EXTERNAL ECONOMIES AND INTERNATIONAL TRADE REDUX*

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Abstract

We study a world with national external economies of scale at the industry level. In contrast to the standard treatment with perfect competition and two industries, we assume Bertrand competition in a continuum of industries. With Bertrand competition, each firm can internalize the externality from production by setting a price below those set by others. This out-of-equilibrium threat eliminates many of the "pathologies" of the standard treatment. There typically exists a unique equilibrium with trade guided by "natural" comparative advantage. And, when a country has CES preferences and any finite elasticity of substitution between goods, gains from trade are assured.

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I. Introduction

External economies hold a central—albeit somewhat uncomfortable—place in the theory of international trade. Since Marshall (1879, 1890) at least, economists have known that increasing returns can be an independent cause of trade and that the advantages that derive from large scale production need not be confined within the boundaries of a firm. Marshallian externalities arise when knowledge and other public inputs associated with a firm's output spill over to the benefit of other industry participants. After Marshall's initial explication of the idea, a lengthy debate ensued on whether an industry with external economies of scale could logically be modelled as perfectly competitive.[1] Even after the matter was finally resolved in the affirmative by Chipman (1970), trade economists continued to bemoan the "bewildering variety of equilibria [that provide] a taxonomy rather than a clear set of insights" (Krugman, 1995, p.1251), the apparent tension between the forces of external economies and other, better-accepted determinants of the trade pattern, and the "paradoxical" implication that trade motivated by the gains from concentrating production need not benefit the participating countries.

The modeling of external economies began at a time when game theoretic approaches to imperfect competition were much less developed than they are today. Consequently, the logical consistency between external economies and perfect competition was seen as a great advantage in allowing the application of familiar tools of general equilibrium analysis. Economists were comfortable in assuming that competitors take prices as given in industries with many small firms. By assuming that small producers contribute little to aggregate industry output, they felt similarly justified in assuming that firms treat industry scale as given when assessing productivity and making decisions about their own output. Firms might recognize the link between industry scale and productivity and nonetheless perceive their own costs to be constant. As Chipman (1970, p.349) noted, "there is no logical contradiction involved in the notion that economic agents do not perceive things as they actually are, and while the idea of treating production functions parametrically may be more subtle and unusual than that of treating market prices in this way, both ideas are of the same logical order."

In application, the competitive models of external economies and trade yielded some discomforting results.[2] A potential for multiple equilibria, noted already by Ohlin (1933, p.55), was shown more formally by Matthews (1949-1950), Kemp (1964), and Ethier (1982a). Graham (1923) argued that scale economies could reverse the trade pattern predicted by "natural" comparative advantage and Ethier (1982a) proved him correct in a competitive model with external economies. Markusen and Melvin (1982) and Ethier (1982a) observed that country size

[1] For a recount of the debate and its protoganists, see Chipman (1965).

[2] There are several good surveys of this literature, beginning with Chipman's (1965) treatment of the earliest contributions. Subsequent overviews include Helpman (1984), Helpman and Krugman (1985, ch.3), Krugman (1995), and Choi and Yu (2003).

might play an independent role in determining the pattern of specialization. Finally, Graham (1923) advanced the possibility of losses from trade, an idea that has been further addressed by Matthews (1949-1950), Melvin (1969), Markusen and Melvin (1982), Ethier (1982a), and others. These "pathologies" are perhaps responsible for the diminished role that external economies now play in thinking about trade, even as such externalities are still considered to be empirically relevant in many contexts.[3]

The pathological outcomes arise in a canonical model of external economies and trade, one that has become well ensconced in the literature and that by now breeds little questioning of its underlying assumptions. Typically, there are two countries and two industries, one with constant returns to scale and the other with increasing returns to scale (IRS) due to national spillovers from production. Often, labor is the only productive factor. Firms in both sectors are price takers and, more importantly, those in the IRS sector take aggregate national output and hence their own potential productivity as given.

In this familiar setting, suppose that the relationship between productivity and national scale is such that zero output implies zero productivity. Then the existence of multiple equilibria is immediate. Firms that take aggregate output as given perceive zero productivity when no other national producers are active in their country. If output in a country is nil, no producer perceives a profit opportunity from entry, even when an absence of local competitors might imply a potential for profit for a firm that evaluates correctly the implications of entry at non-negligible scale. Thus, zero output in any location is self-reinforcing and production of the IRS good can be concentrated in either country for any set of parameter values.[4]

What about the pattern of trade? When production of the IRS good is concentrated in a country, the trade pattern necessarily accords with the ranking of relative productivities when measured at the equilibrium scales of production. That is, each country specializes according to *observed* comparative advantage, where the "observed" productivity of a country that does not produce the IRS good is taken to be zero. However, the pattern of specialization need not conform to the ranking of relative productivities when measured at a common scale of production. The exporters of the IRS good might be less productive than firms in the other country would be, were the latter to produce at a comparable aggregate scale. In this sense, the trade pattern may run counter to the dictates of *natural* comparative advantage. Notice that such a trade pattern violates the requirements for global efficiency in circumstances in which the country with comparative advantage in the IRS industry has sufficient resources to satisfy the first-best level of world demand for that good.

[3] See, for example, Caballero and Lyons (1989, 1990, 1992), Chan, Chen and Cheung (1995), Segoura (1996), Henriksen, Steen, and Ulltveit-Moe (2001) and Antweiler and Trefler (2002).

[4] If productivity is bounded from below even when national output is zero, multiple trade equilibria will exist for a range of parameter values.

In the canonical model, country size can be an independent source of comparative advantage. Suppose, for example, that productivity in the IRS sector is modest when output is small, but not equal to zero. If a small country were to specialize in producing this good, the equilibrium price might be high in light of its meager resource base and a robust level of world demand. Such a high price might invite entry by producers in the large country. In such circumstances, there may be no equilibrium with production of the IRS concentrated in the small country. The large country gains a comparative advantage in the IRS industry by dint of having sufficient resources to produce at large scale.

The canonical model readily yields examples with losses from trade (see, e.g., Helpman and Krugman, 1985, p.55). The autarky equilibrium features underproduction of the IRS good, because firms fail to take into account the spillover benefit from increasing their output on the productivity of others. As is well known, trade can bring harm if it exacerbates a pre-existing market distortion (Bhagwati, 1971). It stands to reason that trade may reduce welfare in a country that shifts substantial resources from the single industry with increasing returns to one with constant returns.

In short, the pathologies that emerge from the canonical model rest on two critical assumptions. First, firms contemplate entry only at such small scale that they believe they can have negligible impact on national output and their own productivity. Second, industries are large in relation to countries' resource endowments, so that a small country may lack sufficient resources to meet world demands for the IRS good at a reasonable price. In such circumstances, the demise of activity in the IRS industry due to the opening of trade can spell dire consequences for a country's productivity and welfare.

In this paper, we modify these two familiar assumptions and show how this dramatically narrows the scope for pathological outcomes. We assume in what follows that firms engage in Bertrand competition and that they correctly anticipate their productivity when they quote a price that would yield non-negligible sales. And we assume that the world economy comprises a large number of (small) industries, so that any industry can be fully accommodated in any country, no matter what its size.[5]

It is well known that perfect competition and Bertrand competition predict similar outcomes in models with homogeneous goods and constant costs. This is true as well—it turns out—in an autarky equilibrium with external economies of scale. However, the predictions of the alternative behavioral assumptions diverge in a world with national external economies and international trade. With Bertrand competition, a firm may be small when its price is the same as that of

[5] In our analysis the scope of the external effects is supposed to be the industry. So it is essential for us to assume that industries are small relative to country sizes. As advocated by Ethier (1982b), one could also think of externalities that transcend both industry and country boundaries. However, external economies at the industry level are plausible in the presence of industry-specific knowledge and other non-rival inputs, and the literature cited in footnote 3 points to their empirical significance.

its (domestic) rivals—as will be the case in the equilibria that we discuss—yet it may recognize that by announcing a price below that offered by other firms in the industry, it can grow to non-negligible scale. We shall see that the different out-of-equilibrium beliefs under perfect and Bertrand competition yield very different conclusions about the possibility of multiple equilibria and the determinants of the trade pattern.

