Competition through the Introduction of New Products*

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With 7 Figures

Summary

A systematic review of the economic problems connected with innovations is given in the introduction. Attention is then focused on inter-firm relations in oligopolistic markets into which new products are introduced.

In Part II competitive situations among oligopolists who intend to introduce similar new products are studied. Game-theoretical models are developed in which one or two of the following parameters are specified as strategic variables: starting time for development research, expenditure for research, size of new equipment, advertising costs and price-supply policies. Equilibria are obtained in several models by the use of the Nash equilibrium point theory for non-cooperative games[1]. For the treatment of competition through the price-supply policies, a dynamic model is developed using the new approach to demand theory as formulated by Morgenstern[2]. In this demand theory, which is especially apt for durable goods, the demand curve does not remain the same after some purchases have occurred. A natural explanation for the frequently observed fall in price for new products is obtained by this theory for the monopolistic case as well.

In Part III cooperation among firms that are willing to introduce similar new goods is considered. Here cooperation takes place in the form of patent releases. In the first model, the starting time for research as well as willingness

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[1] J. F. Nash: Equilibrium points in n-person games. Proceedings of the National Academy of Sciences, U. S. A., *36* (1950), pp. 48—49.

[2] O. Morgenstern: Demand Theory Reconsidered. The Quarterly Journal of Economics, *LXII* (1948), pp. 165 ff.

to cooperate are parameters of action; in the second model the behavior during the negotiations directly influences the payoffs. In both cases non-cooperative equilibria in the sense of Nash are obtained. Furthermore, bargaining schemes for fixing side payments (in this case patent royalties) are studied.

The phonograph record and the chemical industry are examples of industries that introduce large numbers of new products either in a short time period or simultaneously. Part IV is a study of the case in which the number of new products per time unit can be varied. The effects of more or less careful selection (according to the number) as well as the mutual damage of simultaneously emerging successes are considered. In a second model, a further complication is introduced by the fact that the new products can be chosen from several categories having different economic characteristics (i. e., different mutual damage factors and different success probabilities). Throughout Part IV a probabilistic approach to profit maximization is used.

I. Introduction

1. The theoretical framework

The problems of inventions, innovations and new products deserve the attention of the economist. Technological progress has a strong effect on economic development as well as on the market situation for the single businessman. Yet the literature on this subject is relatively small.

Before we get into our main subject, we shall give some definitions which will be used in this paper. The most general term in connection with new products is "innovation". By "innovations" we mean the applications of new methods or new combinations of already existing methods. The term "invention" is mostly used for technological discoveries which are based on new insights into physical facts or on totally new combinations and applications of older insights. Although we shall use the term "invention" in this sense, an exact definition of it is very difficult. The term "new product" is self-explanatory; but a product can be new either in its external appearance or it can be based on a real functional innovation. For example, the new clothes which are poured out by the fashion industry every month are "new products" in their design but not in their function. An electric shaver which has several speeds instead of one is a "new product" in the functional sense. These two examples illustrate that it is quite possible for the design of a product to require more creative work than does its technical conception. We are concerned in this paper mostly with new products in the technical and functional sense.

A distinction between the terms "basic research" and "development research" or "applied research" may not always make much sense. Usually, "basic research" is sponsored only in order to obtain applicable results and very often it already yields the application of its own results. Although such a distinction may not be fruitful if we consider a specific laboratory or a specific scientist, it may be valid if we think of particular projects. Very often all the efforts of a laboratory are concentrated for a time on the goal of transforming a technological achievement into a marketable product. In such cases it makes sense to speak of development research[3].

Let us outline a system of economic theories dealing with innovations. It will then be possible to show the place of this paper within this theoretical framework.

I. *Systems of innovations*
   a) purely theoretical systems
   b) empirical systems

II. *Innovations and the economy as a whole*
   a) macroeconomic effects of innovations
   b) macroeconomic conditions for the emergence of inventions

III. *Innovations and the single firm*
   a) internal problems of a firm maintaining research laboratories
   b) internal shifts within a firm caused by innovations
   c) problem of choosing an innovation

IV. *Innovations and inter-firm relations*
   a) innovations and competition
      1) similar products
         α) obtained by research
         β) obtained by espionage and bribing
      2) different products
      3) large numbers of new products
   b) innovations and the horizontal connection between firms
   c) innovations and inter-firm cooperation
      1) patent release
      2) patent pooling
      3) research specialization
      4) illegal transactions

V. *Innovations and the firm-customer relation*
   a) spontaneous and induced demand for new products
   b) sales promotion of new products

The following explanations may help to clarify the outline presented above.

ad I. a) An abstract system of several types of inventions may be given with an emphasis on those properties which are important for their economic exploitation. As an example we give a system used by Baff[2].

