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ECONOMETRIC ANALYSIS OF THE UNITED STATES MANGANESE PROBLEM

Final Report Part I

Econometric Research Program
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PREFACE

This study contains the Final Report, Part I, of the investigation of programming the supply of manganese for the United States. It is the third technical report distributed on this topic by the Econometric Research Program (formerly Economics Research Project) of Princeton University. The second part of the final report will be distributed at a later date, when the last computations have been concluded.

The first non-stochastic model discussed below was originated and developed by Herman F. Karreman who also bore the main burden of coordinating the entire work. I want to thank him for the devotion and circumspection which he has brought to the entire study.

The computation for the first model was found and developed by Philip Wolfe.

The second stochastic model was originated by Harlan Mills and in part developed by Franklin R. Shupp. The computations of the model were based on a method modified and applied for the purpose by Stuart Dreyfus.

The actual computations for the first model were made on IBM 704's at the RAND Corporation and at the Institute of Mathematical Sciences of New York University. Herman F. Karreman and Philip Wolfe were assisted by Leola Cutler; William Dorn also contributed to the computations at New York University.

The computations for the second model were made on the Maniac computer of Princeton University. They were carried out by Stuart Dreyfus and Franklin R. Shupp. The latter also had a large share in the collection of widely scattered data.

We wish to express thanks to the three mentioned institutions for making their computing facilities so readily available.

Oskar Morgenstern
Director, Econometric Research Program

October 15, 1958
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GENERAL INTRODUCTION

The problem dealt with in this report arises in connection with the American manganese situation. Confronted with the need for manganese in steel production, the subsequent need to import, the availability in this country of various low grade ores, the existence of a technology for upgrading these ores, and the government policy of building a stockpile of imported ores, there is the question of determining the optimal decisions in view of the possibility that various forms of political disturbances might arise. These possibilities are cold war or hot war, each with minor or major blockade, etc. Here is, therefore, a problem in decision making. A decision has to take into account the data, the objective, the acts of nature, the actions of the opponents and the availability of scientific methods in order to arrive at a calculated decision, versus the mere intuitive method of guessing at it by means of common sense and general experience. Furthermore, the decision has to be made for a considerable time interval and conditions may change during this period so that they affect the optimality of the initial decision.

A problem of this type cannot be solved by conventional methods normally used in such cases. Therefore it became worthwhile to study whether new techniques developed over the last few years might lend themselves to the task. Indeed, since these methods are still under active development, one could ask whether additional techniques might be invented still better able to cope with the great amount of complexity encountered.

Results of this study are outlined in the following pages. The investigation is based on a considerable amount of empirical, descriptive work. An attempt was made to come as close to the true data as possible and, where they were lacking, to make estimates upon which it would be hard to improve without fundamentally new information coming to light.

Next, models had to be constructed. It is important to realize what is demanded of a model: (a) it must be similar to reality, but it cannot be identical with it; a model simplifies the infinitely complex real world; (b) it must not be too simple, since it would otherwise be too far away from reality; (c) it must be amenable to thorough logico-mathematical exploration and formulation; (d) it must be possible to solve numerically for the unknowns.

The subsequently described models fulfill all these requirements. Now that they are available it would be possible to expand and improve them, although with considerable difficulty. They could be expanded if more data became available, or if new processes were added; they could be improved if new computing techniques and facilities were developed. As it is, some computations described below went to the limit of the capacity of the most advanced electronic computers now available. In addition, new computing techniques had to be designed in order to cope with the complexity of the present models. It can be surmised that more work along these lines would be promising, probably resulting in the incorporation of a greater variety of possibilities into the models, making them even more useful than they are at present.

A model formulates a scientific problem, and the numerical solution provides as detailed an answer as is possible. But the model does not make recommendations. As a consequence our computations must be seen in this light. We do not recommend that one beneficiation process be preferred over another, or that the stockpile of manganese be reduced or increased, etc. The different models state one or the other of these things as the outcome of the formulation of the problem and the ensuing computations. It is now possible to use our models by putting different data into their framework thus testing what differences this produces in the answers. It will be seen that this is a somewhat delicate process to be done only with the greatest care. An important illustration of this is offered by the two computations made for the econometric model described in the first part below. There, the same constraints, the same import prices were assumed for both, but a different initial stockpile and two different sets of domestic production costs were introduced. The results obtained are quite different.

For the policy maker this spells an important lesson: A great effort should be made to develop useful models of the type shown here. Then he must make sure that he possesses the best possible information to use with the computation. And he must try to formulate his aims as clearly as he can. If uncertainty about the data or about the aims exists, a number of computation runs is easily arranged once the computing codes have been written. The policy-maker may then pick the most suitable procedure, introducing into the final selection also other viewpoints, frequently of a strictly political nature, the existence of pressure groups, geographical considerations, etc., outside most models.

It is interesting, however, to note, as the reader will discover in greater detail below, that the various approaches in our models yield in common one answer in spite of all other differences: the stockpile is played down or consumed rapidly and the improvements in the technology of upgrading manganese ore, as reflected in cost estimates, make themselves felt very strongly over the future years of the assumed emergency. In the details there are wide differences which could never have been guessed at and which could not have been found with conventional means.

The work described in this report, as well as the further refinements which will be the subject of the second part to be published later, are a first step in bringing modern methods to bear on government decision making in an area where advanced tools have not so far been available. From experience gained in other fields it is safe to predict that once exact methods are available the need for their further development and application will rapidly expand. As it happens so frequently with scientific work, the outlines of new problems and new approaches become visible as soon as the first ones have been solved. In this manner progress is achieved.

Specifically, the present report deals with two different situations: the first part makes a fixed forecast of a political emergency-type situation; the second part allows for uncertainty in future political situations. There are advantages to both approaches. The first allows for a greater wealth of detail by distinguishing between various types of alloys, import areas, beneficiation processes and domestic sources. The second has to sacrifice some of this information but gains in flexibility by allowing for many other variants, in particular by assigning an important role to the chance factor. This illustrates some of the more general remarks made above about the possibilities associated with model construction.

A NON-STOCHASTIC MANGANESE MODEL

Case of a fixed political forecast and an extensive description of the manganese technology

INTRODUCTION

In this case the underlying assumption is that one has a good insight in the political situation for the next couple of years. To be more specific, it is assumed in this case that a limited war will break out in the next year and that it will last for about six years. This war will be "limited" in the sense that it will be fought with conventional weapons, in particular submarines, used by the enemy to cut off the supply of foreign ores, i.e., manganese.

The amounts of manganese required to keep the iron and steel industry in running condition under the given circumstances are assumed to follow a certain pattern. The latter is directly related to the demand for steel which is expected to exist in these emergency years.

The supply conditions outside the United States are assumed to be the same throughout all these years. This does not mean, however, that these ores will always be offered at the same prices; on the contrary, it is assumed that the prices of these foreign ores depend to a certain extent on the quantities that will be bought by the United States. The cost of transportation is assumed to reflect the difficulties of getting the ores to the United States under the given circumstances.

Two different situations at the start have been assumed to exist in order to show the dependency of the outcome on the starting conditions. The differences lie in the amounts of ore assumed to be stockpiled in the years previous to the emergency period as well as in the costs at which manganese can be obtained from domestic sources. The reason for making a distinction at this point is that the government can influence, here more than anywhere else, the starting conditions. This is very clear in the case of stockpiling ores, but the government can to a certain extent also influence the rate at which the technology develops inside the country and the domestic production costs decrease.

A distinction was made between the two forms of manganese alloys used in the production of steel, viz., ferro-manganese and silico-manganese. This distinction was necessary because certain types of steel require the use of ferro-manganese whereas silico-manganese is preferred to a combination of ferro-manganese and ferro-silicon in the production of other types of steel.

Another distinction between three import areas: 1) South-Asia 2) South and West Africa 3) Latin America, was made for strategic reasons.

A distinction was also made between the principal sources of domestic manganese namely 1) the Cuyuna deposit in Minnesota 2) the Open-Hearth slags of the steel mills and 3) the Aroostook deposit in Maine. The manganese coming from each of these sources has its own special character and has to be upgraded in a particular way. This led to the selection of 4 upgrading processes, viz., 1) the Sylvester-Dean and 2) the Wright process, both for upgrading Open-Hearth slags 3) the Dean process for Cuyuna and 4) the Udy process for Aroostook ore.[1]

The diagram shown on the following page pictures the whole technology underlying the present model.

THE MODEL

An econometric model of the dynamic operation of the entire manganese system, as shown on the diagram of the following page has been constructed. This model includes variables representing:

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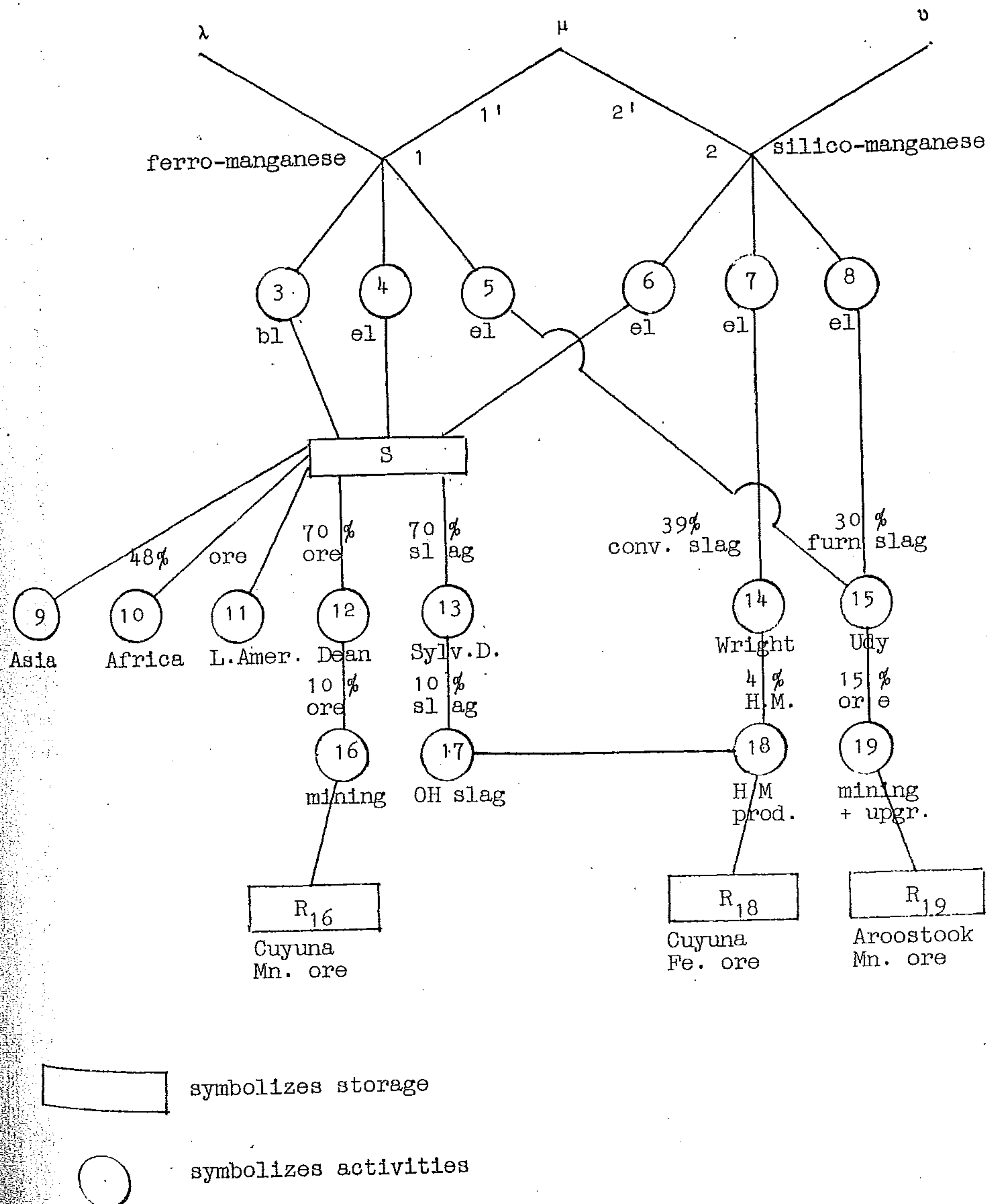
[1] The way in which these 4 processes have been selected out of more than a dozen beneficiation processes is described in "Manganese Model No. 1a", internal paper of February 12, 1958.



symbolizes storage

symbolizes activities

1) quantities of manganese contained in the stockpile and the deposits at the start of the period under consideration
2) yearly manganese consumption requirements of the iron and steel industry
3) yearly importation of manganese
4) yearly domestic mining, beneficiation and processing of manganese
5) capacities of domestic facilities

and relations among these variables representing:

1) material balances at each inventory and stockpile point
2) limitations of electrical energy for manganese beneficiation
3) time to raise domestic capacities
4) limitations of known quantities and qualities of domestic manganese deposits
5) limitations of foreign supplies (influenced by political conditions).