Moreover, the potential role for country size to influence the trade pattern disappears when industries are small in relation to the size of national economies. In a world with many industries à la Dornbusch, Fischer and Samuelson (1977), a small country can realize its comparative advantage in whichever sectors it is strongest by serving world demands in a relatively small range of industries. We will find that country size plays no role in determining a chain of comparative advantage, although it does affect the dividing line between imports and exports along that chain. The assumption of a large number of small industries also helps to justify our assumption that firms take factor prices as given. A Bertrand competitor may entertain the prospect of growing large in its industry. But, even if it does so, it will remain small in relation to the aggregate economy so long as each industry uses a small fraction of the economy's resources.

In summary, we study a model with a continuum of industries and national industry-level external economies. We assume that firms in an industry produce a homogeneous product and engage in Bertrand (price) competition in an integrated world market. Countries may exhibit Ricardian technological differences, but they share the same prospects for scale economies, which are assumed to be a property of the good being produced. We find in this setting that the trade equilibrium typically is unique and that "natural" comparative advantage determines an ordering of industries.[6] Most surprising, perhaps, are our findings about welfare. We consider for this purpose a country whose residents hold CES preferences over the set of goods, with possible asymmetries in the weighting of different goods, and with any finite elasticity of substitution. This includes, of course, the Cobb-Douglas case that has been used in the literature to demonstrate the possibility of losses from trade based on external economies of scale. We find, in contrast, that in spite of the generic inefficiency of the autarky and trade equilibria, gains from trade are assured.

In the main, our analysis follows the older literature in abstracting from two possible aspects of reality. First, we neglect transport costs. Second, we assume that, given productivity—which declines with *aggregate* output—firms face constant returns to scale. How robust are our findings to changes in these assumptions? We will show that the addition of transport costs modifies the analysis somewhat, but important qualitative features survive. When transport costs are small relative to the potential gains from the realization of increasing returns to scale,

[6] If the countries share identical technologies for some or many goods, the model does not predict which goods are produced in each country. But factor prices and consumption levels typically are uniquely determined even in this case.

the introduction of such costs alters the dividing line between goods produced in each country but (unlike in Dornbusch, Fischer and Samuelson) does not create a set of nontraded goods. For larger transport costs, there will be some industries that admit no equilibrium in pure pricing strategies. In any such industry, the mixed-strategy equilibrium has either all production of the good in a well-defined location, or else production everywhere and an absence of trade.

As regards firms' constant returns to scale (given productivity), this is a more critical foundation for our results. Our analysis rests on the assumption that a deviant firm potentially can internalize the externalities from production if an industry is wrongly located. This requires that firms are able to grow large, even if they do not do so in equilibrium. Some small degree of decreasing returns at the firm level would not upset our results. But binding limitations on firms' expansion would restore the potential for multiple equilibria and some of these would have patterns of trade counter to the dictates of natural comparative advantage.

The remainder of the paper is organized as follows. The next section develops our alternative model with Bertrand competition and a continuum of industries. There, we derive our predictions about the pattern of trade. Section III contains a brief review of the efficiency properties of the model, which sets the stage for the gains-from-trade analysis in the subsequent section. In Section V, we introduce trading costs, first assuming that they are small in relation to the strength of scale economies, and then allowing for larger costs. Section VI concludes.

II. A Model of External Economies in a Continuum of Industries

We study an economy with a continuum of industries, as in Dornbusch, Fischer and Samuelson (1977; hereinafter, DFS). Goods are indexed by $i \in [0, 1]$. For the time being, we assume only that preferences in each country are such that a change in the prices of a measure-zero set of goods has a negligible effect on the demand for all goods other than the ones in this set. Demands need not be identical in the two countries, nor need preferences be homothetic.

Goods are produced by a single primary factor, labor. Labor supplies are L and L^* in the home and foreign countries, respectively. Throughout the paper, we take the home wage as numeraire. Technology exhibits constant or increasing returns due to external economies at the local industry level. In the home country, $a_i/A_i(x_i)$ units of labor are required for a unit of output, where x_i is aggregate home production of good i and $A_i(\cdot)$ is non-decreasing, concave, and everywhere has an elasticity smaller than one.[7] In the foreign country, the unit labor requirement is $a_i^*/A_i(x_i^*)$. There are n_i potential producers of good i in the home country and n_i^* potential producers in the foreign country, where $n_i \geq 2$ and $n_i^* \geq 2$. The number of potential producers in each country can be very large, but is assumed to be finite. Firms in each industry engage in price (Bertrand) competition in the integrated world market. Each firm

[7] As in the literature, an elasticity of productivity with respect to output smaller than one is needed to ensure a non-negative marginal productivity of labor.

recognizes that, if it sets a price above that quoted by any other firm, it will consummate no sales, if it sets a price that is the lowest among all quoted prices, it will capture the entirety of world demand, and if it sets a price that is tied for lowest among a group of competitors, it will share demand with these rivals.

II.A. Autarky Equilibrium

Figure I shows the inverse demand curve for good i, labelled DD, for a given level of aggregate spending and a given level of any price aggregate that may affect the demand for this good. Firms in the industry take aggregate spending and the relevant price index (if any) as given.[8] The figure also depicts the average cost curve, which is labelled CC. Intersection points are our candidates for industry equilibrium.

Suppose firms in industry i announce a price above $\tilde{p}_i$ in Figure I. Then a deviant firm could announce a price a bit lower than that offered by the others. The firm would capture all industry sales, and since its price would exceed its average cost at the resulting scale of production, it would earn a profit. Evidently, the price will be bid down at least to $\tilde{p}_i$. But a further price cut by any firm would spell losses, since the deviant would capture the market but fail to cover its costs. When the DD curve is everywhere steeper than the CC curve, as in Figure I, then the unique intersection point depicts the industry equilibrium. In this equilibrium, price equals industry average cost and an arbitrary (and possibly large) number of firms make sales but earn zero profits.

In Figure II, the DD and CC curves have multiple intersections at E', E'', and E. Neither E' nor E'' represents an equilibrium, because if all firms charge the price associated with either of these points, a deviant firm can announce a lower price such that, at the associated demand level, price exceeds average cost. The equilibrium in industry i is at the lowest intersection of DD and CC, so long as DD cuts CC from above.[9]

The general equilibrium requires, of course, that every industry equilibrates and that the labor market clears. There is no need to develop the notation to express this for the general case. Instead, we describe a special case that will prove useful later on when we establish the gains from trade. Suppose utility takes the form

$$U = \left(\int_0^1 b_i c_i^{\rho} di \right)^{1/\rho}, \tag{1}$$

[8] They are justified in doing so, because each industry is small in relation to the aggregate economy. See Neary (2003, 2008), who studies Cournot oligopoly in general equilibrium and similarly uses the existence of many small industries to justify the assumption that when firms compete with rivals in their own sector, they neglect the actions of firms in other industries.

