Lecture 1: The Economic City

WWS 538

Esteban Rossi-Hansberg

Princeton University
What is a City?

- Economics conceptualizes cities as the result of a trade-off between:
  - Agglomeration Effects
    - Can take the form of: Externalities, Amenities, Lack of other Frictions
  - Congestion costs
    - Transport costs of: people, ideas and information, goods, etc.

- Urban public policy depends crucially on what we identify as the key forces
  - Many types of agglomeration and congestion forces lead to externalities and, therefore, inefficient equilibrium outcomes
  - In those cases we need taxes, subsidies, regulation

- Welfare theorems in economics
  - In the absence of externalities, public goods, market power, information frictions, etc. the equilibrium allocation is efficient
Urban Economics

- Most work in the area of urban economics is concerned with measuring and identifying agglomeration and congestion forces.
- We have a good set of empirical papers that point to the importance of many of these forces.
  - Causality problem is the main obstacle.
- However, we need models to understand how policy will affect the allocation and therefore welfare.
  - Policy evaluation requires us to build counterfactuals.
  - Economic policy counterfactuals require theory.
- So large part of the literature is also dedicated to building analytical and quantitative models of cities (and systems of cities) that we can use to evaluate policy.
- There is an important gap between policy analysis and urban economic knowledge: *America 2050 Megaregion Plan*.
The Simplest Model of a City

- Consider a linear monocentric city with density of land equal to 1
- Firms locate at the center, $\ell = 0$
- Use labor to produce an homogenous good according to a production function

$$F(L) = A(\bar{L})L \quad (1)$$

- Agglomeration effect is an externality

$$A(\bar{L}) = a\bar{L}^\alpha \quad (2)$$

  - $\bar{L}$ is the number of workers in a city
  - So the more workers in the city, the more productive are firms (and labor)
  - Marginal product of labor goes up with population size

- Let $w$ be the wage in the city. Firms maximize profits so $\max_L A(\bar{L})L - wL$, and so

$$A(\bar{L}) = w \iff w = a\bar{L}^\alpha \quad (3)$$
The Simplest Model of a City

- Workers are identical and maximize utility, \( U(c) = c \) and can get utility \( \bar{u} \) in any other city
  - So \( \bar{u} \) is the reservation value. Hence they need to get at least \( \bar{u} \) in this city, and in equilibrium exactly \( \bar{u} \)
- Agents live around the center in a unit of land that they rent at cost \( R(\ell) \), where \( \ell \in [-B, B] \) denotes the location of their house
- They need to commute to work at costs \( \tau |\ell| \) in terms of goods (includes both trips)
The Simplest Model of a City: Equilibrium

- All agents in the city get $\bar{u}$ so

$$\bar{u} = w - R(\ell) - \tau |\ell|$$  \hspace{1cm} (4)

- This is the case for all $\ell$

- Land at the boundary of the city can always be used for residential purposes at cost $R_A$. So

$$w - R(\ell) - \tau |\ell| = w - R_A - \tau B$$

- So land rents in the city are given by

$$R(\ell) = R_A + \tau(B - \ell)$$
But $B$ is endogenously determined. In particular, since everyone in the city lives in one unit of land,

$$2B = \bar{L} \iff B = \frac{\bar{L}}{2}$$

and so since by (3)

$$\bar{L} = \left( \frac{w}{a} \right)^{1/\alpha}$$

we obtain that

$$B = \frac{\bar{L}}{2} = \frac{1}{2} \left( \frac{w}{a} \right)^{1/\alpha}$$

(5)
The Simplest Model of a City: Equilibrium

- Hence land rents are given by

\[ R(\ell) = R_A + \tau \left( \left( \frac{w}{a} \right)^{1/\alpha} - \ell \right) \]  

(6)

- Note that \( w \) is also endogenously determined by

\[ \bar{u} = w - R_A - \tau B = w - R_A - \frac{\tau}{2} \left( \frac{w}{a} \right)^{1/\alpha} \]

- Defines a function \( w \left( \bar{u}, \tau, R_A, a, \alpha \right) \). Why? Multiple equilibria?

- The equilibrium city size is then given by

\[ a \left( \bar{L}_E \right)^\alpha - R_A - \frac{\tau}{2} \bar{L}_E = \bar{u} \]  

(7)
Optimal Allocation

- Equilibrium is not optimal
  - Total city output can be improved by adding more workers to the city
- Consider the problem
  \[
  \max_{\bar{L}} A(\bar{L}) \bar{L} - R_A \bar{L} - \tau B^2 - \bar{u}\bar{L} = \max_{\bar{L}} a\bar{L}^{1+\alpha} - R_A \bar{L} - \frac{\tau}{4} \bar{L}^2 - \bar{u}\bar{L}
  \]
- Hence
  \[
  a(1+\alpha)(\bar{L}_O)^\alpha - R_A - \frac{\tau}{2} \bar{L}_O = \bar{u} \tag{8}
  \]
- Compare (7) and (8) to conclude that \( \bar{L}_O > \bar{L}_E \)
  - Since \( a(1+\alpha)(\bar{L}_O)^\alpha - \frac{\tau}{2} \bar{L}_O \) decreasing in \( \bar{L}_O \) by second order condition
  - Sufficient to impose that \( \tau > 2a(1+\alpha)\alpha \)
    - Otherwise optimal city is infinitely large
- Optimal policy is to increase \( w \) by a fraction \( 1+\alpha \)
  - Subsidize employment by firms, or city population, and charge workers in the whole country
Detroit
Detroit