[3] See also the discussion in J. Jewkes, D. Sawers, and R. Stillerman: The Sources of Invention. London: 1958, pp. 17–19.

The two main categories are cost- or time-saving and demand-increasing inventions. The inventions under the first category may save for the producer: (i) capital, (ii) time, (iii) labor. "Time-saving" does not necessarily also mean "cost-saving". (A time-saving invention can be desired as such. The author has described elsewhere the situation of the phonograph record producer who introduces modern machinery which shortens considerably the process of galvanization and record forming. These devices represent high investment costs and save neither material nor labor: they simply save time. But this can be crucial for a record producer when a "smash hit" of a great star singer appears and the producer has to throw thousands of copies of the same record on the market. The strong fluctuations of the short-lived demand necessitates time-saving productions methods, regardless of costs[4].)

The demand increasing effect of an invention may be due to the following advantages which it offers to consumers:

1) stronger performance, higher speed
2) higher accuracy, reliability
3) replacement of a worn-out part (instead of replacement of the whole device)
4) greater durability
5) more comfort
6) convertibility (the device can be used for several purposes)
7) noiselessness
8) time-saving (for consumer)
9) more hygienic
10) simple manipulation
11) needs less space or is less heavy, etc.

Of course, the same invention may have several of these properties or it may be cost- or time-saving for the producer as well as demand increasing. This is especially true in the case of investment goods. The demand for new machines is high if they are cost-saving. Jet planes are bought by airline companies because they are time-saving. If we treat inconveniences of all kinds as negative utilities — which may be called costs if expressed in money — then the reason why consumers accept a new product will always be that it is cost-saving.

ad I. b) A typology of inventions could be obtained by empirical studies of real cases. Jewkes, Sawers and Stillerman present 50 case histories of recent inventions[5].

ad II. a) Several macroeconomic effects of innovations could be studied, e. g., effects on wages and the price level, on income distribution, on employment, and on growth. The best-known book dealing

[4] R. Reichardt: Die Schallplatte als kulturelles und ökonomisches Phänomen. Zürich: 1961.

[5] J. Jewkes, D. Sawers, R. Stillerman: op. cit., pp. 263 ff.

with these problems is probably Schumpeter's *The Theory of Economic Development*[6].

ad II. b) Here the problem is which economic situations are most favorable to the attainment and the application of new technical knowledge. Some authors believe that the pressure for higher wages by labor unions is a strong stimulus for the search for innovations[7, 8]. As is often the case in economic theory, this idea has a microeconomic meaning (decision-making by the individual businessman), as well as a macroeconomic meaning (prevalence of a specific situation).

The question of which kind of firms most frequently exploit inventions is discussed in [9].

ad III. a) Many firms maintain research laboratories in order to develop inventions of new marketable products. The existence of such laboratories poses many organizational questions for a firm. These questions are partly sociological, partly economic in nature. Marcson[10] as well as Jewkes, Sawers and Stillerman[11] treat these problems.

ad III. b) Independently of the existence and the problems of research laboratories, any exploitation of innovations may cause organizational changes within the firm. This is also true if the innovation does not represent the latest stage of technical development, i. e., if a firm changes from an old-fashioned to a more modern but not up-to-date technique. See Bauer[12].

ad III. c) Another approach is the description of all the questions which an entrepreneur has to answer, and of all the problems which he has to consider, before he can decide rationally whether or not he will introduce a new product or exploit an invention and which one he will choose in the case of several inventions. An enumeration of these questions can be found in the check sheet prepared under the supervision of Holleran[13]. A Paper by Oscar Lange[14] also deals with these managerial considerations.

ad IV. a. 1) The introduction of a new product as such can be a means of competition; also the choice of all the strategic variables such as pricing, advertising, research, etc., connected with new products must be considered.

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[6] J. A. Schumpeter: The Theory of Economic Development. Cambridge (Mass.): 1934.

[7] J. R. Hicks: The Theory of Wages. 2nd ed., London: 1935, Chapter VI.

[8] G. F. Bloom: Union Wage Pressure and Technological Discovery. The American Economic Review, *XLI* (1951), pp. 603—617.

[9] J. Jewkes, D. Sawers, R. Stillerman, op. cit.

[10] S. Markson: The Scientist in American Industry. Princeton (N. J.): 1960.

[11] J. Jewkes, D. Sawers, R. Stillerman: op. cit., pp. 127—196.

[12] H. Bauer: Elektronische Automation im Bankwesen. Tübingen: 1961.