The objective of the model is to determine those activities in the manganese system which make it operate at lowest cost. For this reason various costs have been developed for the system, representing:

1) importation costs, which, in the past, have indicated a strong dependence on the imported quantities
2) domestic manganese upgrading costs — for several proposed processes
3) domestic facility expansion costs
4) inventory and stockpile maintenance costs

Altogether, there are 20 variables and 18 relations for each year of operation — for a six year planning model there are 106 variables and 102 relations (slightly less than 6×20 and 6×18 because of "starting and stopping" conditions). The entire expenditure in the manganese system over a six year period is written as an expression in these 106 variables. The problem, then, is to minimize these expenditures, subject to the variables satisfying the 102 relations required of them. This problem turns out to be one of a type called "quadratic programs". The precise mathematical formulation is given in the Appendix.

To solve this large-scale quadratic programming problem, Philip Wolfe, formerly with this project and now with the RAND Corporation, developed a method which is based on George Dantzig's simplex technique.[1] The first computation was performed at the RAND Corporation in April of this year. A second computation, with a different set of data as input, was made at New York University and came to its completion in August.

This second computation was made to find out to what extent changes in the domestic production cost would affect the solution. For, in order to make the problem computable, it had to be assumed that these costs would be constant per unity of activity. This is a rather strong assumption in view of the rapidly developing techniques of upgrading domestic ores. Thus it seemed essential to make a second computation with a set of lower domestic production costs.

THE DATA

The data falls into two groups. The first group is related to the "constraints", being in this case the requirements that have to be met, the quantities of electric energy that will be available, the manganese content of the domestic sources, etc. The second group of data is related to the costs of importing foreign ores and producing upgraded ores and alloys domestically.

a) The Constraints

The two sets of constraints used for the first and second computation are shown on the following two pages.

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[1] The Simplex Method for Quadratic Programming, by Philip Wolfe. RAND Paper P-1205 of 25 October, 1957. (This is an important achievement in computational theory which this project helped motivate.)

The required amounts of alloys during these six years have been assumed to be the same for both computations; they are:

Requirements (in 10^3 NT of Mn)

	Ferro-manganese only λ	Ferro- or silico-manganese μ	Silico-manganese preferred υ
1st year	340	425	85
2nd year	360	450	90
3rd year	370	460	95
4th year	360	440	95
5th year	350	435	90
6th year	350	440	95
	2130	2650	550

The manganese requirements are based on the quantities of iron and steel, assumed to be produced during the years 1959 through 1964. These quantities are well above what would be produced if only the development in the past were continued.[1] The differences reflect the higher production in the emergency assumed to exist in these years. A comparison between the "normal" and "emergency" requirements of manganese is given in the following table:

Estimated Manganese Requirements

	Normal (1000 Nt of Mn)	Normal 1959=100	Emergency (1000 Nt of Mn)	Emergency 1959=100	Emerg. in % of Corr. Norm.
1959	778.5	100	850	100	109.2
1960	798.8	102.6	900	105.9	112.7
1961	819.8	105.3	925	108.8	112.8
1962	841.4	108.1	895	105.3	106.4
1963	863.6	110.9	875	102.9	101.3
1964	887.1	113.9	885	104.1	99.8

[1] See for this "The Bill of Steel Requirements" by Robert E. Kuenne, Econometric Research Program, (unpublished manuscript) June, 1957; this study will be included in the final report.

As can be seen from this table, the requirements at the beginning of the emergency are well above the normal ones. This is in agreement with what one can reasonably expect in such a situation.

The quantities of electricity assumed to be available are, however, not the same:

Electricity (in 10^6 KWH)

	1st Computation		2nd Computation	
	Available to Udy furn. E_1	Available to other furn. E_2	Available to Udy furn. E_1	Available to other furn. E_2
1st year	100	200	200	1540
2nd year	150	215	300	1630
3rd year	195	225	500	1675
4th year	250	235	800	1620
5th year	315	250	1200	1585
6th year	400	265	1600	1610

The available quantities of electricity are much more liberal in the second computation; still, it may be assumed that these more liberal amounts of electric energy are not outside the realm of possibilities in the emergency situation that is here envisaged.

The quantities of manganese assumed to become available in the form of Open Hearth slags are:

Manganese in O. H. slags (in 10^3 NT) R_{17}

	1st Computation	2nd Computation
1st year	285	570
2nd year	300	605
3rd year	310	620
4th year	300	605
5th year	295	590
6th year	295	595
Total	1785	3585

It has been assumed that the quantities of slags discarded by the steel mills will be about twice as much as has been assumed for the first calculation. Actually, the second sequence reflects the present situation much better than the first one, which might be considered as being somewhat on the conservative side.

Furthermore, the following quantities of manganese have been assumed to be available at the beginning of the first year:

Availability of Manganese (in 10^3 NT)

	1st Computation	2nd Computation
Initial stockpile (S)	1950	950
Cuyuna Mn deposits (R_{16})	4320	4320
Cuyuna Fe desposits (R_{18})	1386	1386
Aroostook deposits (R_{19})	8190	8190

The last figure is the highest that was permitted to enter the calculations; the actual figure is higher.

It has been assumed for the second computation that the amount of manganese contained in the initial stockpile is much smaller than what is assumed for the first computation. The latter covers more than the demand of the first two years, while the former is barely sufficient to meet the requirements of the first year. This is more or less the smallest quantity of stockpiled ore that is permissible in view of the uncertain supply of foreign ores under these circumstances and the time involved in constructing the plants and mines.

Finally the following capacities of the beneficiation plants have been assumed to exist at the beginning of the first year:

Initial Capacities (in 10^3 NT of Mn per year)

	1st Computation	2nd Computation
Udy-alloy furn. (K_5)	0	0
Dean benef. plant (K_{12})	4.5	4.5
Sylv. D. benef. plant (K_{13})	0	0
Wright benef. plant (K_{14})	1	10
Udy benef. plant (K_{15})	0	0
Cuyuna mines (K_{16})	6	6
Hot-Metal plants (K_{18})	1.2	12
Aroostook mines (K_{19})	0	0

b) The Costs

The costs of foreign ores consists of two elements, viz., the export prices of the ores and the cost of bringing them to the United States (the latter involves more than merely the freight rates as will be shown later).

As for the export prices the following functions have been used:

1º export price of Indian ore.

$$p_{9,i} = + 0.044x_{9,i} + 0.011x_{9,i-1} - 0.004x_{10,i} - 0.001x_{10,i-1} - 0.004x_{11,i} - 0.001x_{11,i-1} + 48$$

2º export price of African ore.

$$p_{10,i} = + 0.092x_{10,i} + 0.023x_{10,i-1} - 0.008x_{9,i} - 0.002x_{9,i-1} - 0.004x_{11,i} - 0.001x_{11,i-1} + 36$$

3º export price of Latin American ore.

$$p_{11,i} = + 0.172x_{11,i} + 0.043x_{11,i-1} - 0.010x_{9,i} - 0.003x_{9,i-1} - 0.008x_{10,i} - 0.002x_{10,i-1} + 30$$

where the p's are expressed in \$/NT of Manganese and the x's in 1000 NT's of Manganese while the indices 9, 10 and 11 refer to respectively India, South and West Africa and Latin America.

The coefficients have been estimated on the basis of the price-quantity relationships in the years 1952-1956. Although the adopted estimation procedure was rather simple, it is felt that the estimates are sufficiently accurate to be used for the purposes of this study.

Besides these export prices, there are also the costs of bringing the ores to the United States. Normally the freight rates form the main part of these costs, but in the case of an emergency, due account also has to be given to the insurance costs. The following cost figures (in \$/NT of Manganese) have been used:

	India		Africa		Latin-America	
	freight	insurance	freight	insurance	freight	insurance
1st year	\$75.-	\$100.-	\$45.-	\$50.-	\$30.-	\$30.-
2nd year	86.-	90.-	48.-	45.-	32.-	25.-
3rd year	93.-	80.-	52.-	40.-	34.-	20.-
4th year	100.-	60.-	55.-	30.-	36.-	15.-
5th year	107.-	40.-	58.-	20.-	38.-	10.-

The freight rates have been estimated on the basis of information collected for the years 1953-1957 (1st half). It has been assumed that the freight rates during the first year of the emergency will be approximately 1.5 times those of the fall of 1956 (Suez Canal crisis) and from then on will continue the rising trend of the past.

On the basis of past experiences the insurance premiums are assumed to lie in the first year somewhat above the freight rates, but thereafter to dwindle down.

Finally, it should be remarked that the costs of foreign ores have been assumed to be the same in both computations.

However, the domestic production costs used in the first computation differ in many respects from those used in the second one as can be seen from the following two tables. The production costs shown in these tables do not include the cost of the raw material. So, the alloy costs shown here do not include the costs of the imported foreign ores or the upgraded domestic ores; similarly the costs of the upgraded domestic ores do not include their mining costs. This is taken care of by the technical coefficients of which more will be said later. On the other hand, the operating costs shown here include the costs of electric energy used, though the latter are dealt with separately in the actual computations.

In general, it can be stated that the second set of production costs is less conservative than the first one, for more than one reason as will become clear in the following. Still, these costs are not unrealistic at all, obtained as they are from experts who evaluated these processes. On the contrary, these costs might very well prove to be still too high, once these processes are operated on a large scale. In that case, the first set of cost data will appear to be much too high. Still, the general opinion in industry seems to be in favor of the more conservative figures. That is why they were used in the first computation.

As can be seen from the tables on the following pages, the two sets of operating costs differ from each other in two ways.

First, there is a difference in the level of these costs in the sense that the second computation is, in general, based on higher costs of producing the alloys, but on lower costs of upgrading the domestic ores. As for the latter, this is reflected in lower operating as well as lower capital costs of the various beneficiation processes. The level of the mining costs is, however, the same for both computations.

Second, there is the difference that the second set of

Domestic Production Costs in 1st Computation

Description	S	Unit	year 1 ——> year 6
Operating Costs			
Ferro-manganese in blast furnaces	x_3	$/NT of Mn.	96.35 ——> 96.35
Ferro-manganese in electric furnaces	x_4	"" " " "	98.80 ——> 98.80
Ferro-manganese in Udy furnaces	x_5	"" " " "	107.45 ——> 107.45
Silico-manganese in electric furnaces	x_6	"" " " "	157.10 ——> 157.10
Silico-manganese in Wright furnaces	x_7	"" " " "	173.75 ——> 173.75
Silico-manganese in Udy furnaces	x_8	"" " " "	142.95 ——> 142.95
70% ore by Dean's process	x_{12}	"" " " "	92.60 ——> 92.60
70% slag by Sylvester-Dean process	x_{13}	"" " " "	81.60 ——> 81.60
39% conv. slag by Wright process	x_{14}	"" " " "	96.70 ——> 96.70
30% furnace slag by Udy process	x_{15}	"" " " "	87.20 ——> 87.20
10% Manganese ore from Cuyuna mines	x_{16}	"" " " "	21.90 ——> 21.90
10% Open Hearth slags fm. steel mills	x_{17}	"" " " "	—.— ——> —.—
4% Hot-Metal for Wright plants	x_{18}	"" " " "	inc. in conv. slag
15% Mn. ore from Aroostook mines	x_{19}	"" " " "	30.35 ——> 30.35
Capital Costs			
Udy alloy furnaces	y_5	$/NT of Mn.[1]	210.00 ——> 210.00
Dean beneficiation plant	y_{12}	"" " " "	265.70 ——> 265.70
Sylvester-Dean beneficiation plant	y_{13}	"" " " "	391.50 ——> 391.50
Wright beneficiation plant	y_{14}	"" " " "	401.70 ——> 401.70
Udy beneficiation plant	y_{15}	"" " " "	170.25 ——> 170.25
Cuyuna Manganese mines	y_{16}	"" " " "	24.00 ——> 24.00
Hot-Metal plants	y_{18}	"" " " "	inc. in Wright plant
Aroostook Manganese mines	y_{19}	"" " " "	14.75 ——> 14.75

[1] per year.