[9] If no such intersection exists, then the price would tend toward zero and output would tend toward infinity. But then the assumption that firms take wages as given becomes untenable.

where c_i is consumption of good i, with $\rho < 1$ and $\int_0^1 b_i di = 1$. These preference imply a constant elasticity of substitution $\sigma = 1/(1-\rho)$ between every pair of goods. The demand for good i is

$$c_i = \frac{b_i^\sigma p_i^{-\sigma} E}{P^{1-\sigma}} \ ,$$

where E is aggregate spending and $P = \left[\int_0^1 b_i^\sigma p_i^{1-\sigma} di\right]^{1/(1-\sigma)}$ is the exact price index associated with the CES utility function. With these demands, the condition that DD is steeper than CC at output x_i is equivalent to

$$\sigma\theta_i(x_i) < 1,$$

where $\theta_i \equiv A_i'(x_i)x_i/A(x_i)$ is the elasticity of the productivity function with respect to output. A sufficient condition for existence of an industry equilibrium with finite x_i is $\sigma\theta_i(x) < 1$ for all x.[10] We will assume that an equilibrium with finite output exists in every industry, either because these sufficient conditions are satisfied, or otherwise.

In the autarky equilibrium (denoted again by tildes), aggregate spending $\tilde{E}$ equals aggregate income, L. Then product market equilibrium requires

$$\tilde{x}_i = \frac{b_i^\sigma \tilde{p}_i^{-\sigma} L}{\tilde{P}^{1-\sigma}} \text{ for all } i, \tag{2}$$

while price equals average cost implies

$$\tilde{p}_i = \frac{a_i}{A_i(\tilde{x}_i)} \text{ for all } i. \tag{3}$$

This gives a system of equations that jointly determine outputs and prices in all industries.

We note in passing that the autarky equilibrium with Bertrand competition is the same as would arise under perfect competition, were firms to take their own productivity as given. Price in every industry is equal to average cost. Firms earn zero profits. Demand equals supply for all goods. And the labor market clears.

II.B. Trade Equilibrium

We now open the economy to international trade. Again, competition between the potential producers in a country drives price for every good i down to average cost. But now it also must be the case that a potential producer in the "other" country does not wish to shave price further. If home producers in industry i quote the price $wa_i/A(x_i)$, a foreign firm could announce a price a bit lower than that, capture sales of x_i, and achieve labor productivity of $a_i^*/A_i(x_i)$. Such a strategy would be profitable if its per unit cost, $w^* a_i^*/A_i(x_i)$, were less than the quoted price.

[10] Ethier (1982, p.1247) invokes essentially the same assumption as a condition for stabilitiy of equilibrium in his two-industry model. Here, the assumption ensures uniqueness of equilibrium, without need to invoke an *ad hoc* stability analysis.

Therefore, concentration of industry i in the home country requires

$$\frac{a_i}{A_i(x_i)} \leq \frac{w^* a_i^*}{A_i(x_i)}.$$

(Note that $w = 1$, because the home wage is numeraire.) Similarly, if production of good i is concentrated in the foreign country, it must be that

$$\frac{a_i}{A_i(x_i^*)} \geq \frac{w^* a_i^*}{A_i(x_i^*)}.$$

Assuming that each good is produced in only one location (which will be true in equilibrium), the equilibrium price of a good equals the lesser of the two average costs evaluated at the equilibrium scale of world production; i.e.,

$$p_i = \min\left[\frac{a_i}{A_i(\bar{x}_i)}, \frac{w^* a_i^*}{A_i(\bar{x}_i)}\right] \text{ for all } i, \tag{4}$$

where $\bar{x}_i$ is world output of good i in the trade equilibrium.

Now order the goods so that $\alpha_i \equiv a_i/a_i^*$ is increasing in i and define I by $\alpha_I = w^*$. Then (4) implies that goods with $i \leq I$ are produced in the home country and goods with $i > I$ are produced in the foreign country.[11] Figure III—familiar from DFS—depicts this relationship between w^* and I as the curve AA. It depicts as well a second relationship between these variables implied by the labor-market-clearing condition. The greater is I, the fewer are the goods produced in the foreign country and the lower must be the foreign wage for the labor market there to clear. This relationship is labelled BB. The equilibrium marginal good and relative wage are found at the intersection of the two curves. This determination of I and w^* is, of course, exactly the same as in DFS, as extended by Wilson (1980) to include economies with more general demands.

Two observations are in order. First, the pattern of specialization is well determined. The possibility of multiple locations for a given industry disappears once firms recognize that, even if small in equilibrium, each can overcome any coordination failure by becoming large when the opportunity presents itself. Moreover, there is no danger that a country will lack the resources to accommodate a particular industry at the scale required to meet world demand. In a world of many industries, countries can adjust on the extensive margin by hosting greater or fewer numbers of them, without sacrificing the benefits that come from concentrating an industry in a single location.

Second, the pattern of specialization conforms to the pattern of comparative advantage, when

[11] For notational convenience, we adopt the convention that the marginal good is produced in the home country, although this is neither determined by the model nor consequential.

the latter is evaluated at a common scale of production. We have assumed in our formulation (as in the standard approach) that countries share the same scale economies, which are a property of the industry and not the place of production. Countries differ only with respect to a set of Hicks-neutral (and scale neutral) technology parameters. Then, with no risk of coordination failure, the country that enjoys the relative technological advantage in an industry will capture that industry—and its associated scale economies—in the general equilibrium. All that remains to be determined, as in DFS, is the margin between industries that locate at home and those that locate abroad. Indeed, the DFS model is a special case of ours with $A_i(x_i) \equiv 1$ for all i. Here, as there, the dividing line in the chain of comparative advantage is determined by the relative sizes of the countries, the pattern of world demand, and the pattern of technologies in the various industries.

Note that country size plays no role in determining relative productivity. Since every individual industry is small in relation to the size of either economy, any country can produce enough of any good to meet the pressures of world demand. The relative size of the two countries affects the margin I between the set of each country's import goods and its export goods, but it cannot affect the ordering of goods in those sets.

In short, by incorporating many industries and amending firms' perceptions about their own productivity, we have overturned several conclusions from the canonical analysis. Multiple equilibria are not pervasive and "natural" comparative advantage rules.

A pedagogically interesting special case arises when the countries share identical technologies and differ only in labor supplies. This setting has been used by some authors to argue that trade induced by external economies will not equalize factor prices in a pair of countries that differ only in size.[12] However, this conclusion also rests on the assumption that individual industries are large in relation to the sizes of the national economies. In our model, when $a_i = a_i^*$ for all i, if $w^* \neq 1$, then all industries would locate in the low-wage country. This is not consistent, of course, with labor-market clearing in the other country, so factor prices must be equalized. With equal factor prices and identical technologies, the pattern of specialization is not determined. What is determined is only the aggregate employment levels in the two countries which, in equilibrium, matches the exogenous labor supplies. A small country can employ its labor force in equilibrium by producing and exporting a moderate number of goods produced in relatively small quantities or a smaller number of goods that require greater numbers of workers to meet world demand. In any case, differences in country size are accommodated by the number of goods produced in each country and not by the identities of which goods are produced where.

III. Efficiency and the First-Best

Both in autarky and in a trade equilibrium, efficiency requires equality between the marginal

[12] See, for example, Markusen and Melvin (1981) and Ethier (1982a).

rate of substitution and the marginal rate of transformation for every pair of goods. In our setting, utility-maximizing consumers equate as usual the marginal rate of substitution between goods to the relative price. But, in general, the marginal rate of transformation is not equal to the relative price due to the presence of production externalities. Therefore, neither the autarky equilibrium nor the free-trade equilibrium achieves an efficient allocation of labor.