[13] U.S. Department of Commerce: Market Research Series No. 6 (1935). Introduction of New Industrial Products. Check Sheet edited by O. C. Holleran.

[14] O. Lange: A Note on Innovations. The Review of Economic Statistics, *XXV* (1943), pp. 19 ff.

ad IV. a. 2) We consider here the relation between firms whose products are very different. It is possible, e. g., that a strong demand for a new type of TV set may take customers away from a firm which offers an innovation in automobiles, although the two products are substitutes only in the broadest sense of the word. This case represents competition among branches of the economy.

ad IV. a. 3) Some industries, such as the phonograph record or the chemical industry, introduce many new products every month or year. Competition between such firms can be carried out by varying the number of new products as well as by changing the distribution of the new products in several categories or types.

ad IV. b) An innovation may cause a shift in the demand for raw materials. The supplier as well as the buyer of raw materials may therefore change his policy after the emergence of an innovation. This problem is treated in [15].

ad IV. c. Several forms of inter-firm cooperation are possible in connection with innovations. Not only the selling but also exchanging and pooling patents must be considered here. It is also possible for a firm to carry out research for the development of an innovation on a commission basis for one or more other firms. Another possibility is that each firm may agree to specialize on a different aspect of a potential innovation.

All these forms of cooperation are legal. Illegal practices are also possible in connection with innovations, such as price-fixing, assignment of geographical areas for the introduction of a new product, contracts about advertising budgets, etc. As anti-trust legislation differs from country to country, the term "illegal" has to be defined for the specific country under discussion.

ad V. a) The question here is how the demand for new goods comes into existence, how fast it spreads, which income or other social groups first show such a demand, etc.

ad V. b) Sales promotion in connection with a new product may be a weapon in a competitive struggle. This phenomenon could therefore also be treated in the context of *IV. a)*. For a monopolist, however, it is purely a relation with the consumers.

The theoretical system, the skeleton of which we have just presented, could of course be refined by the consideration of more details and complications, as, e. g., the distinction between demand-increasing and cost-saving innovations, or between innovations concerning design and innovations concerning technical functions. Other questions which lie outside economic theory and which are not mentioned above, such as the sociology of invention, the role of inventions in human history[16], etc., may also be of interest to the economist.

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[15] S. L. Anderson and G. D. Edwards, jr.: Product Venture Analysis by Computer Simulation. Wilmington (Delaware): 1961.

[16] B. A. Fiske: Invention. The Master Key to Progress. New York: 1921.

2. Purpose of this paper, methods used

In this paper we will focus our attention on a relatively small fraction of the above described system, namely on the problems listed under *IV. a. 1. a), IV. a. 3)* and *IV. c. 1):* competition after the introduction of similar new products obtained by research, competition through the number of products, and cooperation based on the release of patents. Furthermore, we will consider only demand-increasing and not cost-saving innovations, and we will confine ourselves to oligopolistic markets. This restriction will enable us to give a system of formal treatments yielding some sort of oligopolistic theory for the introduction of new products. For the study of competitive situations, we consider several models in which the time for research, the size of equipment, the advertising budget, and price policy are the strategic variables. We are studying only those cases in which one or two of these parameters are variable. In this way it is possible to understand the intuitive meaning of the results. It did not seem advisable to construct more general models, because the necessary assumptions contain too many unrealistic simplifications. We share in this respect the opinion of Baumol, that "generalization is very costly and that these costs are often not given adequate attention"[17].

An important problem is that of the decision whether or not a new product should be developed and introduced. We could try to analyze the situation in which this decision is the strategy choice of a firm. We would then have to know what the future profits would be if the firm has the new product or not for all possible cases in which the competitors have this product or not. But it would only make sense to write down future profits if we knew the optimal strategy after the introduction or non-introduction of the new good. It is easily seen that this would require an even more general theory than would any of the most complicated models based on the previously mentioned parameters. Therefore, we did not attempt to formulate such a model since it would probably turn out to be too unrealistic.

One objective of this paper is to leave the trodden paths of oligopoly theory, which is mostly concerned with price policies. But here we also emphasize the importance of other variables. On the other hand, it is possible just in the domain of price policy to obtain some nice results applying the hitherto rather neglected new approach to demand theory of Morgenstern[18]. Therefore, Section II.7 may be of special interest to the economist.

The theory of games will be applied in the different models. We assume a linear utility on money, but no comparison of utilities among the firms. As we consider mainly the differences of profits, the linearity

[17] W. J. Baumol: Business Behavior, Value and Growth. New York: 1959, p. 3.