Domestic Production Costs in 2nd Computation

Description	S	Unit	year 1 ⟶ year 6
Operating Costs			
Ferro-manganese in blast furnaces	x_3	\$/NT of Mn.	105.40 ⟶ 105.40
Ferro-manganese in electric furnaces	x_4	"" " " "	109.75 ⟶ 109.75
Ferro-manganese in Udy furnaces	x_5	"" " " "	112.15 ⟶ 100.40
Silico-manganese in electric furnaces	x_6	"" " " "	168.05 ⟶ 168.05
Silico-manganese in Wright furnaces	x_7	"" " " "	183.75 ⟶ 176.15
Silico-manganese in Udy furnaces	x_8	"" " " "	171.60 ⟶ 153.60
70% ore by Dean's process	x_{12}	"" " " "	88.40 ⟶ 80.10
70% slag by Sylvester-Dean process	x_{13}	"" " " "	76.90 ⟶ 69.40
39% conv.slag by Wright's process	x_{14}	"" " " "	96.70 ⟶ 87.10
30% furnace slag by Udy process	x_{15}	"" " " "	71.25 ⟶ 64.05
10% Mn. ore from Cuyuna mines	x_{16}	"" " " "	21.90 ⟶ 19.70
10% Open-Hearth slag fm. steel mills	x_{17}	"" " " "	—.— ⟶ —.—
4% Hot Metal for Wright plants	x_{18}	"" " " "	inc. in conv. slag
15% Mn. ore from Aroostook mines	x_{19}	"" " " "	30.95 ⟶ 27.95
Capital Costs			year 1 ⟶ year 5
Udy alloy furnaces	y_5	\$/NT of Mn.[1]	153.00 ⟶ 141.00
Dean beneficiation plants	y_{12}	"" " " "	203.00 ⟶ 203.00
Sylvester-Dean beneficiation plants	y_{13}	"" " " "	294.00 ⟶ 270.40
Wright beneficiation plants	y_{14}	"" " " "	236.65 ⟶ 217.65
Udy beneficiation plants	y_{15}	"" " " "	188.50 ⟶ 173.50
Cuyuna Manganese mines	y_{16}	"" " " "	24.00 ⟶ 22.00
Hot Metal plants	y_{18}	"" " " "	inc. in Wright plants
Aroostook Manganese mines	y_{19}	"" " " "	14.75 ⟶ 13.55

[1] per year

REMARK: If there is a difference in the costs of the 1st and those of the 6th year, then it is equally spaced over the intervening years.

costs, insofar as they refer to non-conventional processes, decrease slightly over time. This reflects the technological progress which can be expected to take place if those processes would be operated on a commercail basis. This decrease in the operating costs has in general been put at 2% per year, which corresponds more or less with the overall increase in the industrial productivity of the United States throughout the years. On the other hand, the costs of producing the alloys along conventional lines have been kept constant during all six years.

As for the costs of stockpiling the ores, they consist of the rent for the land upon which the high-grade ores will be stored as well as the costs of handling these ores (if any). The total of these costs have been estimated to be approximately $1.00 per N.T. of manganese per year.

It should be remarked that all future costs of importation, stockpiling and domestic production (operating as well as capital costs) are discounted at a rate of 5% per year. Consequently there is no need for including the interest in the amounts invested in the stockpile in the costs of the stockpile.

Besides production costs also technical coefficients, denoted by c's in the model, have to be taken into account. Such a coefficient is defined as the amount of manganese in the raw material that is necessary to produce 1 N.T. of manganese in the product. The following technical coefficients have been used for both computations (the c's in brackets are those of the model; compare also the diagram):

1º for the production of

75% ferro-manganese from		
48% imported ores		
in blast furnaces	(c_3)	1.111
in electric furnaces	(c_4)	1.111
30% furnace slag	(c_5)	1.176

65% silico-manganese from

48% imported ores	(c_6)	1.111
39% converterslag	(c_7)	1.143
30% furnace slag	(c_8)	1.176

2º for the beneficiation of

10% Cuyuna ore into 70% ore	(c_{12})	1.250
10% O.H. slag into 70% slag	(c_{13})	1.250
4% Hot Metal into 39% slag	(c_{14})	1.200
15% Aroostook ore into 30% slag	(c_{15})	1.053

3º for the production of 4% Hot Metal

from 40% Open-Hearth slag and 4.5% Cuyuna Ferro ore	(c_{19})	1.124

These technical coefficients have been assumed to be the same for all six years of the period under consideration.

These technical coefficients have to be interpreted in such a way, that, for instance, 1.11 N.T. of manganese in the form of high grade ore is needed to produce 1 N.T. of manganese as ferro-manganese or that 1.25 N T. of manganese in the form of 10% O.H. slag is needed to produce 1 N.T. of manganese in the form of high-grade ore. Consequently, $1.11 \times 1.25 = 1.39$ N.T. of manganese in the form of O.H. slag is needed to produce 1 N.T. of manganese as ferro-manganese.

These technical coefficients in combination with the production costs form the total cost of obtaining the manganese in the form of ferro- or silico-manganese. Let us assume, for example, that the O.H. slags will be upgraded by the Sylvester-Dean process and then processed in blast furnaces into ferro-manganese (this is the simplest case). Let us further assume that the costs of producing ferro-manganese in blast furnaces is $96.35 per N.T. of manganese and the cost of upgrading O.H. slags by the Sylvester-Dean process is $81.60

per N.T. of manganese; the cost of 1 N.T. of manganese in the O.H. slag will be put at $1.00 for the moment. Furthermore, let us neglect the cost of constructing the Sylvester-Dean plant in which this upgrading has to take place; this means that we assume that such a Sylvester-Dean plant already exists (this makes it really simple to determine the costs). The total cost of producing 1 N.T. of manganese in this way will then be $96.35 + 1.11 × $81.60 + 1.39 × $1.00 = $188.30 approximately. This then is the cost of obtaining 1 N.T. of manganese as ferro-manganese from O.H. slags if we assume that the production facilities already exist.

Whether such a production facility should or should not be constructed and particularly what the capacity of such a facility should be in each of the six years under consideration does not depend merely on a comparison of total operating costs but on the other aspects of the problem as well. The entire problem, in all its complexity, has to be considered in one sweep and it is exactly for the purpose of finding a solution to such a complicated problem that the technique of mathematical programming has been developed.

THE RESULTS

The results can be grouped into three parts.

First, the solution shows to which extent each activity has to be carried out to meet the requirements at lowest cost. Second, the solution shows what the total costs of the program are, divided into payments for importation, into costs of operating the domestic plants as well as of constructing them. Third, the solution tells us also how much has to be added to these total costs if the requirements are somewhat increased as well as how much can be subtracted from them if the constraints are somewhat released. All these aspects will be discussed one at a time; moreover, to enhance the exposition all the results of the first computation will be given first and thereafter those of the second one.

1) First computation

a) Activity levels.

The solution indicates that the cheapest way of obtaining the ferro-manganese, required or preferred to silico-manganese, is by producing this alloy from high-grade ores in blast furnaces only. In particular nothing will be produced in this case by the Udy process, in contrast with the solution of the second computation, as will be seen later.

As for the silico-manganese, this too should be produced from high-grade ores with the exception of a small quantity to be produced from "Hot Metal". The latter corresponds with the initial capacity of the Wright plants assumed to be present at the beginning of the first year of the program.

The specification over the years (in 1000 N.T. of manganese) is given in the following table:

	Ferro-manganese		Silico-manganese		
	Total ($\lambda_i + \mu_i$)	from high-grade ores (x_3)	Total (υ_i)	from high-grade ores (x_6)	from Hot-Metal (x_7)
1st year	765	765	85	84.1	.9
2nd year	810	810	90	89.1	.9
3rd year	830	830	95	94.1	.9
4th year	800	800	95	94.1	.9
5th year	785	785	90	89.1	.9
6th year	790	790	95	94.1	.9

The way the necessary quantities of high-grade ores, beyond what is already in the stockpile at the beginning of the first year, should be obtained is indicated in the table on the following page (all quantities are in 1000 N.T. of manganese):

	Importation			Domestic Production		
	Total	Africa (x_{10})	Latin America (x_{11})	Total	Dean (x_{12})	Sylvester-Dean (x_{13})
1st year	234.7	30.9	203.8	4.5	4.5	——.—
2nd year	414.4	131.5	282.9	4.5	4.5	——.—
3rd year	371.5	105.5	266.0	361.0	4.8	356.2
4th year	349.3	93.1	256.2	621.9	265.7	356.2
5th year	360.4	82.2	278.2	621.9	265.7	356.2
6th year	——.—	——.—	——.—	621.9	265.7	356.2

It should once more be mentioned that in this model the imports in a certain year i affect the position not of the same year but that of the following year, year $i + 1$ (that is also the reason why there are no quantities imported in the 6th year in this model). This becomes clearer in the following table, it shows the position of the stockpile at the beginning of each year, what is added to it as well as what is taken from it during each year, and what the position is at the end of each year.

Changes in the Stockpile
(all quantities in 1000 N.T. of manganese)

	a)	b)	c)	d)	e)	f)	g)	h)
1st year	1950	943.4	850.0	93.4	4.5	——.—	4.5	1011.1
2nd year	1011.1	999.0	900.0	99.0	239.2	234.7	4.5	251.3
3rd year	251.3	1026.7	922.2	104.5	775.4	414.4	361.0	——.—
4th year	——.—	993.4	888.9	104.5	993.4	371.5	621.9	——.—
5th year	——.—	971.2	872.2	99.0	971.2	349.3	621.9	——.—
6th year	——.—	982.3	877.8	104.5	982.3	360.4	621.9	——.—

a) Position at the beginning of the year
b) Total of requirements $[1.11(x_3 + x_6)]$
c) Required for ferro production $(1.11 \cdot x_3)$
d) Required for silico production $(1.11 \cdot x_6)$
e) Total of additions $(x_{10} + x_{11} + x_{12} + x_{13})$

f) Added from importation in previous year $(x_{10} + x_{11})$

g) Added from domestic production in same year $(x_{12} + x_{13})$

h) Position at end of the year.

As can be seen from this table, there is at the beginning of the 2nd year still enough manganese in the stockpile to cover the needs of that year. Nevertheless the solution indicates that one should already start in the 2nd year to add ore to the stockpile rather than first to use up all the ore that is in it. This is an interesting feature since there is only a very small cost element (the carrying charges) associated with the ore that is in the stockpile. In other words, as far as the program is concerned, the ore in the stockpile is almost costless. Still, the solution indicates that in order to keep the costs for all six years of the period as low as possible, it is better not to exhaust the stockpile already in the 2nd year. — The consequence of the latter would have been that in the later years more foreign ore had to be imported or more domestic ore had to be upgraded. The first alternative would have resulted in higher cost per N.T. of manganese contained in the foreign ores; the second alternative would have made it necessary to increase the capacity of the domestic plants, which would have increased the construction costs and consequently also the cost of depreciation for which the program is charged. — It is clear that, without the program, it would have been hard to guess correctly how much should be imported and produced domestically in each year to keep the costs for the whole period at a minimum.

b) Computed Costs.

Five groups of costs can be distinguished:

1) The amounts to be spent on importation are shown in the table on the following page. These costs include the linear as well as the quadratic terms of the objective function that are related to these import activities. The linear terms only amount to \$158.2 mln; the quadratic terms add another \$63.8 mln to this amount.

	Total costs of importation	Import from India	Import from Africa	Import from Latin America
1st year	$31.4 mln	———	$ 4.2 mln	$27.2 mln
2nd year	$57.1 mln	———	$18.0 mln	$39.1 mln
3rd year	$48.5 mln	———	$13.6 mln	$34.9 mln
4th year	$42.7 mln	———	$11.3 mln	$31.4 mln
5th year	$42.3 mln	———	$ 9.3 mln	$33.0 mln
	$222.0 mln	———	$56.4 mln	$165.6 mln

2) The costs of mining and upgrading domestic ores and slags are:

	Total costs of Upgrading	Mining and upgrading domestic ores $x_{12} + x_{16}$	Upgrading O-H slags x_{13}
1st year	$ .5 mln	$ .5 mln	$—.– mln
2nd year	$ .5 mln	$ .5 mln	$—.– mln
3rd year	$26.5 mln	$ .5 mln	$26.0 mln
4th year	$51.2 mln	$26.7 mln	$24.5 mln
5th year	$48.2 mln	$25.2 mln	$23.0 mln
6th year	$45.5 mln	$23.8 mln	$21.7 mln
	$172.4 mln	$77.2 mln	$95.2 mln

These costs do not include the costs of building the necessary capacities. In other words, these are only the costs of operating both types of plants.