To see this, note that the marginal product of labor in industry i in the autarky equilibrium is $A_i(\tilde{x}_i)/\left[a_i\left(1-\theta_i\right)\right]$.[13] It follows that the autarky marginal rate of transformation between good i and good j is

$$\widetilde{MRT}_{ij}=\left[\frac{1-\theta_i(\tilde{x}_i)}{1-\theta_j(\tilde{x}_j)}\right]\frac{a_i/A_i(\tilde{x}_i)}{a_j/A_j(\tilde{x}_j)}=\left[\frac{1-\theta_i(\tilde{x}_i)}{1-\theta_j(\tilde{x}_j)}\right]\frac{\tilde{p}_i}{\tilde{p}_j},$$

where the second equality follows from (3); i.e. from price equals average cost. Therefore, the autarky relative outputs of goods i and j can be efficient only if it happens that $\theta_i(\tilde{x}_i)=\theta_j(\tilde{x}_j)$. Similarly, the free-trade relative outputs can be efficient only if $\theta_i(x_i)=\theta_j(x_j)$. With relative prices equal to relative average costs, they will not generally be equal to relative *marginal* costs. A set of Pigouvian subsidies could be used to achieve the autarky first best. A similar set of subsidies, together with a set of optimum tariffs, could do likewise for a country that trades.

It should be clear from the discussion that the autarky equilibrium achieves the first best in the special case in which all industries bear a constant and common degree of scale economies.[14] If $\theta_i(x)\equiv\theta$ for all x and i, the equality between price and average cost in every industry ensures equality between relative prices and marginal rates of transformation. With trade, the first best still requires a set of optimal tariffs to improve the terms of trade. But gains from (free) trade can be established for this special case in the usual way.[15]

IV. Gains from Trade

To study the prospects for gains from trade with more general production technologies, we posit CES preferences for the home country. Domestic preferences are described by the utility function in (1). We make no assumptions about the weighting of the various goods in utility,

[13] Let L_i be employment in industry i. Then $x_i=A_i(x_i)L_i/a_i$, so $dx_i=(A_i/a_i)\,dL_i+(A_i'/a_i)\,dx_i$ or

$$\frac{dx_i}{dL_i}=\frac{A_i}{a_i}\left(\frac{1}{1-\theta_i}\right).$$

[14] See Chipman (1970) for an, early, formal demonstration of this point.

[15] For the case of constant and common elasticities θ, the free-trade output vector maximizes the value of domestic output given prices. We can prove gains from trade using the standard, revealed-preference argument: The value of free-trade consumption vector at the prices of the trade equilibrium equals the value of the free-trade output vector at those prices, which exceeds the value of autarky output vector at those prices, which in turn equals the value of the autarky consumption vector at those prices. So, consumers could buy the autarky consumption bundle in the trade equilibrium with the income available to them. If they chose not to do so, it must be that they prefer the trade consumption bundle. This argument assumes, of course, that aggregate demand can be represented as the outcome of maximizing a well-behaved utility function, which will be true, for example, with homothetic preferences.

about foreign preferences, or about the productivity functions $A_i(x_i)$ (except for the maintained assumption that $A_i(x_i)$ is non-decreasing, concave, and has an elasticity smaller than one). In particular, we allow for varying degrees of scale economies across industries and non-constant output elasticities in any or all of them.

IV.A. Elastic Demands

We begin with $\sigma \geq 1$, which generates demands that are price elastic. Let us suppose that there are losses from trade; i.e., $U < \tilde{U}$. Since the home wage is numeraire in both the autarky and trade equilibria, $U < \tilde{U}$ requires $P > \tilde{P}$; i.e., a lower *real* income with trade than without implies a higher price index.

In Figure IV, we show the autarky equilibrium in some industry i. It is found at $\tilde{E}$, the intersection between $\tilde{D}\tilde{D}$ and $\tilde{C}\tilde{C}$, and has

$$\tilde{x}_i = \frac{b_i^\sigma \tilde{p}_i^{-\sigma} L}{\tilde{P}^{1-\sigma}}$$

as we have noted before (see Equation 2).

In a world with trade, producers of good i face the world demand

$$\frac{b_i^\sigma p_i^{-\sigma} L}{P^{1-\sigma}} + c_i^* \left(\mathbf{p}, w^* L^*\right),$$

where $c_i^*(\mathbf{p}, w^* L^*) > 0$ is the foreign demand for good i in the trade equilibrium (an arbitrary function of the vector of prices $\mathbf{p}$ and foreign income, $w^* L^*$). The hypothesis that $P > \tilde{P}$ implies that $P^{1-\sigma} < \tilde{P}^{1-\sigma}$ under the assumption that demand is elastic ($\sigma \geq 1$). The intuition is that the substitution effect of the higher price level outweighs the income effect on the demand for good i, so the inverse demand curve with trade—DD in the figure—lies to the right of $\tilde{D}\tilde{D}$ under the maintained hypothesis. The average cost curve with trade is depicted by CC and is given by

$$\min\left[\frac{a_i}{A(\bar{x}_i)}, \frac{w^* a_i^*}{A(\bar{x}_i)}\right]$$

where $\bar{x}_i$ again is the total world output of good i. This curve coincides with $\tilde{C}\tilde{C}$ for $i \leq I$; i.e., for goods that are produced by the home country in the trade equilibrium. It lies below $\tilde{C}\tilde{C}$ (as depicted in Figure IV) for goods that are produced abroad ($i > I$); these goods are imported by the home country only because their average cost is less than it would be with domestic production. In either case, the intersection of DD and CC lies below and to the right of $\tilde{E}$.

As the figure makes clear, our hypothesis that $P > \tilde{P}$ implies $p_i < \tilde{p}_i$ for an arbitrary good i, be it one that is exported in equilibrium or one that is imported. Therefore, all prices are lower in the trade equilibrium than in the autarky equilibrium (relative to the domestic wage). But this is a contradiction, because the price index cannot rise if each element in the index falls. We conclude that $P < \tilde{P}$ and thus $U > \tilde{U}$; i.e., the home country gains from trade!

Since $P < \tilde{P}$ and $\sigma \geq 1$, the inverse demand curve DD may, in fact, be to the left or to the right of $\tilde{D}\tilde{D}$. Apparently, aggregate output in some industries might fall as the result of the opening of trade. Nonetheless, the impact of these cases cannot be so great as to negate the overall gains that come from international specialization according to comparative advantage and the realization of greater scale economies due to the expansion of the market.

IV.B. Inelastic Demands

Next consider the case of $\sigma < 1$; i.e., inelastic demands. Since there are gains from trade as σ approaches one from above[16], and the model is continuous in the parameter σ, trade losses for some σ would imply the existence of a $\hat{\sigma}$ such that $P(\hat{\sigma}) = \tilde{P}(\hat{\sigma})$, in the obvious notation. Let us suppose this to be true, and consider further the autarky and trade equilibria that would arise when $\sigma = \hat{\sigma}$.

