

Price Movements in the Stock Market

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Summary

This paper contains an analysis of the number of stock prices which advanced and declined on The New York Stock Exchange on each trading day during a four year period.

The analysis confirms the findings of a number of other authors, that price movements in a stock market are of an essentially random nature.

It appears, however, that prices of individual stocks do not move independently. There is a strong tendency for prices to move in unison, either up or down.

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1. Introduction

1.1. To a casual observer of the stock market prices will probably seem to move in a completely random fashion. Stock brokers and others operating in the market appear, however, to believe very firmly that there is some system in these movements, and that people who know or can guess how the system works, stand to make a profit. So far there is little or no statistical evidence to support such assumptions, and Granger and Morgenstern (4) seem quite justified in referring to these widely held beliefs as "stock-market folklore."

1.2. There is on the other hand an increasing amount of statistical evidence which indicates that price movements may well be completely random.

The first substantial piece of evidence is probably Cowles' study from 1933. Cowles (2) showed that by buying and selling at random one would do just as well as by following the advice of recognized (and highly paid) professional investment advisers.

Twenty years later Kendall (5) found that a number of stock price indices behaved as if generated by a random walk process. Kendall comments as follows on his findings: "Investors can, perhaps, make money on the Stock Exchange, but not, apparently, by watching price-movements and coming in on what looks like a good thing." However, he adds, a little sadly: "But it is unlikely that anything I say or demonstrate will destroy the illusion that the outside investor can make money by playing the markets, so let us leave him to his own devices." Alexander (1) who analyzed Kendall's data, using different statistical techniques, arrived at

substantially the same conclusion. Osborne (6) found that prices in the New York Stock Exchange seemed to behave in a random manner, similar to that of Brownian motion in statistical mechanics. Recently Granger and Morgenstern (4) used some of the most advanced available statistical techniques to analyze a vast amount of stock market statistics, and found practically nothing which could throw doubt on the random walk hypothesis.

1.3. In the light of the evidence just cited, it appears to be of some significance just to find something in the stock market which is not purely random. This paper has no higher aspiration level than doing just this.

2. Description of the Data

2.1. For the period July 1, 1958 - June 29, 1962 we recorded the number of stock prices which advanced, declined or remained unchanged on each trading day at the New York Stock Exchange. There were 1007 trading days in the period, and the number of stocks listed was above 1500, so in a sense we obtained a sample of more than 1.5 million observations.

It is obviously not practical to reproduce the data in full, but a sample is given in Table 1. This extract includes the latter half of May 1962 when prices fell dramatically, and the first half of June 1962 which showed the beginning of a recovery.

Table 1

Price changes on the New York Stock Exchange

May 14 — June 15, 1962

<u>Date</u>	<u>Number of Stocks</u>			
	Advanced	Declined	Unchanged	Not Traded
May 14	635	468	211	227
" 15	936	209	149	253
" 16	468	575	236	262
" 17	316	696	251	278
" 18	420	554	255	312
" 21	382	599	233	327
" 22	141	982	174	244
" 23	135	1005	156	245
" 24	292	777	214	258
" 25	170	1004	160	207
" 28	74	1212	89	166
" 29	630	637	132	142
" 31	1071	190	96	184
June 1	554	556	225	206
" 4	105	1110	120	206
" 5	507	581	229	224
" 6	895	199	185	262
" 7	528	499	240	274
" 8	543	454	233	311
" 11	305	766	193	277
" 12	123	1022	141	255
" 13	196	935	185	225
" 14	186	965	166	224
" 15	837	303	192	204

2.2. In the crudest possible form we can summarize the data as follows:

505 095	Advances
519 494	Declines
227 089	Unchanged

The Standard and Poor's index of 500 common stocks stood at 45.98 in July 1958, and at 55.63 in June 1962. This index includes almost half the common stocks of U. S. companies which are listed on the New York Stock Exchange. Preferred stocks and stocks in foreign companies may not behave in the same way as common U. S. stocks, but it seems nevertheless safe to assume that stock prices in general have increased by about twenty per cent during the four year period covered by our data. Under this assumption it is a little surprising that we should find about 14,000, or three per cent more declines than advances in our data. This means of course that the average price advance was considerably greater than the average decline during the four years under consideration.