It has also been shown that the quantities of upgraded ore produced from the 2nd year (Sylvester-Dean process) and the 3rd year (Dean process) to the sixth year are constant. Consequently, the decrease in annual cost in those years is, apart from a discount factor, due to the technological improvements assumed to manifest themselves in those years.

3) The cost of processing the high-grade ores and slags into the alloys:

	Total Costs of Producing Alloys From upgrading ores	Production of ferro-manganese in blast furnaces x_3	Production of silico-manganese in electric furnaces x_6
1st year	\$78.7 mln	\$66.5 mln	\$12.2 mln
2nd year	\$80.1 mln	\$67.7 mln	\$12.4 mln
3rd year	\$79.3 mln	\$66.8 mln	\$12.5 mln
4th year	\$74.0 mln	\$62.0 mln	\$12.0 mln
5th year	\$69.5 mln	\$58.6 mln	\$10.9 mln
6th year	\$67.8 mln	\$56.8 mln	\$11.0 mln
	\$449.4 mln	\$378.4 mln	\$71.0 mln

4) The costs of processing low grade ores and slags into the alloys, i.e., the costs of the integrated processes. In this case the Wright process is the only one that is active and to the extent as permitted by the initial capacity (.9 ths. N.T. of manganese per year). The costs amount to \$.2 mln in each of the six years.

5) The costs of building the capacities necessary to mine and upgrade the Cuyuna ores as well as the ores needed to upgrade the Open-Hearth slags. The program indicates that the Cuyuna mines and the Dean plants should be built in the third year, costing altogether \$20.0 mln, of which \$2.0 mln is for developing the mines and \$18.0 mln for constructing the plants. On the other hand, the Sylvester-Dean plants for upgrading the O-H slags should be constructed in the second year, which would involve \$50.6 mln.

The table shown on the following page shows the total costs of the program by these five groups (all costs are in \$ mln).

These are the computed costs of the program; they do not include the cost of the high-grade ores that were in the stockpile at the beginning of the first year. These ores will be used up in

	a)	b)	c)	d)	e)	f)	g)
1st year	31.4	.5	78.7	.2	110.8		110.8
2nd year	57.1	.5	80.1	.2	137.9	50.6	188.5
3rd year	48.6	26.5	79.3	.2	154.6	20.0	174.6
4th year	42.6	51.2	74.0	.2	168.0		168.0
5th year	42.3	48.2	69.5	.2	160.2		160.2
6th year	—.—	45.5	67.8	.2	113.5		113.5
	220.0	172.4	449.4	1.2	845.0	70.6	915.6

a) Importing foreign ores
b) Upgrading domestic ores
c) Producing alloys from upgraded ores
d) Producing alloys from low grade ores
e) Total Operating Costs
f) Total Capital Costs
g) Operating and Capital Costs

carrying out the program; consequently something has to be added to these computed costs in order to arrive at the total cost.

The value to be attributed to this initial stockpile is rather arbitrary. One could argue that its value is very high, since without it the alloys needed in the first year of the emergency could not be produced. On the other hand, one of the results of this computation is, as will be shown later, the "shadow-price" of this stockpiled ore, being the decrease in the total cost of the program as a result of an additional N.T. of manganese in the stockpile at the start. This shadow-price is in this case \$143.06 per N.T. of manganese and the manganese in the stockpile used up in the program has been evaluated at this price.

The total cost of the program is then the sum of:

Computed Costs	\$915.6 mln
Cost of initial stockpile 1950 ths. × \$143.06 =	\$279.0 mln
	\$1194.6 mln

c) Shadow-prices.

Besides the various activity levels and the costs involved in them, the computation shows also by how much the total costs of the program would change if the requirements were slightly increased or the constraints slightly relaxed. The changes in these costs are indicated by what is commonly called "shadow-prices", by-products of the computation.

In this case there are two types of shadow-prices, indicating:

1º increases in the total cost resulting from higher alloy requirements. These cost increases (in $/NT of manganese) are:

	Production of ferro-manganese	Production of silico-manganese	Difference
1st year	249.9	310.4	60.5
2nd year	246.4	304.1	57.7
3rd year	243.2	298.1	54.9
4th year	230.4	282.7	52.3
5th year	218.1	267.9	49.8
6th year	205.9	253.3	47.4

It should here be mentioned that these costs are brought forward to the beginning of the first year of the program. This explains the downward trend in these costs.

2º decreases in the total cost resulting from a relaxation of the constraints as a consequence of an increase in the quantities of available power, of more high-grade ore in the initial stockpile, of a higher starting capacity of the plants, etc.

These cost decreases (in $/NT of manganese) are:

	a)	b)	c)	d)	e)	f)	g)	h)
1st year	—.—	—.—	19.1	—.—	—.—	—.—	9.3	—.—
2nd year	—.—	15.6	25.7	—.—	38.9	16.8	.5	—.—
3rd year	8.7	19.7	28.8	2.6	43.5	23.3	—.—	—.—
4th year	8.4	18.0	26.3	2.5	39.2	20.5	—.—	—.—
5th year	8.3	16.4	24.0	2.2	35.0	17.7	—.—	—.—
6th year	117.7	14.8	21.8	1.8	30.8	14.8	—.—	18.6
	143.1	84.5	145.7	9.1	187.4	93.1	9.8	18.6

a) 1 N.T. of manganese more in initial stockpile (S)
b) Higher starting capacity of Udy-alloy furnace K_5
c) Higher starting capacity of Dean upgrading plant K_{12}
d) Higher starting capacity of Cuyuna manganese mines K_{16}
e) Higher starting capacity of Sylvester-Dean upgrading plant K_{13}
f) Higher starting capacity of Wright upgrading plant K_{14}
g) Higher starting capacity of Wright Hot Metal plant K_{18}
h) Higher quantity of steel mill slags R_{17}

These shadow-prices differ in character from those of the alloys, stated under $1^{\underline{o}}$, to the extent that the latter refer to yearly changes in the requirements while in this case the shadow-prices refer to changes in the situation at the beginning of the program. For instance, 1 N.T. of manganese more in the stockpile would reduce the costs of the program in the third year by $8.7, in the fourth year by $8.4, in the fifth year by $8.3 and in the sixth year by $117.7. Consequently, the costs of the program, taken over all years, will decrease by $143.1 if there were 1 N.T. of manganese more in the stockpile at the beginning of the program than is here assumed. The same holds for the initial capacities of some of the beneficiation plants as well as for the alloy furnaces needed in the Udy process. The shadow-price of the slags indicates that the total cost of the program would decrease by $18.6 if 1 N.T. of manganese more would become available in the form of slags than is here assumed. This, in turn, means that all the slags assumed to become available will be used up in this program. On the other hand, the absence of shadow-prices for the manganese contained in the deposits means that not all this manganese will be utilized in this program.

Comparing now the overall decreases in the costs, we see that most will be gained by an increase of the initial capacity of the Sylvester-Dean plants by 1 N.T. of manganese per year. This is followed by an increase in the initial capacity of the Dean upgrading plant combined with that of the Cuyuna mines. An increase of 1 N.T. of manganese in the initial stockpile comes apparently after that. This strongly suggests that, under the given price-and cost functions, it would be preferable to give the Sylvester-Dean plants some initial capacity and to increase the initial capacity of the Dean plants and Cuyuna mines rather than to add more manganese to the initial stockpile. This is certainly an interesting result.

It should, however, be remarked that these shadow-prices indicate the effect of marginal increases in the capacities of these plants and mines on the total costs of the program. However, they do not indicate what the initial capacities of these plants should be in order to obtain the absolute minimum of these costs. In order to find that out, several other computations have to be made based on various combinations of higher starting capacities of these plants; or, still better, on various combinations of initial capacities and initial stockpile. For this the reader is referred to the last part of this paper, where the results of this sort of computation, made on the basis of a stochastic model, are described.

2) Second Computation

a) Activity levels

In this case the cheapest way of obtaining the ferro-manganese that is required or preferred to silico-manganese is not the one by which everything is obtained from high-grade ores. Contrary to the result of the first computation, a continuously increasing part of the ferro-manganese is obtained from low-grade ores by Udy's process. Undoubtedly this is due to the fact that Udy's processing costs are already in the first year assumed to be lower than those of the first computation; moreover, it has been assumed here that these costs will decrease over time.

As for the silico-manganese, this again should be obtained mainly from high-grade ores, though the quantity to be produced from "Hot Metal" is in this case somewhat higher. This is due to the fact that the initial capacity of the Wright plants has been assumed to be a multiple of that of the first computation.

The specification over the years (in 1000 N.T. of manganese) is given in the following table:

	Total requirements ($\lambda + \mu$)	from high-grade ores (x_3)	from Udy Slag (x_5)	Total requirements (υ)	from high-grade ores (x_6)	from Hot-Metal (x_7)
1st year	765	765	—.—	85	76.3	8.7
2nd year	810	704.2	105.8	90	81.3	8.7
3rd year	830	650.1	179.9	95	86.3	8.7
4th year	800	539.4	260.6	95	86.3	8.7
5th year	785	524.4	260.6	90	85.7	4.3
6th year	790	522.5	267.5	95	95.0	—.—

The necessary quantities of high-grade ores should be obtained as shown in the table on the following page (all quantities in 1000 N.T. of manganese).

Here again it should be mentioned that the imports in year i affect the position in year i + 1. In this case the importation shows a declining trend with a marked downward shift from the 3rd to the 4th year; this reflects sharply the increased proportion of total requirements to be met by Udy-slag. Another deviation from the solution of the first computation is that the annual quantity of ore to be upgraded by the Dean process does not exceed the initial capacity of that plant.

	Importation			Domestic Production		
	Total	Africa (x_{10})	Latin America (x_{11})	Total	Dean (x_{12})	Sylvester-Dean (x_{13})
1st year	280.1	35.5	244.6	4.5	4.5	——.—
2nd year	245.4	21.8	223.6	572.9	4.5	568.4
3rd year	122.3	——.—	122.3	572.9	4.5	568.4
4th year	105.0	——.—	105.0	572.9	4.5	568.4
5th year	113.3	——.—	113.3	572.9	4.5	568.4
6th year	——.—	——.—	——.—	572.9	4.5	568.4

The position of the stockpile at the beginning of each year and the changes in it during the years are:

Changes in the Stockpile
(all quantities in 1000 N.T. of manganese)

	a)	b)	c)	d)	e)	f)	g)	h)
1st year	950.0	934.8	850.0	84.8	4.5	——.—	4.5	19.7
2nd year	19.7	872.7	782.4	90.3	853.0	280.1	572.9	——.—
3rd year	——.—	818.3	722.3	95.9	818.3	245.4	572.9	——.—
4th year	——.—	695.2	599.3	95.9	695.2	122.3	572.9	——.—
5th year	——.—	677.9	582.7	95.2	677.9	105.0	572.9	——.—
6th year	——.—	686.2	580.6	105.6	686.2	113.3	572.9	——.—

a) Position at beginning of year
b.) Total of requirements $1.11\ (x_3 + x_6)$
c) Required for ferro-production $1.11\ x_3$
d) Required for silico-production $1.11\ x_6$
e) Total of additions $x_{10} + x_{11} + x_{12} + x_{13}$
f) Added from importation in previous year $x_{10} + x_{11}$
g) Added from domestic production in same year $x_{12} + x_{13}$
h) Position at end of year.

As for the capacities of the plants, the only two that show increases are the Sylvester-Dean plants and the Udy plants. These increases in the capacities (in 1000 N.T. of manganese per year) are the following:

	Sylvester-Dean plants	Udy plants		
	producing high-grade ore	producing ferro-manganese	producing furnace slag	producing 15% ore
1st year	568.5	105.8	124.4	130.6
2nd year	——.—	74.1	87.1	91.7
3rd year	——.—	80.7	95.0	100.0
4th year	——.—	——.—	——.—	——.—
5th year	——.—	6.7	7.9	8.4

There is here a marked difference in tempo of building these plants: the Sylvester-Dean plants should be given their maximum capacity right away in the first year, while the capacity of the Udy plants should be extended more gradually. More will be said on this topic later on.

b) Computed Costs

The five groups of costs are:

1) The amounts to be spent on importation in these years are:

	Total Cost of Importation	Import from India (x_9)	Import from Africa (x_{10})	Import from Latin America (x_{11})
1st year	$36.9 mln	—.—	$4.9 mln	$32.0 mln
2nd year	$31.2 mln	—.—	$2.7 mln	$28.5 mln
3rd year	$12.6 mln	—.—	—.—	$12.6 mln
4th year	$ 9.2 mln	—.—	—.—	$ 9.2 mln
5th year	$ 9.1 mln	—.—	—.—	$ 9.1 mln
	$99.0 mln	—.—	$7.6 mln	$91.4 mln

Again, these amounts include the effect of the linear as well as the quadratic terms of the objective function; the latter amount altogether to $26.9 mln.