[18] O. Morgenstern, op. cit.

of utilities of money is easier to justify than it would be if we were to deal with the entire wealth of a firm. Normally, future profits are taken here to be discounted back to the present moment — considering a reasonably distant future. Thus, managerial decisions are thought of as based on an evaluation of future earnings.

Throughout a large part of this paper, we shall assume a unique price for the new product. One has to think of the case in which several firms put new products on the market which fulfill the same functions and are therefore perfect substitutes. We think that these models are also suitable for cases in which there are slight differences in design and price, but no essential functional differences among the several brands. In such cases, we neglect the price difference and assume that it is essentially the advertising efforts which lead to the several market shares. However, other cases, such as price differentiation as a market strategy, will be discussed in other sections of this paper.

The mathematical models which will be developed are nothing more than the reformulation of what is presented in a descriptive manner. But they will be helpful tools for the demonstration of some properties of the several game situations. In most cases, the functions and examples are chosen in a way which minimizes the computational difficulties while still preserving the essential properties of the description.

We shall conclude this section with some remarks on the notation used in this paper. If a firm evaluates earnings and costs with an internal interest rate i, the discount factor can be written as $v = \frac{1}{1+i}$. We now need the sum of a finite and of an infinite progression over v^n. By $\varphi(n)$ we denote the value of a series of payments of the amount "one" starting at time zero and ending at time $n-1$, and by ψ the same value for $n \to \infty$. We then have

$$\varphi(n) = \frac{1-v^n}{1-v} \text{ and } \psi = \frac{1}{1-v}, \; 0 < v < 1. \tag{I.1}$$

Generally, capital letters will be used for costs or earnings per time unit, small letters for the length of periods of time measured in units, and small Greek letters for constants in functions. The subscripts will denote the player.

II. Non-cooperative Situations for the Introduction of New Products

1. Choice of starting time for research as a parameter of action

In this model the choice of starting time for research is the only parameter of action. We consider only development research: this is the situation in which a new technical concept is known, but not patented. There are situations in which the firms know that a specific invention lies "just around the corner". It can happen, therefore, that several firms are working simultaneously on the same technical problem. To mention just one example: color television. Only for development research it makes sense to assume a given time span which can be estimated by the entrepreneur.

We use the following notation:

R = cost per time unit for research (II.1.1.)
E = cost per time unit for the construction of the equipment
A = incremental cost per time unit for advertising
P = expected incremental earnings per time unit due to the new product
r = number of time units required for research
e = number of time units required for the construction of new equipment
f = total length of period considered by the manager
t = number of time units of delay from zero until the start of research

It must be noted that we are considering only the extra-costs and extra-earnings which occur due to the introduction of the new product. If we assign a probability to each conceivable economic success of the new merchandise, we can then mutiply the earnings by the corresponding probability. By summation — or in the case of a continuous probability distribution, by integration — we get the value of P. All data except t are given for the firm by technology and the economy; but one can also think of them as random variables. (In this latter case, the firm merely maximizes an expected value U.)

Research is carried out during r periods of time, following a so-called "dead time" during which the machinery for the fabrication of the new product is installed. Then advertising and selling begin. f merely indicates how far into the future the manager projects his plans.

From the previous statements, we get as the payoff to the firm:

$$U = -v^t \varphi(r) R - v^{t+r} \varphi(e) E - v^{t+r+e} \varphi(f-t-r-e) A + $$
$$+ v^{t+r+e} \varphi(f-t-r-e) P. \quad \text{(II.1.2)}$$

We do not need subscripts here, as we are considering only one firm. It is immediately clear that if the expected and discounted earnings are greater than the discounted costs, i. e., if the product is profitable, the best policy for the firm is to start its research as early as possible. $t = 0$ will be chosen, regardless of what other firms do. The decision problem for an oligopolist under these circumstances will be the same as that for a monopolist.

2. Length of research time as a parameter of action

The previous considerations have shown that a firm may have an incentive to enter the market with a new product as soon as possible. This can be achieved not only by starting research early, but also by shortening the time spent in development research. Of course, such a saving of time will normally result in higher research costs per time unit.

It is difficult to get a clear insight into the functional relation between the costs and the length of research work. Such a relation will be different for different kinds of technical problems. But it can be said with certainty that the total cost of a program will increase considerably if the research work is vigorously concentrated into a shorter period of time. One might think in this connection of "crash-programs" for which new laboratories have to be built, new people trained, etc. Examples of such "crash-programs" can be found especially in the field of military armaments. The development of guided missiles starting from the desperate efforts of Nazi Germany on its V-2 to the present day's arms race illustrates this kind of problem. (See the case history in [19], where literature on this subject is also mentioned.)