This result has little significance in itself, but it is worth noting that it is a direct contradiction of the findings of Cowles and Jones (3).

2.3. On each day there are a number of listed stocks which are not traded and it seems natural to group these with the class "Unchanged." However, the number of listed stocks varies from time to time. New stocks are admitted, and stocks are removed, either as a result of mergers, or because they are not subject to very active trading. It would be possible from published information to determine the exact number of listed stocks for each trading day, but this does not seem worth while for our present purposes.

The number of stocks listed at the end of each of the four years covered by our data were:

End 1958	1507
End 1959	1507
End 1960	1528
End 1961	1541

If we take the arithmetic average, 1521, and assume that this is the average number of listed stocks during the period, we can summarize our findings as in Table 2.

Table 2

Daily changes in Stock Prices at NYSE — July 1959 - June 1962

Advances	505 095
Declines	519 494
Unchanged or not traded	<u>507 058</u>
Total	1 531 647

2.4. Table 2 seems to confirm the various random hypotheses referred to in Section 1. The Table may be taken to suggest that on any day the probability that an arbitrary stock shall advance, decline or remain unchanged is about $1/3$. We shall examine this rather drastic hypothesis in some detail in Section 3.

2.5. Our data can be considered as consisting of 1007 observations of a pair (x,y) , where x and y respectively stand for the number of advances and declines on the day observed. Table 3 gives the frequencies of the observations in a rather coarse grouping. Casual inspection of the

Table suggests that our sample may have been drawn from a bivariate normal population.

The usual statistics computed from our observations are:

$$\bar{x} = \frac{1}{n} \Sigma x = \frac{505\ 095}{1007} = 501.6$$

$$\bar{y} = \frac{1}{n} \Sigma y = \frac{519\ 494}{1007} = 515.9$$

$$s_x^2 = \frac{1}{n} \Sigma (x - \bar{x})^2 = 20\ 498$$

$$s_y^2 = \frac{1}{n} \Sigma (y - \bar{y})^2 = 22\ 209$$

$$s_x = 143.2$$

$$s_y = 149.0$$

$$c_{xy} = \frac{1}{n} \Sigma (x - \bar{x})(y - \bar{y}) = -19\ 819$$

$$r = \frac{c_{xy}}{s_x s_y} = -0.923$$

These statistics, together with an inspection of Table 3 suggest that our sample may have been drawn from a bivariate Normal population with very high negative correlation. We shall try to throw some further light on this suggestion in the following section.

3. Complete Independence and Randomness

3.1. We shall now consider a simple theoretical model. Let

p = the probability that an arbitrary stock shall advance on a particular day

q = the probability that the same stock shall decline

Table 3

Number of advances and declines in stock prices at NYSE July 1958 - June 1962

<u>Declines</u>	<u>Advances</u>										<u>Total</u>
	0	-199	200-299	300-399	400-499	500-599	600-699	700-799	800-899	900-	
0	-	-	-	-	-	-	-	-	6	3	9
200-299	-	-	-	-	-	11	21	6	1	39	
300-399	-	-	-	-	36	85	36	2	-	159	
400-499	-	-	-	-	49	183	62	2	-	296	
500-599	-	-	-	20	187	69	3	-	-	279	
600-699	-	9	35	70	38	1	1	-	-	119	
700-799	1	6	19	21	-	-	-	-	-	59	
800-899	6	19	1	1	1	-	-	-	-	27	
900-	20	-	-	-	-	-	-	-	-	20	
<u>Total</u>	27	63	112	277	289	162	59	14	4	1007	