Comparing this with the corresponding results of the first computation, we see that there is in this case far less dependence on importation than before. This is the case, despite the fact that the initial stockpile contains now only half as much manganese as before. Consequently, a much larger share of the requirements must be met out of domestic production of manganese. This, in turn, is a consequence of the lower production costs of the domestic upgrading processes, which has here been assumed.

2) The cost of mining and upgrading domestic ores and slags are:

	Total cost of mining and upgrading	Mining and upgrading domestic ores $x_{12} + x_{16}$	Upgrading O-H slags x_{13}
1st year	$ 0.5 mln	$0.5 mln	$—.— mln
2nd year	$40.8 mln	$0.5 mln	$40.3 mln
3rd year	$37.8 mln	$0.5 mln	$37.3 mln
4th year	$34.7 mln	$0.5 mln	$34.2 mln
5th year	$32.1 mln	$0.5 mln	$31.6 mln
6th year	$29.9 mln	$0.5 mln	$29.4 mln
	$175.8 mln	$3.0 mln	$172.8 mln

Once more the remark should be made that these amounts do not include the costs of building the plants. In other words, these are only the costs of operating both types of plants.

Comparing this result with that of the first computation, we see that in this case the annual production of the Dean plants does not exceed its initial capacity. Apparently this process has been replaced by one with lower costs, i.e., the Udy process, as we will see later. The higher production of the Sylvester-Dean plants is due to the fact that a higher quantity of O-H slags has been assumed to become available.

3) The costs of processing high-grade ores into the alloys are:

	Total Costs of producing alloys from upgrading ores	Production of ferro-manganese in blast furnaces x_3	Production of silico-manganese in electric furnaces x_6
1st year	$85.6 mln	$73.7 mln	$11.9 mln
2nd year	$77.5 mln	$65.3 mln	$12.2 mln
3rd year	$70.4 mln	$58.0 mln	$12.4 mln
4th year	$58.2 mln	$46.3 mln	$11.9 mln
5th year	$54.6 mln	$43.3 mln	$11.3 mln
6th year	$52.9 mln	$41.0 mln	$11.9 mln
	$399.2 mln	$327.6 mln	$71.6 mln

It should here be mentioned that only part of the ferro- and silico- requirements are met by processing high-grade ores into alloys.

4) The costs of processing low-grade ores and slags into the alloys are (in $ mln):

	Alloys from Aroostook ore				Alloys from "Hot-Metal"		
	a)	b)	c)	d)	e)	f)	g)
1st year	2.5	—.—	—.—	—.—	2.5	1.5	1.0
2nd year	21.6	19.3	10.5	8.8	2.3	1.4	0.9
3rd year	32.9	30.7	16.7	14.0	2.2	1.3	0.9
4th year	43.5	41.5	22.7	18.8	2.0	1.3	0.7
5th year	39.7	38.8	21.1	17.7	0.9	0.6	0.3
6th year	37.2	37.2	20.1	17.1	—.—	—.—	—.—
	177.4	167.5	91.1	76.4	9.9	6.1	3.8

a) Total costs of producing alloys from low grade ores

b) Total costs of Udy process

c) Costs of producing alloys from furnace slag (x_5)

d) Costs of producing furnace slag from Aroostook ore ($x_{15} + x_{19}$)

e) Total costs of Wright process
f) Cost of producing alloys from converter slag (x_7)
g) Costs of producing converter slag from Hot-Metal $(x_{14} + x_{18})$.

Again it should here be remarked that these amounts do not include the cost of building the plants.

Comparing this result with that of the first computation, we see a marked difference. In this case, the Udy plants contribute substantially to meeting the requirements for ferro-manganese. This is due to the assumption that the technology of the Udy process will be improved considerably in the near future resulting in lower costs per unit of activity. In this case, it replaces the Dean process which had a much more important role in the first solution.

5) The cost of building the necessary capacities are:

	Total of construction costs	Sylvester-Dean plant producing high-grade ores K_{13}	Udy plant producing ferro-manganese K_5	Udy plant producing furnace slag K_{15}	Udy plant producing 15% ore K_{19}
1st year	\$ 99.4 mln	\$79.6 mln	\$ 7.7 mln	\$11.2 mln	\$0.9 mln
2nd year	\$ 10.3 mln	\$—.— mln	\$ 4.0 mln	\$ 5.8 mln	\$0.5 mln
3rd year	\$ 7.9 mln	\$—.— mln	\$ 3.1 mln	\$ 4.4 mln	\$0.4 mln
4th year	\$ —.— mln	\$—.— mln	\$—.— mln	\$—.— mln	\$—.— mln
5th year	\$ 0.2 mln	\$—.— mln	\$ 0.1 mln	\$ 0.1 mln	\$—.— mln
	\$117.8 mln	\$79.6 mln	\$14.9 mln	\$21.5 mln	\$1.8 mln

A marked difference in tempo of constructing these two types of plants appears from these figures: the whole capacity of the Sylvester-Dean plants is built in the very first year, while that of the Udy plants is more gradually expanded over time.

All computed costs are summarized in the following table (all amounts are in $ mln):

	a)	b)	c)	d)	e)	f)	g)
1st year	36.9	0.5	85.6	2.5	125.5	99.4	224.9
2nd year	31.2	40.8	77.5	21.6	171.1	10.3	181.4
3rd year	12.6	37.7	70.4	32.9	153.6	7.9	161.5
4th year	9.2	34.6	58.2	43.5	145.5	—.—	145.5
5th year	9.1	32.0	54.6	39.7	135.4	0.2	135.6
6th year	—.—	39.8	52.9	37.2	119.9	—.—	119.9
	99.0	175.4	399.2	177.4	851.0	117.8	968.8

a) Costs of importing foreign ores
b) Costs of upgrading domestic ores
c) Costs of producing alloys from high-grade ores
d) Costs of producing alloys from low-grade ores
e) Total of operating costs
f) Total of Capital Costs
g) Sum of Operating and Capital Costs.

As argued before, these computed costs have to be increased with the "value" of the initial stockpile. Its "shadow-price" is in this case $135.50 per N.T. of manganese as will be shown later. Hence the total costs of the program are:

computed costs	$968.8 mln
value of initial stockpile 950 ths × $135.50 =	128.7 mln
Total cost of the program	$1097.5 mln

The results of both computations will be discussed in the summary.

c) Shadow-prices

1° increases in the cost as a result of higher alloy requirements:

These costs (in $/N.T. of manganese) are:

	Production of ferro-manganese	Production of silico-manganese	Difference
1st year	247.0	306.6	59.6
2nd year	243.3	300.1	56.8
3rd year	228.2	282.3	54.1
4th year	197.0	248.6	51.6
5th year	180.5	229.6	49.1
6th year	167.3	214.1	46.8

Comparing these shadow-prices with those of the first computation, we see that up to the second year the production costs of both alloys are about the same. However, from the third year on, the second computation registers a much sharper drop in these production costs than the first one. This, again, is the influence of the improvement of the technology in upgrading domestic ores that has been assumed in this case.

2° decreases of the costs
as a result of a relaxation of the constraints.

These costs (in $/N.T. of manganese) are:

	a)	b)	c)	d)	e)	f)	g)	h)
1st year	—.—	—.—	—.—	—.—	21.8	—.—	11.6	—.—
2nd year	10.4	18.4	22.7	1.8	30.0	47.6	19.0	—.—
3rd year	25.0	16.3	20.1	1.6	27.2	42.7	17.4	—.—
4th year	12.0	14.4	18.5	1.4	9.4	22.9	3.6	—.—
5th year	8.3	12.0	14.8	1.2	4.3	15.6	—.—	—.—
6th year	79.8	11.1	13.6	1.0	1.7	11.1	—.—	13.5
	135.5	72.8	89.7	7.0	94.4	139.9	51.6	13.5

a) More ore in initial stockpile S
b) Higher initial capacity of Udy alloy-furnace K_5
c) Higher initial capacity of Udy slag plants K_{15}
d) Higher initial capacity of Aroostook mines K_{19}
e) Higher initial capacity of Dean upgrading plant K_{12}
f) Higher initial capacity of Sylvester-Dean upgrading plant K_{13}
g) Higher initial capacity of Wright Hot-Metal plant K_{18}
h) Higher quantity of steel mill slags R_{17} .

The Sylvester-Dean plants have again a higher shadow-price than the initial stockpile. As before this indicates that it would be preferable, under the given price- and cost-functions, to give the Sylvester-Dean plants some initial capacity rather than to add more manganese to the stockpile.

Some initial capacity of the Udy upgrading plants and Aroostook mines would make it possible to upgrade the ores already in the first year of the program, which would reduce the total costs by $89.7 + $7.0 = $96.7 per N.T. of manganese required. Moreover, if this could be combined with giving the alloy furnaces some starting capacity (or if we could assume that these furnaces had already some capacity at the start then the total costs of the program would decrease still further. It is even conceivable that they might drop more than in the case of some initial capacity of the Sylvester-Dean plants.

Moreover, there are shadow-prices for the electricity in the Aroostook region, amounting to $1.59 per 1000 KWH in the first and $1.87 per 1000 KWH in the second year. This indicates that there will be a shortage of electricity in that region for the first two years. Thereafter, there is apparently enough electricity available for the Udy plants since there are no shadow-prices for electricity in the third and following years.

The minor role of the Dean plants in this second computation is indicated by the lower shadow-price: $94.4 against $145.7

in the first computation. Still, its shadow-price in the second computation is slightly below that of the Udy plant and Aroostook mine combined. This indicates that increasing the initial capacity of the Dean plants is a good alternative too, even in the second computation.

As for Wright's process, this too plays a minor role in this second computation as indicated by the lower shadow-price: \$93.1 + 9.8 = \$102.9 in the first and only \$51.6 in the second computation.

The shadow-price of the Open Hearth slags indicates that there will be a shortage of this material also in this second computation. The other deposits will, however, still contain considerable quantities of manganese at the end of the six years considered here.

SUMMARY

The results of both computations cannot be compared without first paying some attention to the differences in the conditions on which they are based.

As for the constraints, it should be remarked first of all that the alloy requirements that have to be met in both cases, have been taken to be the same. But larger quantities of Open Hearth slags have been assumed to become available in the second case, containing 3585 - 1785 = 1800 ths. N.T. of manganese more taken over all six years. Since this is a rather large quantity, it would not be correct to evaluate this at \$18.6 per N.T. of manganese, the shadow-price of O-H slags in the first computation, since shadow-prices apply to marginal changes in the constraints and not to an increase of the available quantity of manganese as is considered here. On the other hand, the shadow-price of O-H slag in the second computation turns out to be \$13.5 per N.T. of manganese. The average of both shadow-prices, being \$16.– per N.T. of manganese, has now been used for the evaluation of this difference in the available quantity of manganese in the O-H slags. In other words, the second computation had on this score an initial "benefit" of 1800 ths. × \$16.– = \$28.8 mln. Furthermore, there is also a difference in the initial capacity of the Wright plants of 9 ths. N.T. of manganese per year and in that of the Hot-Metal plants of 10.8 ths. N.T. of manganese per year. Since these are rather small differences, the shadow-price of these two initial capacities of the first computation have been applied to evaluate these differences. They gave the second computation another initial "benefit" of 9 ths. × \$93.1 + 10.8 ths. × \$9.8 = \$0.9 mln. approximately. Finally, there was a difference in the available quantities of electric energy. However, electric energy had no shadow-price in the first computation, in other words, there was no shortage of electric energy in that case. Consequently, the higher availability of this source of power does not matter in comparing the results of both computations. The differences in the constraints give therefore the second computation an initial benefit of \$28.8 + 0.9 = \$29.7 mln. altogether.