The demand function with trade and the price-equals-average cost relationship (4) imply

$$\begin{aligned} \frac{\bar{x}_i}{A_i(\bar{x}_i)} &= \frac{b_i^{\hat{\sigma}} p_i^{1-\hat{\sigma}} L}{P^{1-\hat{\sigma}} \min[a_i, a_i^* w^*]} + \frac{c_i^*\left(\mathbf{p}, w^* L^*\right)}{A_i\left(\bar{x}_i\right)} \\ &> \frac{b_i^{\hat{\sigma}} p_i^{1-\hat{\sigma}} L}{P^{1-\hat{\sigma}} a_i}, \end{aligned}$$

where the inequality in the second line follows from the fact that $a_i \geq \min[a_i, a_i^* w^*]$ and $c_i^*\left(\mathbf{p}, w^* L^*\right) > 0$. Now suppose that $p_i > \tilde{p}_i$. Then, with $\hat{\sigma} < 1$ and $P = \tilde{P}$,

$$\frac{b_i^{\hat{\sigma}} p_i^{1-\hat{\sigma}} L}{P^{1-\hat{\sigma}} a_i} > \frac{b_i^{\hat{\sigma}} \tilde{p}_i^{1-\hat{\sigma}} L}{\tilde{P}^{1-\hat{\sigma}} a_i} = \frac{\tilde{x}_i}{A_i(\tilde{x}_i)}.$$

Therefore $\bar{x}_i/A_i(\bar{x}_i) > \tilde{x}_i/A_i(\tilde{x}_i)$. But the function $x_i/A_i(x_i)$ is increasing in x_i by the assumption that $\theta_i(x_i) < 1$. So the string of inequalities implies $\bar{x}_i > \tilde{x}_i$, which in turn implies $p_i < \tilde{p}_i$. This contradicts our supposition that $p_i > \tilde{p}_i$. We conclude that $p_i < \tilde{p}_i$ for all i, which in turn implies $P < \tilde{P}$. It follows that there can exist no $\hat{\sigma}$ for which the price index under autarky is the same as the price index with trade. Again, the home country gains from trade!

Figure V illustrates the autarky and free-trade equilibrium in a typical industry i when $\sigma < 1$. With $P < \tilde{P}$ and $\sigma < 1$, the DD curve necessarily lies to the right of the $\tilde{D}\tilde{D}$ curve. The CC curve is identical to $\tilde{C}\tilde{C}$ for $i \leq I$ and strictly below it for $i > I$. It follows that $\bar{x}_i > \tilde{x}_i$ for

[16] The limiting case of the CES as $\sigma \to 1$ is equivalent to Cobb-Douglas preferences. Gains from trade for these preferences can be established directly. With a constant spending share b_i on good i in the home country,

$$\frac{\bar{x}_i}{A(\bar{x}_i)} = \frac{b_i L}{\min[a_i, w^* a_i^*]} + \frac{c_i^*(\mathbf{p}, w^* L^*)}{A(\bar{x}_i)} > \frac{b_i L}{a_i} = \frac{\tilde{x}_i}{A(\tilde{x}_i)}.$$

But this implies $\bar{x}_i > \tilde{x}_i$ for all i and therefore $p_i < \tilde{p}_i$ for all i. The home worker's real income increases in terms of every good.

all i. In this case, trade expands the world output of every good. All prices fall relative to the home wage, and domestic workers benefit as a result.

We have established gains from trade for the home country when its residents have CES preferences, independent of the elasticity of substitution, the pattern of comparative advantage, the pattern of scale economies, and the details of foreign preferences. This goes some way toward overturning the "paradoxical" welfare implications that derive from the standard analysis, and the discomfort generated therefrom.

V. Transport Costs

Until now, we have followed the literature on trade with external economies in neglecting transportation costs. In our setting, it might appear that such costs would limit the scope for a potential producer in a country with natural comparative advantage in an industry to undermine the perverse location of that industry in the other country. Might transport costs reintroduce the potential for multiple equilibria with different patterns of trade? We will show that, when transport costs are small relative to the strength of scale economies, the equilibrium in every industry is unique and the direction of trade is determined by a combination of natural comparative advantage, relative market sizes, and transport costs. For larger trading costs, multiple equilibria can arise, but these do not involve alternative directions of trade. Rather, the multiplicity exists in industries that lack a pure-strategy equilibrium and it concerns the probability that a good is produced in a single, well-defined location versus the probability that it is produced everywhere and not traded at all. For every good, trade can flow in only one direction.

Before proceeding, we need to specify the market structure that we will consider. Models with oligopolistic competition and trade are distinguished by whether markets are deemed to be *segmented*, so that firms can announce arbitrarily different prices in different geographic locations, or rather *integrated*, so that price differences cannot exceed the cost of shipping between the two markets.[17] In our setting, the two market structures yield identical outcomes in the absence of transport costs, so we did not need to address this issue before now. However, equilibria for segmented and integrated markets differ in the presence of trading costs. We will pursue here the slightly simpler case of segmented markets, although the interested reader can readily apply our methods to verify that outcomes with integrated markets are qualitatively similar.

We assume from now on that $t_i > 1$ units of good i must be shipped from any location in order to deliver one unit of the good to the other country. We also assume that scale economies are strictly positive (i.e., $A_i'(\cdot) > 0$ for all i). We consider first the case in which t_i is small for all i, in a sense that will become clear. We then turn to larger trading costs.

[17] See, for example, Venables (1994), who compares outcomes for a Bertrand duopoly with constant production costs and ad valorem trading costs under the alternatives of segmented and integrated markets.

V.A. Small Transport Costs

The addition of transport costs introduces the possibility that it will be attractive for a firm or firms to produce only for their local market. Such firms would enjoy an advantage relative to foreign producers inasmuch as they could serve the market without incurring the shipping costs. However, they would face a disadvantage relative to firms that serve both markets from their smaller scale of production. The case of "small" transport costs arises when, for every good, the former potential advantage from local production is outweighed by the latter cost.

Let us ignore for a moment the possibility that a firm may target a single market. If all firms contemplate selling both locally and abroad, competition among producers in a given location will bid the pair of prices down to the firms' cost of serving the respective markets. If home firms succeed in capturing both markets, these costs are $wa_i/A_i(x_i + x_i^*)$ and $wa_it_i/A_i(x_i + x_i^*)$ for the home and foreign markets, respectively.[18] Then, no deviant firm in the foreign country can profit by shaving these prices and capturing both markets if

$$\left[\frac{wa_i}{A_i(x_i + x_i^*)} - \frac{w^*a_i^*t_i}{A_i(x_i + x_i^*)}\right]x_i + \left[\frac{wa_it_i}{A_i(x_i + x_i^*)} - \frac{w^*a_i^*}{A_i(x_i + x_i^*)}\right]x_i^* \leq 0$$

or if

$$\frac{a_i}{a_i^*} \leq \frac{w^*}{w}\frac{t_ix_i + x_i^*}{x_i + t_ix_i^*} \ . \tag{5}$$

If, on the other hand, the inequality in (5) runs in the opposite direction, then foreign firms can sell at their costs of serving the home and foreign markets, $w^*a_i^*t_i/A_i(x_i + x_i^*)$ and $w^*a_i^*A_i(x_i + x_i^*)$ respectively, and no deviant home firm can profit by undercutting this pair of prices.

Suppose now that (5) is satisfied and consider a foreign deviant that seeks to undermine production in the home country by capturing only its local market. It could do so by announcing a pair of prices such that its local price were a bit below $wa_it_i/A_i(x_i + x_i^*)$ and its price for exporting to home consumers were prohibitively high. This deviation is unprofitable if and only if

$$\left[\frac{wa_it_i}{A_i(x_i + x_i^*)} - \frac{w^*a_i^*}{A_i(x_i^*)}\right]x_i^* \leq 0$$

or

$$\frac{a_i}{a_i^*} \leq \frac{w^*}{w}\frac{A_i(x_i + x_i^*)}{A_i(x_i^*)\,t_i} \ . \tag{6}$$

Analogously, if (5) is violated, then a deviate home firm will not be able to profit by undercutting foreign producers in (only) the home market if and only if

$$\left[\frac{w^*a_i^*t_i}{A_i(x_i + x_i^*)} - \frac{wa_i}{A_i(x_i)}\right]x_i \leq 0$$

[18] Although we continue to take the home wage as numeraire, we will retain w in the expressions in this section, because it improves the clarity of the arguments.

or

$$\frac{a_i}{a_i^*} \geq \frac{w^*}{w} \frac{A_i\left(x_i\right) t_i}{A_i\left(x_i + x_i^*\right)} \ . \tag{7}$$

Now notice that, when t_i is close to one, (5) implies that (6) is satisfied while the opposite inequality from (5) implies that (7) is satisfied.[19] In other words, a deviation to serve only one market can never pay when transport costs are small, because such a deviation requires the sacrifice of scale economies.