Detroit MI By Home Tract

- Detroit City Outline
- Downtown Tract Outline
- County Outline
- Major Highway
- Lake St Clair

Employed Residents per SqMi

- 2 - 199
- 200 - 399
- 400 - 699
- 700 - 899
- 900 - 1,199
- 1,200 - 1,499
- 1,500 - 1,999
- 2,000 - 4,843

Total number of workers in the census tract normalized by square mile in tract.
The Wall and Berlin

Panel A: Greater Berlin Land Prices 1936
Panel B: West Berlin Land Prices 1936
Panel C: West Berlin Land Prices 1986
Panel D: Greater Berlin Land Prices 2006
Panel E: West Berlin Land Prices 2006

Figure 2: The Evolution of Land Prices in Berlin Over Time
Generalizations

- Model can be generalized in multiple ways
- Two important ones are:
  - Circular city
  - Firms can use land for production and so business areas emerge
  - Density of employment and residents could be endogenous
  - External effect that depends on distance to other workers
- The equilibrium and optimal allocations are studied in Lucas and Rossi-Hansberg (2002) and Rossi-Hansberg (2004)
Monocentric city is an equilibrium only for small commuting costs

Higher commuting costs (τ) result in mixed areas at the center
- Areas in which both firms and residences coexist
  - Realistic feature of many cities: residents in downtown areas commute to work by foot

If externality decays fast, possibility of many business areas
Employment Density in LA in 1990

Figure 1. Employment Density, Los Angeles County, 1990, at Different Resolutions.
Source: Authors' plots of data from Southern California Association of Governments.

Thus the fractal has a similar shape no matter what scale is employed for viewing it. If the original element is one-dimensional, the fractal's length becomes infinite as one measures it at a finer and finer resolution; the classic example is a coastline. One plus the elasticity of measured length with respect to resolution is known as the fractal dimension.

For example, a coastline might have length \( L \) when measured on a map that can just resolve 100-meter features, and \( L \times 10^{D-1} \) when 10-meter features can be seen; its fractal dimension would then be \( D \), at least within that resolution range. A perfectly straight coastline has fractal dimension one, since its length does not increase with the level of resolution.

Geographers have used fractals to examine the irregularity of the line marking the outer edge of urban development in a particular urban region. Michael Batty and Paul Longley (1994, pp. 174-79) use data on land development in Cardiff, Wales, to define such a boundary to an accuracy as fine as 11 meters. Their best estimates of the fractal dimension of this boundary are between 1.15 and 1.29. (By way of comparison, Britain's coastline has fractal dimension 1.25, Australia's 1.13.) Surprisingly, they find that the fractal dimension of Cardiff's outer edge of development declined slightly over the time period examined (1886 to 1922), a period of significant transport improvements, mainly in the form of streetcars. They conclude that "the traditional image of urban growth becoming more irregular as tentacles of development occur around transport lines is not borne out" (p. 185).

More significantly, one can use fractals to represent two-dimensional development patterns, thereby capturing irregularity in the interior as well as at the boundary of the developed area. For example, a fractal can be generated mathematically by starting with a large filled-in square, then selectively deleting smaller and smaller squares so as to create self-similar patterns at smaller and smaller scales. Such a process simulates the existence of undeveloped land.
Optimal Allocation of Generalized Model

- The optimal allocation has no mixed areas
- Location specific subsidies can implement optimal allocation
- High commuting costs result in multiple business areas

![Diagram of optimal allocation with phases:]
- Initial phase: High density of firms at the center, low commuting costs.
- Transition phase: As commuting costs increase ($\tau \uparrow$), the equilibrium shifts to a more dispersed pattern with firms at the center and residences at the boundary.
- Final phase: High commuting costs result in a ring of businesses and residences at the center and boundary.
Implication for NYC in 1992

Business land is more concentrated in the efficient allocation than in equilibrium, both in terms of area and densities.

A labor subsidy can eliminate this disparity by reducing the cost of labor for firms in particular areas.

Lorenz Curves of Employment: New York City
Optimum with Low Commuting Costs

Fig. 4. Optimal allocation for $\kappa = 0.001$. 

Fig. 4. Optimal allocation for $\kappa = 0.001$. 

ERH (Princeton University) Lecture 1: The Economic City
Fig. 5. Optimal allocation for $\kappa = 0.01$. 
Comparing the Equilibrium and Optimal Allocations

Fig. 6. Optimal density of workers.
Policy Examples: Labor Subsidies

Fig. 8. Labor subsidy.

![Graph showing labor subsidy](image-url)
Policy Examples: Zoning Restrictions

Fig. 9. Optimum and equilibrium with zoning restrictions.