Then there are differences in the costs of processing the high-grade ores into alloys and the costs of upgrading and mining the domestic ores. The following tables give a summary of these costs (in \$ per N.T. of manganese):

	Operating Costs		
	1st comp.	2nd computation	
Processing			
Ferro in blast furnaces	96.35	105.40 ⟶	105.40
Ferro in electric furnaces	98.80	109.75 ⟶	109.75
Ferro in Udy furnaces	107.45	112.15 ⟶	100.40
Silico in electric furnaces	157.10	168.05 ⟶	168.05
Silico in Wright furnaces	173.75	183.75 ⟶	176.15
Silico in Udy furnaces	142.95	171.60 ⟶	153.60
Beneficiation			
Dean process	92.60	88.40 ⟶	80.10
Sylvester-Dean process	81.60	76.90 ⟶	69.40
Wright process	96.70	96.70 ⟶	87.10
Udy process	87.20	71.25 ⟶	64.05
Mining			
Cuyuna ore	21.90	21.90 ⟶	19.70
Aroostook ore	30.35	30.95 ⟶	27.95

	Capital Costs	
	1st comp.	2nd computation
Processing		
Ferro in blast furnaces		
Ferro in electric furnaces		
Ferro in Udy furnaces		
Silico in electric furnaces	210.00	153.00 ⟶ 141.00
Silico in Wright furnaces		
Silico in Udy furnaces	383.00	264.50 ⟶ 243.75
Beneficiation		
Dean process	265.70	203.00 ⟶ 203.00
Sylvester-Dean process	391.50	294.00 ⟶ 270.40
Wright process	401.70	236.65 ⟶ 217.65
Udy process	170.25	188.50 ⟶ 173.50
Mining		
Cuyuna ore	24.00	24.00 ⟶ 22.00
Aroostook ore	14.75	14.75 ⟶ 13.55

From these tables it appears that the costs used for the second computation are in general higher for processing and lower for beneficiation and mining. It should, however, once more be stressed that these lower beneficiation and mining costs cannot be regarded as being unrealistic. They only reflect some improvement in the technology of upgrading domestic ores, an improvement which can be expected to materialize in lower costs once the processes are put in actual operation.

There is, however, no difference in the costs of importing foreign ores; they have been assumed to be the same in both cases.

The outcome of both computations (in millions of dollars) for all six years is summarized in the table on the following page.

The second program turns out to be $97.1 mln cheaper, despite $61.2 mln higher costs of processing high-grade ores into alloys. In other words, the costs of obtaining the required quantities of high-grade ores are $158.3 mln lower in the second computation (this is the difference between the totals of the "value" of the initial stockpile, the costs of importing foreign ores and

	1st computation			2nd computation		
Total costs of the program			1194.6			1097.5
"Value" of initial stockpile		279.0			128.7	
Costs of importation		222.0	501.—		99.—	227.7
			693.6			869.8
Cost of mining and beneficiation						
Operating						
Dean process	77.2			2.6		
Sylvester-Dean process	95.2			172.8		
Wright process	0.5			3.8		
Udy process	—.—	172.9		76.4	255.6	
Capital						
Dean process	20.0			—.—		
Sylvester-Dean process	50.6			79.6		
Udy process	—.—	70.6	243.5	23.3	102.9	358.5
Cost of alloy production						
Operating						
ferro in blast furnaces	378.4			327.6		
ferro by Udy process	—.—			91.1		
silico in electric furnaces	71.0			71.6		
silico by Wright process	0.7	450.1		6.1	496.4	
Capital						
ferro by Udy process	—.—	—.—	450.1	14.9	14.9	511.3

the costs of mining plus upgrading domestic ores in both computations.) True, the second computation had an initial "benefit" of \$29.7 mln on account of a more liberal set of constraints as shown before. Consequently, the costs of obtaining the required quantities of high-grade ores are actually \$128.6 mln lower in the second computation.

Furthermore, it appears from the last table that \$501.— minus \$227.7 = \$273.3 mln less is spent on foreign ores (under the assumption that the initial stockpile consists of foreign ores only). Of the latter amount \$97.1 mln is saved and \$176.2 mln more is spent in the United States itself. All this illustrates clearly what the consequences are of a slight improvement in the technology of upgrading domestic ores.

As for the individual process, there seems to be a strong preference for the Sylvester-Dean process in both programs. As a matter of fact, the shadow-prices indicate that, in both cases, the total costs would decrease more by adopting this process than by increasing the initial stockpile! Moreover, the first program prefers also the Dean process to an increase in the stockpile, while the second program is more in favor of the Udy process (particularly if we could assume that the alloy furnaces had some initial capacity). This, despite the fact that the second computation has been based on the assumption that there is barely enough manganese in the initial stockpile to enable the production of the quantities of alloys that are needed in the first year. All this seems to lead to the conclusion that it is good to practice restraint in adding foreign ore to the stockpile if one is faced with a rapidly developing technology of upgrading domestic ores.

It must, however, be borne in mind that much depends on the starting conditions in computations of the type as described here. For instance, if we could have assumed that some of the domestic beneficiation plants had already a sizeable capacity at the start of the program, then the solution would probably have been quite different. This becomes clear if we see that in the solution of both computations the Wright plant is, in almost all years, used to its full initial capacity despite the fact that it is the least favored process of all four considered here. This leads to the tentative conclusion that in an emergency situation as envisaged here probably every process will be utilized to the full extent of its initial capacity and by doing so will eliminate to the same extent other processes from being adopted.

All this leads up to the interesting problem of which starting conditions would be the best in terms of quantities of ore in the initial stockpile and initial capacities of the various plants. It is clear that this will depend on the given political situation and the then prevailing expectation about its future development. Chapter III of this paper deals with this type of problem.

A STOCHASTIC MANGANESE MODEL

Case of a probabilistic political forecast and a gross description of the manganese technology

INTRODUCTION

One dimension of difficulty, in considering problems of a strategic material such as manganese, is that of the complexity and interactions of importation, domestic production, stockpiling, and emergency availabilities. The concepts and techniques of linear and quadratic programming, as illustrated in the preceding section, handle these difficulties quite well.

Another dimension of difficulty is that of uncertainty. Estimates of future demand and availabilities of manganese must be based on assumptions about uncertain future political events and conditions. For this reason, another model, addressed to these uncertainties, rather than the complexities of the manganese system, has been constructed. This model emphasizes the dynamics of the acquisition of new information over time as the political future unfolds. This emphasis is achieved at the expense of the "fine structure" of the preceding model. No comparisons of domestic processes are made in it; instead, the gross relations of domestic production, importation, stockpiling, and emergency availabilities are studied under conditions of future uncertainties. For this reason, this model considers only manganese up to high-grade ore — not in the form of alloys.

Conceptually, the problem of uncertainty is handled by "embedding" a decision maker in the manganese problem. Our problem is not one of constructing a decision but one of constructing a conceptual decision maker — a policy. That is, by a policy we shall mean a collection of rules which determine for any given set of conditions, the current decisions for operating the manganese system (for the next year, say). A policy, thus, takes information into account as it becomes available.

For example, the adoption of a "five year plan", based on one's best estimate of the next five years (but, in year four, using the plan based on the estimates of conditions made four years previously, etc.) is a decision. A set of rules for constructing "one year plans", each based on all information available up to their adoption is a policy. On the surface, it might appear that a "five year plan" could be constructed with more long range objectives in mind than the rules for constructing "one year plans" might allow. This is not the case at all; long range objectives can be built into policies as well (in fact, better than) plans.

It is possible, then, to pit policies against uncertainties of the future, where it is not possible to so test decisions. This is the objective of this section: to describe a model for testing the performance of policies under uncertainty.

THE MODEL

A year's operation (say for year t) of the entire manganese system will be summarized into three kinds of information:

Internal State — a pair of numbers

- S_t: the amount of manganese stockpiled at the beginning of year t
- C_t: the (yearly) capacity of domestic manganese production facilities at the beginning of year t

External State — a pair of numbers

- D_t: industry demands for manganese during year t
- A_t: the amount of manganese available through importation during year t

Activities — a triple of numbers

- I_t: the amount of manganese imported during year t
- P_t: the amount of manganese produced domestically during year t

B_t: the amount of additional domestic production capacity built during year t.

We take these numbers, given for each year, as a description of the system in operation.

The problem of decision can be described this way: At the beginning of year 1959, we know the Internal State of stockpile and domestic production facilities — S_{59}, C_{59}, we can make a good estimate of the External State of manganese requirement and foreign availabilities — D_{59}, A_{59}, and we must make yearly Activity decisions on importation, domestic production, and facility expansions — I_{59}, P_{59}, B_{59}. During the year 1959, operations continue (as do political fortunes), and year 1960 begins with a new S_{60}, C_{60} (which can be calculated, in general, as $S_{60} = S_{59} + I_{59} + P_{59} - D_{59}$, $C_{60} = C_{59} + B_{59}$, etc.), and new estimates of D_{60}, A_{60}, with new decisions I_{60}, P_{60}, B_{60} to be made, and so on, year after year.

In order to study these decisions under uncertainty, we shall suppose that the External States (demands and availabilities, as governed by political conditions) are not known in the future with certainty — that political conditions change (or endure) with various probabilities. For illustrative purposes, five distinct political conditions, governing manganese demands and availabilities, have been selected, being, briefly:

	industry requirements	importation limits
p - peace	800,000 NT	none
c - cold war, no blockade	1,000,000 NT	none
cb - cold war, minor blockade	1,000,000 NT	1,000,000 NT
hb - hot war, minor blockade	1,200,000 NT	1,000,000 NT
hB - hot war, major blockade	1,200,000 NT	600,000 NT

Given that the political situation in year t, is any one of these, we suppose that definite probabilities are known concerning the situations to be in year $t + 1$. We take, for illustration, the following probabilities:

If the political situation is:	in the following year, the probabilities are:				
	p	c	cb	hb	hB
p	.7	.2	.1	0	0
c	.1	.6	.2	.1	0
cb	.1	.2	.4	.1	.2
hb	.4	0	0	.4	.2
hB	.5	0	0	.2	.3

As a check on their plausibility, if these probabilities were in effect over a long period of time, we would expect the following frequencies for political conditions:

p: 39%
c: 28%
cb: 16%
hb: 10%
hB: 7% .

We will call this table of probabilities a <u>forecast</u> of the political environment. As the probabilities in the p column are increased and the hb, hB columns decreased, the forecast becomes more "optimistic" and conversely, more "pessimistic" forecasts can also be made.

Summarizing, then, our model has the following specifications (beginning with 1959, say):

S_{59}, C_{59} are determined from the past operation
D_{59}, A_{59} are determined by the political situation
I_{59}, P_{59}, B_{59} are selected by a policy

S_{60}, C_{60} are determined from the past operation

D_{60}, A_{60} are determined by the political situation

I_{60}, P_{60}, B_{60} are selected by a policy

etc. In this way, any given policy will generate (tried over and over) a set of future histories of the operation. Costs and emergency availabilities of these histories can be computed and the policy judged accordingly.

In order to assess the performance of policies we need to specify costs of importation, of domestic production, of domestic plant buildup, of shortage, etc. With such constants inserted into the model we are in a position to find policies, under various conditions of uncertainty, which minimize the total expected cost (of operation and shortages) of the system over a given period of time. These costs are, by and large, aggregates of the costs of the previous model; (The Dean process has been used for domestic production costs); these costs are not the lowest estimated but are generally considered the most reliable estimates of a domestic process which are available. The model and its costs will be derived in the final report, of course, and discussed in more detail.

ANALYSIS OF THE MODEL

Fortunately, it is possible, through mathematical analysis, to discover a policy which minimizes costs directly, accounting for the long run effects of the uncertainties one has chosen for the model without trying various ones out. A technique for doing this is called "dynamic programming"; for practical computation, the technique relies heavily on the modern high speed internally programmed electronic computer.

This model has been analyzed by means of a special computer code written by Mr. Stuart Dreyfus, of the Mathematics Department of Princeton University, in the Maniac computer belonging to the University. We have been most fortunate in acquiring the service of Mr. Dreyfus, who has had unique and unusual experience with the coding of this technique as a staff member of the RAND Corporation, and in the cooperation of the University in obtaining the services of the Maniac.

The mathematical technique of dynamic programming builds up intrinsic valuations for each possible condition the manganese system may find itself in. This valuation is taken to be the following:

> Given a political situation (and its induced External State) and an Internal State for the manganese system, that composite condition is assigned the minimum possible costs (including those of operating, capital investment, and shortages) which would accrue over the next ten years, averaged over the uncertainties of the forecast of the political environment.