For t_i small, the equilibrium in each industry is unique. In this case, good i is produced only in the home country if (5) is satisfied and good i is produced only in the foreign country if (5) is violated. Competition selects the location for production that minimizes the total cost of producing and shipping the equilibrium quantities. For $t_i \gtrapprox 1$, this corresponds to a ranking by natural comparative advantage.[20] But for t_i somewhat larger, a good that ranks highly in natural comparative advantage for the home country may nonetheless be produced in the foreign country, if the aggregate demand for the good is sufficiently greater there. What the foreign producers lose in their comparative disadvantage they potentially can make up with their lesser shipping costs.

V.B. Large Transport Costs

We have just seen that small trading costs do not threaten trade. As transport costs grow larger, however, the strategy of pricing to serve only local consumers becomes increasingly attractive. For t_i sufficiently large, a deviant can undermine any pure-strategy equilibrium in which output is concentrated in a single country. Consider, for example, an industry i with trading costs and other parameters such that (5) is satisfied but (6) is violated. In such circumstances, good i cannot be produced only at home, because the implied price to foreign consumers would leave room for a profitable deviation by local producers in the foreign country.

We might hypothesize, then, that there exists an equilibrium with production of good i in both countries and no trade. Indeed, this is correct when t_i is sufficiently large such that

$$\left[\frac{wa_i}{A_i\left(x_i\right)} - \frac{wa_i}{A_i\left(x_i + x_i^*\right)}\right] x_i + \left[\frac{w^* a_i^*}{A_i\left(x_i^*\right)} - \frac{wa_i t_i}{A_i\left(x_i + x_i^*\right)}\right] x_i^* \leq 0 \ . \tag{8}$$

The first-term in (8) represents the profit that a home firm selling to both markets could earn by undercutting slightly the local price in a potential equilibrium with no trade. The second term represents the loss that such a firm would suffer by selling abroad despite the high transport costs. If the sum of these two is negative, an equilibrium with local production in both countries and no trade in good i would be immune to a profitable deviation by producers in the home

[19] This follows from the fact that $\left(t_i x_i + x_i^*\right) / \left(x_i + t_i x_i^*\right) \approx 1$, $A_i\left(x_i + x_i^*\right) / A\left(x_i^*\right) t_i \approx A_i\left(x_i + x_i^*\right) / A\left(x_i^*\right) > 1$, and $A_i\left(x_i\right) t_i / A_i\left(x_i + x_i^*\right) \approx A_i\left(x_i\right) / A_i\left(x_i + x_i^*\right) < 1$ when $t_i \approx 1$.

[20] We use the symbol $\gtrapprox$ to mean "greater than but approximately equal to".

country that might seek to capture both markets.

But suppose that (6) is violated only slightly; i.e., $a_i/a_i^* \gtrapprox w^* A_i\left(x_i + x_i^*\right)/wA_i(x_i^*)t_i$. Then, an equilibrium with dispersed production of good i would be undermined by a deviant in the home country that shaves prices in both markets. The losses that such a deviant would suffer on exports—represented by the second term in (8)—would be quite small. These would be more than compensated by its strictly positive profits on local sales—the first term in (8)—that would result from its greater productivity compared to its rivals. In short, if (6) is violated only slightly, then (8) must be violated as well. This situation admits no industry equilibrium in pure pricing strategies.

In such circumstances, there does exist an equilibrium in mixed strategies. In this equilibrium, firms in one country price at the unit cost they would achieve by serving only their local market while firms in the other country randomize among a pair of pricing strategies, one that yields only local sales and another that ensures sales in both markets. For clarity, we will refer to these respectively as the "local-pricing strategy" and the "global-pricing strategy." We proceed now to characterize the symmetric, mixed-strategy equilibrium; i.e., we describe an equilibrium in which all firms in a given country pursue the same (mixed) strategies.

For concreteness, suppose that (5) is satisfied. Recall that there are $n_i \geq 2$ home firms and $n_i^* \geq 2$ foreign firms. The latter firms price so as to compete only for their local market; i.e., they set a high price for export sales and a local price of $w^* a_i^*/A_i\left(x_i^*\right)$. The home firms randomize. With probability q_i, they too compete only for local sales, with a high price for sales in the foreign country and a local price of $wa_i/A_i\left(x_i\right)$. With probability $1 - q_i$, they contest both markets, shaving the price $w^* a_i^*/A_i\left(x_i^*\right)$ to capture the foreign market, while setting a price $p_i > wa_i/A_i\left(x_i + x_i^*\right)$ for home sales. The latter strategy entails losses on foreign sales, but profits on home sales that potentially can compensate.

What values of p_i and q_i are consistent with equilibrium? First note that p_i cannot exceed $wa_i/A_i\left(x_i\right)$. If it did, a deviant firm could announce a price slightly below p_i for home sales, and a very high price for foreign sales, and thereby earn positive expected profits.[21] Also, p_i must be such that each home firm is indifferent between the alternative strategies over which it randomizes. The local-pricing strategy yields zero profits in all states of the world.[22] The

[21] With such a strategy, the deviant would earn zero profits whenever any of its rivals quoted the price $wa_i/A_i\left(x_i\right)$. But, with probability $\left(1 - q_i\right)^{n_i - 1}$, all would quote the price $p_i > wa_i/A_i\left(x_i\right)$ for home sales and $w^* a_i^*/A_i\left(x_i^*\right)$ for foreign sales. The latter would ensure these firms of the entire foreign market. The deviant would capture the benefits of large scale without suffering any losses on exports, and it would profit from its home sales.

[22] If some rival announces $p < wa_i/A_i\left(x_i\right)$, then a firm quoting $wa_i/A_i\left(x_i\right)$ makes no sales and no profits. If no rival announces this price, then the firm makes positive local sales, but sells at cost.

alternative strategy yields profits proportional to

$$\pi_i(p_i) = \left[p_i - \frac{wa_i}{A_i(x_i + x_i^*)}\right] x_i + \left[\frac{w^* a_i^*}{A_i(x_i^*)} - \frac{wa_i t_i}{A_i(x_i + x_i^*)}\right] x_i^*$$

where the factor of proportionality reflects the number of firms that realize the global-pricing strategy and thereby share the two markets. Since expected profits from the global-pricing strategy are proportional to $\pi_i(p_i)$, a home firm will be indifferent between the alternative strategies if and only if

$$\left[p_i - \frac{wa_i}{A_i(x_i + x_i^*)}\right] x_i + \left[\frac{w^* a_i^*}{A_i(x_i^*)} - \frac{wa_i t_i}{A_i(x_i + x_i^*)}\right] x_i^* = 0 \ . \tag{9}$$

Note that (9) pins down p_i. The second term in (9) is negative, because (6) is violated in the case under consideration. These are the losses that home firms suffer on their foreign sales as a result of the high transport costs. The first term reflects the positive profits that the home firms collectively earn on local sales if they succeed in capturing the world market. Competition bids the home price down to the point where the home profits just offset the foreign losses.