We shall assume that these probabilities are the same for all the m listed stocks, and we further assume that any arbitrary stock will advance, decline or stay put independently of how other stocks behave. The probability that we shall observe x advances and y declines is then

$$f(x,y) = \frac{m!}{x! y! (m-x-y)!} p^x q^y (1-p-q)^{m-x-y}$$

For large m we have approximately

$$f(x,y) = K \exp \left\{ - \frac{(1-p)(1-q)}{2(1-p-q)} \left[u^2 + \frac{2\sqrt{pq}}{\sqrt{(1-p)(1-q)}} uv + v^2 \right] \right\}$$

where

$$u = \frac{x - mp}{\sqrt{mp(1-p)}}$$

$$v = \frac{y - mq}{\sqrt{mq(1-q)}}$$

$$K = \frac{1}{2 \pi m \sqrt{pq(1-p-q)}}$$

3.2. In our example we have $m = 1521$. Under the drastic assumption that

$$p = q = 1/3$$

our formula reduces to

$$f(x,y) = K \exp \left\{ - \frac{2}{3} \left[\left(\frac{x - 507}{13\sqrt{2}} \right)^2 + \left(\frac{x - 507}{13\sqrt{2}} \right) \left(\frac{y - 507}{13\sqrt{2}} \right) + \left(\frac{y - 507}{13\sqrt{2}} \right)^2 \right] \right\}$$

This theoretical distribution does clearly differ substantially from the sample distribution studied in para. 2.4. and tabulated in Table 3. Firstly, the variance of the theoretical distribution is much smaller than in our sample, and secondly, the theoretical correlation coefficient is $r = - 0.5$, while in the sample we found $- 0.923$, i.e., almost perfect negative correlation.

3.3. Our sample does in no way upset the hypothesis that price movements in a stock market have the nature of a random walk. However, the sample also shows that individual stock prices do not move independently. If the hypothesis of complete independence was true, we should get a far stronger concentration around the central values than shown by Table 3, in fact, the odds would be less than 1 to 1000 of finding any observations at all outside the four central cells of Table 3.

Our sample shows that there is a strong tendency for stock prices to move together — either up or down. This is not really surprising. Concepts like "rally" and "crash" are familiar in most stock markets. However, rallies and crashes would be extremely unlikely if all stock prices moved independently in a random manner.

4. Concluding Remarks

4.1. The evidence that stock prices move in a random manner is obviously — to put it mildly — disturbing to security analysts, investment counselors and economists wedded to the more orthodox ways of thinking. These groups have objected, often in very strong terms, Weintraub (7) for instance, states: "The random-walk hypothesis flies in the face of common sense and the facts and, moreover, suggests a degree of naivete' on the part of its

advocates as to the rules of the game which professional speculators are playing." Such strong statements are however, usually backed by weak statistics.

With the present state of our knowledge it seems quite fair to consider security analysts, at least those who belong to the "technical" school, as the astrologers of our century. The horoscopes they prepare for individual stocks are based on price behavior in the past, but they may equally well be based on the constellation of the stars when the companies were founded. There is no scientific proof that this will make any difference. Security analysts are respected members of our society, like astrologers once were, and they are amply rewarded when their predictions prove right — as they are bound to be in about half the cases.

4.2. The random walk hypothesis does not mean that it is impossible to devise a "system" which will be profitable in the stock market. The hypothesis in fact implies that it is virtually certain (i.e., probability equal to one) that any stock will, some time in the future, sell at a price higher than today's. The hypothesis even implies that this will happen infinitely often. This means that one is virtually certain to make a profit by using any system of the type: "Buy stock X every time the price is lower than 10, and sell it every time the price is higher than 20."

In order to evaluate a system of this kind it is usual to consider an index or another stock, and calculate the profits one would have made by buying and selling the index at the times when according to the system one should have bought or sold stock X. If the index is stochastically independent of the price of stock X, the expected profit of buying and

selling the index will obviously be zero. If there is some stochastic dependence, as indicated earlier in this paper, the index operations can be expected to give a positive profit, but lower than the profit of the system based on stock X.

Most proofs that the various systems work are of this nature, and they can obviously not discredit the random walk hypothesis. Whether these proofs have any other significance is just a matter of taste.

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