This valuation system, when it is built up, provides the raw material for building policies which achieve these minimum costs in the operation of the manganese system. For this model and the political forecast given above the valuations have been computed, and an illustrative, though incomplete, sampling of them is described below. The figures are in millions of dollars, required to support the manganese system for ten years, averaged over the political uncertainties. A "year's stockpile", "50% domestic facilities", etc. refers to peace time requirements (a year's peace time requirements in stockpiles, 50% of a year's peace time requirements in domestic facilities, etc.). The conditions in which the manganese system may find itself have three dimensions — the political situation, the stockpile, and the domestic facilities. It should here be remarked that these costs are only those of acquiring the high-grade ores, not the alloys; so, they are not directly comparable with those of the econometric model.

Political Situation	% Domestic Capacity	years of stockpile 0	1	2	3
p	0	900	805	715	630
	50	780	700	625	560
	100	740	695	680	685
c	0	1000	895	795	710
	50	870	775	690	580
	100	800	730	695	685
cb	0	1070	925	810	715
	50	910	790	680	615
	100	815	735	695	685
hb	0	1130	965	850	735
	50	975	830	715	620
	100	855	750	695	685
hB	0	1355	1000	835	720
	50	1050	830	705	610
	100	875	740	695	685

Notice that these valuations relate the political situation to the value of domestic capacity and stockpiles. With political situation c (cold war, no blockade) and no domestic capacity the value of the first year of stockpile is 1070 - 925 = 145. I.e., the first year of stockpile reduces the ten year costs of 145 (million dollars). Similarly the value of the second year of stockpile is 925 - 810 = 115. In contrast, in political situation hB, with no domestic capacity, the value of the first year of stockpile is 1355 - 1000 = 355 — more than twice the value above, showing the effect of an emergency. Similarly the value of domestic capacity changes under changing political situations and stockpile conditions.

As a rule of thumb, in the table above, it costs (as opposed to its value) about 120 to move down a row (to increase domestic capacity by 50%) and costs about 120 (depending, to some extent, on the domestic capacity available) to move a column to the right (to increase stockpile by a year). These rules of thumb give indications, for each political situation where in the table it is profitable to be: if one can trade more value for less cost it would be profitable to do so. For example, it is worth having the first year's stockpile in political situation c with no domestic capacity, since the value differential 1070 - 925 = 145 is greater than the cost differential 120. By debiting each internal state with these differential costs, we obtain a new "rule of thumb" table of directly comparable "adjusted costs": (adding to each entry, 120 for each year of stockpile and 120 for each 50% of domestic capacity).

Political Situation	% Domestic Capacity	years of stockpile 0	1	2	3
p	0	900	925	955	990
	50	900	940	985	1040
	100	980	1055	1160	1285
c	0	1000	1015	1035	1070
	50	990	1015	1050	1060
	100	1040	1090	1175	1285
cb	0	1070	1045	1050	1055
	50	1030	1030	1040	1095
	100	1055	1095	1175	1285
hb	0	1130	1085	1090	1095
	50	1095	1070	1075	1100
	100	1095	1110	1175	1285
hB	0	1355	1120	1075	1080
	50	1170	1070	1065	1090
	100	1115	1100	1175	1285

With this valuation system, the problem of decision making under uncertainty is reduced to the following:

1) The decision maker finds the manganese system in one of the conditions indicated by the chart in a given political situation, with a given domestic capacity, and a given stockpile.

2) Depending on the political situation, the decision maker has some degree of choice in

moving from the Internal State (a position within the subtables) to another Internal State at the beginning of the following year.

3) Depending on the political situation, the new political situation emerges (by chance in our model — at least partly by design from other viewpoints) and determines a new condition of the manganese system.

Example: Suppose we are at:

political situation cb
no domestic capacity — 1040 in last chart
2 years stockpile

and that an allowable strategy is to use up a year of the stockpile (an increase the domestic capacity to 50%). Then, referring back to the probability table by which probability situation cb is transformed into next year's situation, we see that we shall be at:

1 year stockpile
50% domestic capacity

and

Political Situation	with probability	adjusted cost
p	.1	940
c	.2	1015
cb	.4	1030
hb	.1	1070
hB	.2	1070

with a long run average adjusted cost of

$$(.1)940 + .2(1015) + .4(1030) + .1(1070) + .2(1070) = 1030.$$

We then associate the value 1030 with that decision — and, compute, for each possible decision, a similar associated value, picking the decision which gives a lowest value. Repeating this entire analysis for each possible condition in which the manganese situation may be found, we build up, then a policy which at each point in time selects a decision to minimize the long run average of the adjusted costs.

It should be remarked that the above illustrative analysis is one of "rule of thumb". We must define costs of proceeding from one Internal State to another more accurately, and provide for many more possible conditions (1 1/2 years stockpile, 25% domestic capacity, etc.) for a more reasonable representation of the situation. But the idea is precisely that of the rule of thumb analysis, above, carried out in more detail.

RESULTS

It must be borne in mind that these results are more illustrative of what kinds of information can be obtained by the analysis methods developed here than of actual advice in the problem of manganese. The tables above are only a sample of the information generated by the computer runs. And these runs are based on assumptions which require further substantiation.

In order to illustrate the kind of information the analysis provides, we use the tables above to demonstrate the character of an optimal policy for the stochastic manganese model. As indicated above, in any given situation, an optimal decision can be found; the employment of optimal decisions over a period of time will result in tendencies for the system to locate itself in certain general positions. Thus, each political state, under an optimal policy will tend to produce a certain Internal State. By carrying out the required analyses, we find these states to be:

Political Situation	tends to produce	Internal State: Domestic Capacity	Internal State: Year's Stockpile
p		50%	0
c		50%	0
cb		50%	1
hb		50%	1
hB		50%	1

With finer distinctions in the valuating system we would find greater variability in these Internal States. (As it is, we only have a choice of 0, 50, or 100% Domestic Capacity — in p, say, 50% may be better than 0 or 100% but not better than 25% — finer distinctions will bring out these points.) Actually in the full fledged analysis, both Domestic Capacity and Year's Stockpile will tend to increase as the political situation gets graver.

It remains to explore the sensitivity of these results on the specific data used in the model. Two phenomena stand out in such an analysis:

1) the differences, in the adjusted costs table, between low stockpiles and high stockpiles are not at all large even with 50% domestic capacity present, and the differential between importation costs and domestic costs will have to be correspondingly greater to indicate really strong effects.

2) the relative value of stockpiling is highly dependent on domestic capacity: roughly, they are competing devices — if one is employed, the other loses value rapidly, and the indicated policy chooses domestic capacity over stockpiling as its primary device for handling the problem of emergency.

Point 1) suggests that a relatively small stockpile is indicated with its actual size not too sensitive. From this it might seem that technological developments in manganese may be much more crucial than the economic problem.

APPENDIX

Mathematical Formulation of the Manganese Problem (Case of Fixed Political Forecast)

The objective is, as stated before, to provide the iron and steel industry with required amounts of manganese alloys at minimum cost by a combination of importing ores, stockpiling ores and producing high grade ores domestically.

A distinction has been made between the part λ of the manganese requirements that has to be supplied in the form of ferro-manganese, the part μ for which both alloys can be used and the part υ for which silico-manganese is preferred to a combination of ferro-manganese and ferro-silicon.

Denoting the total amount of manganese required in year i by Q_i, we have the equality:

$$Q_i = \lambda_i + \mu_i + \upsilon_i \qquad \text{for } i = 1, 2, \ldots, 6$$

The parameters λ, μ and υ can be found at the top of the diagram in the text.

Looking at that diagram, we can state that, in order to meet the requirements for ferro-manganese, it is necessary that

$$x_{3,i} + x_{4,i} + x_{5,i} \geq \lambda_i + x_{1',i} \qquad \text{for } i = 1, 2, \ldots, 6$$

where $x_{1'}$ stands for the amounts of ferro-manganese that will be used for that part of the manganese requirements for which ferro- as well as silico-manganese can be used; x_3, x_4 and x_5 stand for the amount of ferro-manganese to be produced in blast furnaces, in electric furnaces and in "Udy"-furnaces.

Similarly, in order to meet the requirements for silico-manganese, it is necessary that

$$x_{6,i} + x_{7,i} + x_{8,i} \geq x_{2',i} + \upsilon_i \qquad \text{for } i = 1, 2, \ldots, 6$$

where x_6, x_7 and x_8 stand for the quantities of silico-manganese that can be obtained by one of the various silico-manganese processes.

Moreover, we have the equality

$$x_{1',i} + x_{2',i} = \mu_i \qquad \text{for } i = 1, 2, \ldots, 6.$$

Elimination of the $x_{1',i}$ variables leads to the following set of inequalities:

$$(1) \quad x_{2',i} + x_{3,i} + x_{4,i} + x_{5,i} \geq \lambda_i + \mu_i \qquad \text{for } i = 1, 2, \ldots, 6$$

$$(2) \quad - x_{2',i} + x_{6,i} + x_{7,i} + x_{8,i} \geq \upsilon_i \qquad \text{for } i = 1, 2, \ldots, 6$$

$$(3) \quad x_{2',i} \leq \mu_i \qquad \text{for } i = 1, 2, \ldots, 6.$$

To produce the required quantities of ferro- and silico-manganese in the normal way, a sufficient quantity of ore with a certain minimum content of manganese has to be available. This high-grade ore can be obtained from the stockpile, from importing foreign ores and from up-grading domestic ores and slags by the Dean and the Sylvester-Dean process.[1] For conceptual reasons we make now the hypothesis that all high-grade ores that will be imported or produced domestically go into the stockpile and all ore that is needed for the production of these alloys is taken out of the stockpile.

Now the statement can be made that so much ore must be stockpiled at any time as is necessary to meet the requirements up to that time; of course more ore of this type might be stockpiled at

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[1] The other two processes produce as a first step slags with a lower manganese content than required for the production of the alloys along conventional lines: these slags have to be processed in a special way and are therefore left out of the discussion for the moment.

this time. This is expressed as follows:

$$c_3 \sum_{j=1}^{i} x_{3,j} + c_4 \sum_{j=1}^{i} x_{4,j} + c_6 \sum_{j=1}^{i} x_{6,j} \leq S_i \quad \text{for every } i = 1, 2, \ldots, 6.$$

The coefficients c_3, c_4 and c_6 stand for the amounts of manganese in the ores sufficient to produce 1 N (et) T(on) of Manganese in the form of alloys; in other words, they are technical ratios. The meaning of the last expression is now to guarantee that there is sufficient high-grade ore stockpiled at the beginning of every year i as is required for the production of the alloys from the very first year up to the end of year i.

The quantity of ore that is stockpiled up to the beginning of year i is now by definition equal to what there was in the stockpile at the beginning of year 1 plus what has been imported or produced domestically in the previous years. This is the consequence of the hypothesis that all imported and domestically produced ore goes into the stockpile. Stated more exactly:

$$S_i = S_1 + \sum_{j=1}^{i-1} x_{9,j} + \sum_{j=1}^{i-1} x_{10,j} + \sum_{j=1}^{i-1} x_{11,j} + \sum_{j=1}^{i-1} x_{12,j} + \sum_{j=1}^{i-1} x_{13,j} \quad \text{for } i = 1,2,\ldots,6.$$

Combining the last two expressions gives us the fourth inequality of our system:

$$(4) \quad c_3 \sum_{j=1}^{i} x_{3,j} + c_4 \sum_{j=1}^{i} x_{4,j} + c_6 \sum_{j=1}^{i} x_{6,j} - \sum_{j=1}^{i-1} x_{9,j} - \sum_{j=1}^{i-1} x_{10,j} - \sum_{j=1}^{i-1} x_{11,j} - \sum_{j=1}^{i-1} x_{12,j} - \sum_{j=1}^{i-1} x_{13,j} \leq S_1 \quad \text{for } i = 1,2,\ldots,6.$$

It should be remarked that in the actual computations the j-index of the last two sigma signs was permitted to run from 1 to i instead of i - 1. This means that the quantities of high-grade ores produced in year i by the Dean and Sylvester-Dean processes were also used to meet the requirements of year i. For one can always manage to produce these quantities, while in the case of importation one cannot always be sure that the foreign ores will come into the country.