The common mixing probability q_i must be such that no home firm has any incentive to deviate. Two sorts of potential deviations come into play. First, a home firm might try to avoid the losses associated with foreign sales, while still enjoying profits from home sales when some of its rivals play the global-pricing strategy. Second, a home firm might deviate to a local price higher than p_i to capitalize on the event that all of its rivals realize the local-pricing strategy. We address each of these deviations in turn.

Suppose that $n_i - 1$ firms pursue the mixing described above and consider a deviant that sets a price just below p_i for local sales and a very high price for export sales. Such a deviant would never sell abroad and thus would never suffer losses on any export sales. If one or more of its rivals happen to realize the global-pricing strategy, then these rivals will capture the foreign market. The deviant home firm would then capture the home market (thanks to its slightly lower price), while benefiting from the external economies generated by its rivals' foreign sales. In such states, the deviant would earn a positive profit. However, if all of its home rivals instead realize the local-pricing strategy, the deviant will be left with the home market and with a productivity level that implies losses. The probability of the latter outcome must be sufficiently high so that expected profits from the deviation are non-positive. In particular, we require

$$\left(1 - q_i^{n_i - 1}\right)\left[p_i - \frac{wa_i}{A(x_i + x_i^*)}\right] x_i + q_i^{n_i - 1}\left[p_i - \frac{wa_i}{A_i(x_i)}\right] x_i \le 0 \ . \tag{10}$$

The term in the first square brackets in (10) represents the gain from capturing the entirety of the home market when at least one home rival realizes a price low enough for export sales. The term in the second square brackets represents the losses that result for the deviant when none of its home rivals export. Each term is weighted by its respective probability of occurrence. Since the term in the first square bracket is positive and that in the second bracket is negative, there must be a finite range of values of q_i, including $q_i = 1$, that satisfies (10).

Now consider the deviation that entails a price just below $wa_i/A_i(x_i)$ for home sales and one just below $w^*a_i^*/A_i(x_i^*)$ for foreign sales; that is, the deviant prices to ensure itself of export sales and hopes for an especially large profit in the home market by pricing above p_i. If no other home firm realizes the global-pricing strategy, the deviant will earn positive profits. But if at least one other home firms realizes this strategy, the deviant will capture no home sales (due to its higher price) and yet it will share in the foreign losses. The mixing probabilities for the others must be such that, overall, this strategy yields the deviant non-positive expected profits, or that[23]

$$q_i^{n_i-1}\left[\frac{wa_i}{A(x_i)}-\frac{wa_i}{A(x_i+x_i^*)}\right]x_i+\frac{1-q_i^{n_i}}{n_i(1-q_i)}\left[\frac{w^*a_i^*}{A_i(x_i^*)}-\frac{wa_it_i}{A_i(x_i+x_i^*)}\right]x_i\leq 0\ . \qquad (11)$$

The left-hand side of (11) is increasing in q_i in the range where (6) is violated, which implies that if (11) is ever satisfied, it will be so for a range of q_i that includes $q_i = 0$.

Any q_i that satisfies (10) and (11) can be sustained in a mixed-strategy equilibrium. This suggests the possibility of multiple equilibria that share certain qualitative features but differ in their mixing probabilities. In all such equilibria, either good i is produced and exported by the home country or it is not traded at all.

V.C. Implications for the Trade Pattern

We have seen that, when (6) and (7) are satisfied—as they must be when t_i is close to one—the equilibrium in industry i is unique and always involves trade. If (5) is satisfied, good i is produced only in the home country; otherwise, it is only produced in the foreign country. Notice how this conclusion differs from that in DFS, where transport costs always imply a range of nontraded goods. In DFS, comparative advantage is the sole motivation for trade. Then, small transport costs are sufficient to choke off trade in industries where comparative cost advantages are slight. Here, trade takes place not only for reasons of natural comparative advantage, but also due to scale economies. Small transport costs will not kill trade when external economies are strictly positive.

When trading costs are larger, industries fall into one of three categories: (i) industries with a unique pure-strategy equilibrium characterized by production in only one country; (ii) industries with a unique pure-strategy equilibrium characterized by dispersed production and an absence

[23] The first term in (11) represents the profits that the deviant earns on home sales when it alone captures export sales times the probability that this event occurs. The second term represents the deviant's expected losses on export sales, considering that it may share the foreign market with zero, one, two, etc. of the other home firms.

of trade; and (iii) industries with a mixed-strategy equilibrium in which either production is concentrated or the good is not traded.

To see how these possibilities fit together, let us reconsider a good for which (5) holds, i.e., a good in which the home country has a sufficiently strong natural comparative advantage to make this country the sole candidate for exporting. Recall that the smaller is a_i/a_i^*, the stronger is the home country's natural comparative advantage in industry i. If a_i/a_i^* is so small that (6) also holds, then the industry will be concentrated in the home country, as we have already seen. No firm in the foreign country can offer a price to local consumers that generates sales and covers its per-unit costs. If a_i/a_i^* is not very small, but rather large enough that (8) holds, then production takes place in both countries.[24] Let $\bar{a}_i$ denote the critical value of a_i/a_i^* such that (8) holds as an equality. Note that $\bar{a}_i$ may or may not be larger than the left-hand side of (5); if it is not, then there are no values of a_i/a_i^* for which good i is always nontraded. Finally, for all values of a_i/a_i^* that satisfy (5) but not (6) or (8), the industry equilibrium must involve mixed strategies.[25] At the lower end of this range, where (6) is just violated, (11) holds only for $q_i \gtrapprox 0$. At the upper end of the range (if one exists), where $a_i/a_i^* \lessapprox \bar{a}_i$, (10) holds only for $q_i \lessapprox 1$. In other words, for parameters such that the condition for an equilibrium with specialization and trade just fails, the home firms almost surely choose the global-pricing strategy, whereas for parameters such that the condition for an equilibrium with dispersed production just fails, the home firms almost surely choose the local-pricing strategy.

VI. Concluding Remarks

External scale economies need not overturn orthodox presumptions about the pattern and welfare consequences of international trade. By assuming a continuum of industries in which producers behave as Bertrand competitors, we have found that the pattern of trade is uniquely determined and that "natural" productivity advantage governs the chain of comparative advantage. We have also identified a class of preferences, namely CES, for which gains from trade are

[24] For (8) to be satisfied, we must have

$$\frac{x_i}{A_i\left(x_i+x_i^*\right)}-\frac{x_i}{A_i\left(x_i\right)}+\frac{t_i x_i^*}{A_i\left(x_i+x_i^*\right)}>0$$

and

$$\frac{a_i}{a_i^*} \geq \frac{w^*}{w} \frac{A\left(x_i+x_i^*\right)}{A\left(x_i^*\right) t_i}\left[\frac{t_i x_i^*}{t_i x_i^*-x_i\left(\frac{A\left(x_i+x_i^*\right)}{A\left(x_i\right)}-1\right)}\right] \equiv \bar{a}_i .$$

[25] The fact that

$$\frac{t_i x_i^*}{t_i x_i^*-x_i\left(\frac{A\left(x_i+x_i^*\right)}{A\left(x_i\right)}-1\right)}>1$$

implies that the right-hand side of (6) is less than $\bar{a}_i$. This in turn means that, except when transport costs are small, the set of values of a_i/a_i^* at which both (6) and (8) are violated is non-empty.

assured.

The extensive literature on external economies typically posits a competitive environment with many small firms. Researchers have used the small size of producers in a competitive equilibrium to justify an assumption that firms neglect the impact of their output decisions on aggregate scale and productivity. We have argued that it is not the equilibrium size of a firm that matters for its assessment of productivity, but rather the nature of industry competition and, in particular, whether a firm can foresee growing large as a result of its actions. When industry participants believe that they can grow large, the potential for internalization is enough to coordinate locational decisions even when externalities remain pervasive in equilibrium.

