimately require careful design. Careful design requires simulation. Simulation requires models that make sense, and making models that make sense takes time. Sideways-looking pseudoscience can be created in a week, but models which look forward require research, collaboration, reflection, experimentation, and mathematical advances. The current crisis must spur the remaining financial institutions to invest seriously in such R&D. The problem on Wall Street has hardly been the over-influence of quantitative modeling, as some have claimed, but rather, in some derivatives markets, its near absence.