Ferro-manganese can be produced in blast furnaces, using coal as source of energy as well as in electric furnaces. Silico-manganese can be produced in electrical furnaces only. So, it is quite conceivable that the amount of electric energy that will be needed to produce these alloys will exceed what is available for it in an emergency. To make sure that this required amount of electric energy will always be within bounds, the following two inequalities have been included in the model:

$$(5) \quad (e_5 + e_{15} \cdot c_5)x_{5,i} + (e_8 + e_{15} \cdot c_8)x_{8,i} \leq E_{1,i} \quad \text{for } i = 1, 2, \ldots, 6$$

$$(6) \quad e_4 x_{4,i} + e_6 x_{6,i} + e_7 x_{7,i} \leq E_{2,i} \quad \text{for } i = 1, 2, \ldots, 6.$$

The coefficients e_4, e_5, e_6, e_7 and e_8 stand here for quantities of electric energy needed for the production of the alloys in the various ways.

The reason for including two rather than one inequality in this model is that the location of the Udy plants does not allow them to draw on the existing sources of electric energy. On the contrary, these plants will have to obtain the necessary quantity of electric energy from power plants in their neighborhood which are now under construction.

So far, we ignored the fact that in many instances the plants and mines necessary to carry out this program do not exist as yet. Hence, the model must provide for the construction of these facilities to the extent as they will be needed in the successive years.

It can be assumed that the necessary alloy plants exist except for those needed in the Udy process. To provide for the latter, electric furnaces have to be constructed in which the ferro- as well as the silico-manganese can be produced. This is expressed by the following formulae:

$$x_{5,i} + f_c x_{8,i} \leq K_{5,i} \qquad \text{for} \quad i = 1, 2, \ldots, 6.$$

In this case the coefficient f_c stands for the ratio between the period of time needed to produce 1 N.T. of manganese in the form of silico-manganese and that needed to produce it in the form of ferro-manganese; this ratio is approximately 1.7.

$K_{5,i}$, the capacity of the furnaces at the beginning of any year i, is by definition equal to the capacity at the start $K_{5,1}$ (which is zero in this case) plus what is added to it in the previous years, which additions will be denoted by the symbol $y_{5,i}$. The exact formulation for this is:

$$K_{5,i} = K_{5,1} + \sum_{j=g_5}^{i-1} y_{5,j}$$

g_5 stands here for the gestation period, i.e., the time that is needed to give the plant its initial capacity. If one starts with constructing the plant in the second year, for instance, and the gestation period is one year, then it is clear that one cannot expect the plant to produce prior to the third year.

Combining the last two expressions gives us the 7th inequality of our system:

$$(7) \quad x_{5,i} + f_c x_{8,i} - \sum_{j=1}^{i-1} y_{5,j} \leq K_{5,1}$$

for $i = 1, 2, \ldots, 6$.

Having considered the alloy plants, we now turn to the beneficiation plants, that have to be built to enable the upgrading of the domestic ores and slags. They lead to expressions similar to the last one:

$$(8) \quad x_{12,i} - \sum_{j=g_{12}}^{i-1} y_{12,j} \leq K_{12,1}$$

(Dean's process) $i = 1, 2, \ldots, 6$

$$(9) \quad x_{13,i} - \sum_{j=g_{14}}^{i-1} y_{13,j} \leq K_{13,1}$$

(Sylvester-Dean process) $i = 1, 2, \ldots, 6$

$$(10) \quad c_7 x_{7,i} - \sum_{j=g_{14}}^{i-1} y_{14,j} \leq K_{14,1}$$

(Wright's process) $i = 1, 2, \ldots, 6$

$$(11) \quad c_5 x_{5,i} + c_8 x_{8,i} - \sum_{j=g_{15}}^{i-1} y_{15,5} \leq K_{15,1}$$

(Udy's process) $i = 1, 2, \ldots, 6$.

In addition, the mines have to be developed and the facilities for producing the "Hot Metal" that is needed in the Wright process have to be created. But the "Open Hearth" slags are automatically produced by the steel mills and no facilities are needed there. Hence, there are only three expressions in this case, namely:

$$(12)\quad c_{12}x_{12,i} - \sum_{j=g_{16}}^{i-1} y_{16,j} \leq K_{16,1}$$

(Cuyuna mines) $i = 1, 2, \ldots, 6$

$$(13)\quad c_{14} \cdot c_7 x_{7,i} - \sum_{j=g_{18}}^{i-1} y_{18,j} \leq K_{18,1}$$

(Hot Metal plant) $i = 1, 2, \ldots, 6$

$$(14)\quad c_{15} \cdot c_5 x_{5,i} + c_{15} \cdot c_8 x_{8,i} - \sum_{j=g_{19}}^{i-1} y_{19,j} \leq K_{19,1}$$

(Aroostook mines) $i = 1, 2, \ldots, 6.$

Finally, we had to take into account that no more ores can be extracted from the deposits than is contained in them and that no more slag will be used than is actually available. This is expressed in the following inequalities:

$$(15)\quad c_{12} \sum_{j=1}^{i} x_{12,j} \leq R_{16,1}$$

(Cuyuna Mn. ore) $i = 1, 2, \ldots, 6$

$$(16)\quad c_{13} \sum_{j=1}^{i} x_{13,j} + c_7'' \sum_{j=1}^{i} x_{7,j} \leq R_{17,i}$$

(O.H. slags) $i = 1, 2, \ldots, 6.$

$$(17)\quad c_7''' \sum_{j=1} x_{7,j} \leq R_{18,1}$$

(Cuyuna Fe ore) $i = 1, 2, \ldots, 6$

$$(18) \quad c_{15} \cdot c_5 \sum_{j=1}^{i} x_{5,j} + c_{15} \cdot c_8 \sum_{j=1}^{i} x_{8,j} \leq R_{19,1}$$

(Aroostook ore) $i = 1, 2, \ldots, 6.$

The coefficients c_7'' and c_7''', used here, are compound ratios.

The model presented so far contains in general per time period 18 inequalities, involving 12 activity- and 8 capacity-variables, so 20 variables altogether. Some of these inequalities and variables drop out in the first period because the capacities necessary to produce do not exist yet. And in the sixth and last period there will be no increases in the capacities since these would only effect the production in the seventh period which is not to be provided for by the model. The same holds for the 3 import activities in the sixth year; they too would not effect the solution for the 6 periods considered here.

The exact number of inequalities and variables for the six periods here considered are:

	Inequalities	Variables Activity	Variables Capacity	Total
1st period	12	9	8	17
2nd period	18	12	8	20
3rd period	18	12	8	20
4th period	18	12	8	20
5th period	18	12	8	20
6th period	18	9	—	9
Total	102	66	40	106

With each of these 106 variables is associated a cost; together they form the total cost of the program. In general, it can be assumed that the cost per unit of activity is constant. The cost of such an activity is then a linear function of that activity. And the total expenditure of the program is then the sum of the costs of all activities. Stated more exactly:

$$E_1 = \sum_{i=1}^{6} \sum_{k=1}^{12} \alpha_{k,i} x_{k,i} + \sum_{i=1}^{5} \sum_{k=1}^{8} \alpha'_{k,i} y_{k,i} \quad .$$

The right-hand expression, as it stands, is a shorthand for $6 \times 12 = 72$ activity and $5 \times 8 = 40$ capacity variables. However, 3 activity variables in the first period drop out because the necessary capacity does not yet exist and another 3 activity variables in the last period. viz., the import activities drop out too. Hence, there are in total $72 - 6 + 40 = 106$ linear terms altogether.

In the case of the foreign ores, however, it may not be assumed that the cost per unit of activity, i.e., the export price, is a constant. Experience in the past has shown that the export price of Indian ore, for instance, does depend on the quantity bought by the United States from India as was to be expected. And what is more, it must be assumed that this price depends also on the quantities bought by the United States from the other two areas, i.e., South and West Africa as well as Latin America. This is a mere reflection of the dominant position of the United States as a buyer of this ore on the market of each area as well as on the market of all areas together. In addition to this, it must be assumed that the export price of Indian ore in year i will depend not only on the quantities bought by the United States from all three areas in year i, but also on those bought in the previous year $(i - 1)$.

Denoting the export price of Indian ore in year i by $p_{9,i}$, this can be expressed as follows:

$$p_{9,i} = \beta_{9,i} x_{9,i} + \beta_{9,i-1} x_{9,i-1} - \beta_{10,i} x_{10,i} - \beta_{10,i-1} x_{10,i-1} - $$
$$- \beta_{11,i} x_{11,i} - \beta_{11,i-1} x_{11,i-1} + \alpha_{9,i}$$

where α's and β's are constants. There are similar price functions for the other two export prices. Together, they can be formulated as follows:

$$p_{\ell,i} = \sum_{k=9}^{11} (\beta_{k,i}x_{k,i} + \beta_{k,i-1}x_{k,i-1}) + \alpha_{\ell,i} \qquad \text{for } i = 1, 2, \ldots, 6$$

$k = 9, 10$ and 11, $\ell = 9, 10$ and 11, $\beta_{k,i}$ and $\beta_{k,i-1}$ are positive for $k = \ell$ and negative for $k \neq \ell$.

Now, these prices have to be multiplied by their corresponding quantities $x_{\ell,i}$ ($\ell = 9$, 10 and 11) to give the costs of the imports from a certain area ℓ in a certain year i. For instance, the costs of importing ore from India ($\ell = 9$) in year i is:

$$x_{9,i}p_{9,i} = x_{9,i}\left[\sum_{k=9}^{11} (\beta_{k,i}x_{k,i} + \beta_{k,i-1}x_{k,i-1}) + \alpha_{9,i}\right]$$

The costs of importing ores from all three areas in year i is then:

$$\sum_{\ell=9}^{11} x_{\ell,i}p_{\ell,i} = \sum_{\ell=9}^{11} x_{\ell,i}\left[\sum_{k=9}^{11} (\beta_{k,i}x_{k,i} + \beta_{k,i-1}x_{k,i-1}) + \alpha_{\ell,i}\right] =$$

$$= \sum_{k=9}^{11}\sum_{\ell=9}^{11} (\beta_{k,i}x_{k,i}x_{\ell,i} + \beta_{k,i-1}x_{k,i-1}x_{\ell,i}) + \sum_{\ell=9}^{11} \alpha_{\ell,i}x_{\ell,i}$$

for $i = 1, 2, \ldots, 6$ and the same restrictions on the $\beta_{k,i}$ and $\beta_{k,i-1}$.

The expenditure of importing ores from all three areas in the first five years of our program is then:

$$E_2 = \sum_{i=1}^{5} \sum_{\ell=9}^{11} x_{\ell,i} p_{\ell,i}$$

$$= \sum_{i=1}^{5} \sum_{k=9}^{11} \sum_{\ell=9}^{11} (\beta_{k,i} x_{k,i} x_{\ell,i} + \beta_{k,i-1} x_{k,i-1} x_{\ell,i}) +$$

$$+ \sum_{i=1}^{5} \sum_{\ell=9}^{11} \alpha_{\ell,i} x_{\ell,i}$$

with the same restrictions on $\beta_{k,i}$ and $\beta_{k,i-1}$ as before.

The expression under the 3 Σs is a shorthand for $5 \times 3 \times 3 = 45$ terms related to the period i and $4 \times 3 \times 3 = 36$ terms related to the period $(i - 1)$; so there are altogether 81 quadratic terms here.

The expression under the 2 Σs contains 15 linear terms already included in the formulas for E_1 stated before. The expression for the total expenditures of the program is therefore:

$$E = E_1 + E_2 = \sum_{i=1}^{6} \sum_{k=1}^{12} \alpha_{k,i} x_{k,i} + \sum_{i=1}^{5} \sum_{k=1}^{8} \alpha'_{k,i} y_{k,i} +$$

$$+ \sum_{i=1}^{5} \sum_{k=9}^{11} \sum_{\ell=9}^{11} (\beta_{k,i} x_{k,i} x_{\ell,i} + \beta_{k,i-1} x_{k,i-1} x_{\ell,i}) .$$

Two points have still to be mentioned here:

a) As it now stands, the program is charged with the full cost of building the plants and developing the mines. But it is highly probable that at the end of the six years considered here, the plants and mines will still have some value as production factors.

The more so, since the normal life span of these plants and mines is assumed to be approximately ten years. It is therefore reasonable to charge the program with only a fraction of these capital costs. This is achieved by multiplying these costs with an appropriate depreciation factor.[1]

b) Since these expenditures occur in various years, they have to be brought on a common basis. This is achieved by bringing all expenditures forward to the beginning of the first year by multiplying each expenditure with an appropriate discount factor.

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[1] These points are more fully discussed in Technical Report No. 2, of 12 September, 1957, page 19/20 and 23.