The literature also typically focuses on an environment with only two industries. Often the assumption is made for convenience, although it might be appropriate if external scale economies spread throughout broad sectors. If, instead, external economies are generated by specific activities, a model with many activities may be more appropriate. Then, country size no longer plays the role that it does in the standard analysis, because a small country can accommodate all of some activities so long as it hosts relatively few of them.

In this paper, we have considered a static environment in which productivity in an industry and country depends only on current local output. Our results can readily be extended to settings with dynamic externalities wherein productivity depends also on past experience. Suppose, for example, that industry productivity is determined by fundamentals and by output last period. Then, natural comparative advantage (given by fundamentals as well as initial conditions) determines allocations in the first period, as in DFS. In the second period, the initial advantages are strengthened by the specialization pattern of the prior period, and the pattern of trade gets 'locked-in', as in Krugman (1987). Hence, initial comparative advantage uniquely determines the subsequent evolution of the economy.[26] Moreover, the logic of the present paper indicates that the outcome will be the same if productivity depends both on current and past output. Equilibrium in the first period would follow natural comparative advantage —as we have shown— and then the dynamic externalities would lock in this trade pattern forever.

These observations may have more general applicability than our simple Ricardian model. For example, Grossman and Rossi-Hansberg (2008) study task trade between similar countries. They assume that firms perform a continuum of tasks to generate output and that productivity in each task depends on how frequently it is performed by all firms in that same location. In that setting, a firm can grow large by serving as a potential supplier of a task to others. Although no outsourcing takes place in equilibrium, the potential for same is enough to coordinate locational decisions for each task. Moreover, the fact that each task is small means that any task might be concentrated in a small country.

[26] Such an economy could still have several steady states, but the initial conditions will determine a unique dynamic equilibrium path. See Matsuyama (1991) for a detailed discussion of a similar argument.

Another relevant application can be found in urban economics. The existence of cities seems evidence of agglomeration economies that are, at least partially, external to the firm. As in our setup, a site can have small or zero productivity if no firm locates there, but could be as productive as New York if ten million people reside nearby. Multiple equilibria are pervasive in models of systems of cities, as sites may or may not host cities depending on whether agents coordinate to live there. To address this problem, the urban economics literature has invoked the actions of large 'city corporations' (Henderson, 1974) that can subsidize factors to encourage the internalization of externalities and thereby coordinate location decisions.[27] Our theory provides an alternative mechanism for coordination in naturally productive sites. If firms engage in Bertrand competition, each will recognize that by locating alone in a "good" site it can potentially achieve efficient scale as a "one-company town." Such a producer might still be small in relation to the aggregate economy, if the number of towns or cities is large. In equilibrium, all towns may house numerous firms, yet the equilibrium allocation of activities across space will be unique. In short, our conclusions about trade with many industries may apply as well to agglomeration with many cities.

Our paper demonstrates that the size of industries (or cities) and the nature of the competition ought to be carefully considered in future modeling of external scale economies.

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[27] Durable housing investments can also serve as a coordinating mechanism in dynamic models, as shown by Anas (1992) and Henderson and Venables (2009).

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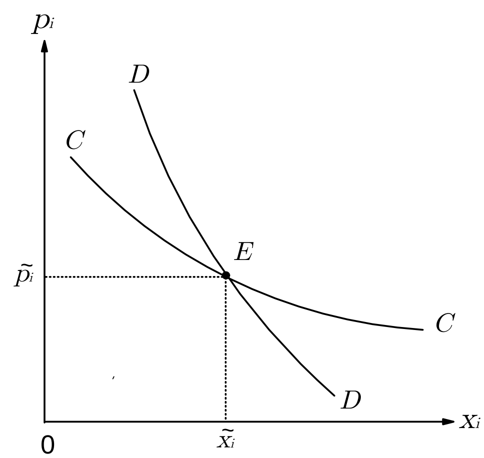
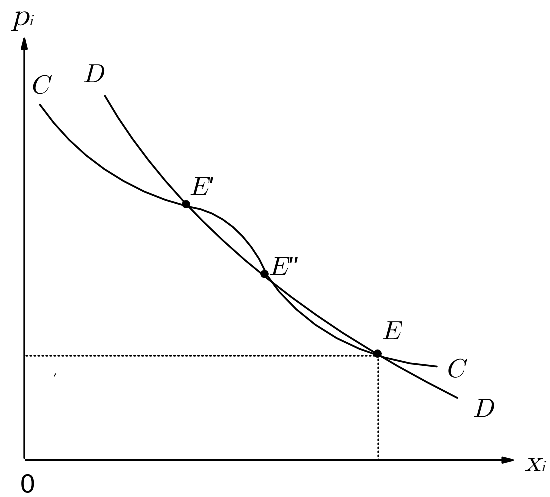
Figure I



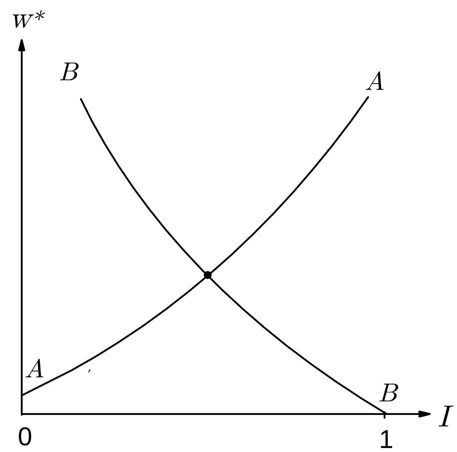
Unique Intersection of DD and CC

Figure II



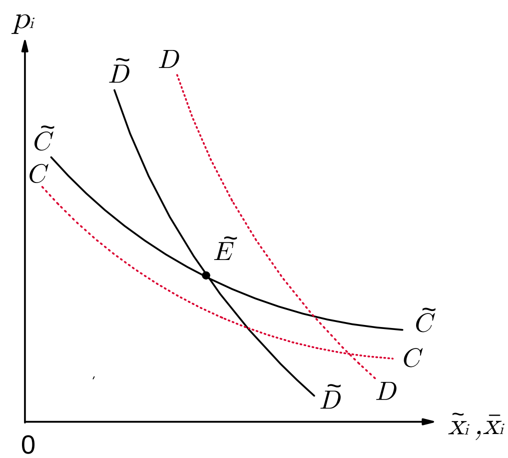
Multiple Intersections of DD and CC

Figure III



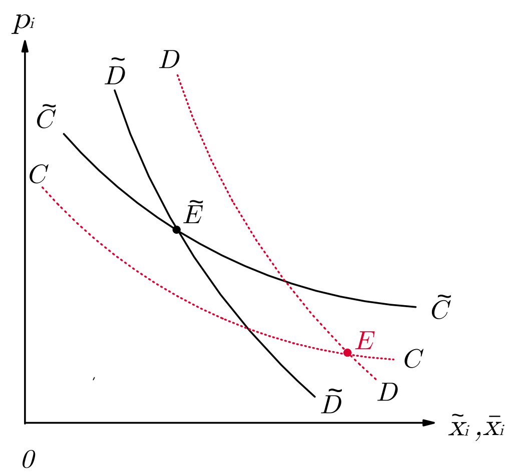
Free Trade Equilibrium

Figure IV



Hypothesized Industry Equilibrium Assuming
Losses from Trade

Figure V



Autarky and Trade Equilibrium for $\sigma < 1$