To bring out this property mathematically, we assume that the total cost for research equals $\sigma + \frac{\varrho}{r}$, where ϱ and σ are constants and r is the length of research time. We neglect the discount factor in this case. This function is taken to be defined for values of r which vary between some reasonable boundaries r' and r''.

In order to describe the fact that it is advantageous to enter into the market early, we define the total earnings due to the new product as follows for the i-th firm.

$$\gamma_i + [\sum_{\substack{j=1 \\ j \neq i}}^{n} r_j - (n-1)\, r_i]\, \delta_i - \varepsilon\, r_i, \tag{II.2.1}$$

where all the constants are taken as positive and n is the number of oligopolists in the market. This formula can be interpreted as follows: If r_i is greater than the average of the other r' s, i. e., if the i-th firm enters into the market later than the other firms on the average, then it loses a certain amount, expressed by the second term. But the absolute value of r_i also has an influence on earnings as expressed in the last term. Of course, we could construct a more complicated function, replacing (II.2.1) by another expression which decreases monotonically with a growing r_i, but this simpler function adequately shows the important relationship.

We now consider this market situation — in which the starting point for research is identical for all firms and the length of research-time is the only parameter of action — as an infinite game. (It would perhaps be more realistic to say that the cost of research per time unit is variable. But we assume that the interdependence between these costs and the length of research time is fully understood by the managers, so that it

[19] J. Jewkes, D. Sawers, R. Stillerman: op. cit., pp. 355—359.

does not matter which parameter is considered as the independent variable.) Then the payoff for the i-th firm is:

$$U_i = -\sigma_i - \frac{\varrho_i}{r_i} + \gamma_i + [\sum_{j \neq i} r_j - (n-1)\, r_i]\, \delta_i - \varepsilon_i\, r_i \qquad \text{(II.2.2)}$$

$$\gamma_i,\ \delta_i,\ \varepsilon_i,\ \varrho_i,\ \sigma_i > 0,\ r' \leqq r_i \leqq r'',\ n \geqq 2.$$

This game is easily treated.

$$\frac{d\,U_i}{d\,r_i} = + \frac{\varrho_i}{r_i^2} - (n-1)\,\delta_i - \varepsilon_i, \qquad \text{(II.2.3)}$$

$$\frac{d^2\,U_i}{d\,r_i^2} = - \frac{2\,\varrho_i}{r_i^3}. \qquad \text{(II.2.4)}$$

The second derivative is negative for all positive values of r_i; therefore, U_i attains a maximum if we put (II.2.3) equal to 0. We then get:

$$r_i{}^* = \sqrt{\frac{\varrho_i}{(n-1)\,\delta_i + \varepsilon_i}}, \qquad \text{(II.2.5)}$$

where $r_i{}^*$ denotes the optimal choice of the free parameter, whether $r' \leqq r_i{}^* \leqq r''$, will always have to be decided. In this game, for a single player one strategy dominates all other strategies. Thus, a certain length of research time (or a certain amount for research costs) turns out to be optimal under all circumstances.

If all players choose a value $r_i{}^*$ according to (II.2.5), $r_i = r_i{}^*$ (for $i = 1, 2, \ldots, n$), we get a Nash equilibrium point for noncooperative games[20, 21]. This result is quite in accordance with the predilection of economic theory for equilibria. In this case, an equilibrium can be defined for an oligopolistic market model.

A simple discontinuous numerical example for the two-person-game case may illustrate the previous considerations. Suppose the parameters are such that we obtain as payoff functions:

$$U_1 = 100 + (r_2 - r_1)\,10 - \frac{80}{r_1}, \qquad \text{(II.2.6)}$$

$$U_2 = 120 + (r_1 - r_2)\,15 - 5\,r_2 - \frac{50}{r_2}.$$

We then get the following game matrix:

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[20] R. D. Luce and H. Raiffa: Games and Decisions. New York: 1957, p. 106.

[21] J. F. Nash: Equilibrium points in n-person games. Proceedings of the National Academy of Sciences, U. S. A., *36* (1950), pp. 48—49.

Table 1

		Player 2			
		$r_2 = 1$	2	3	4
Player 1	$r_1 = 1$	20 (65)	30 (70)	40 (58)	50 (42.5)
	2	50 (80)	60 (85)	70 (73)	80 (57.5)
	3	53 (95)	63 (100)	73 (88)	83 (72.5)
	4	50 (110)	60 (115)	70 (103)	80 (87.5)

We have to find the equilibrium points in this game, i. e., a payoff pair, in which no player can get more, if he changes his strategy, whereas the opponent keeps his original strategy unchanged. This is especially simple in this case, as for player 1 $r_1 = 3$ dominates all other strategies, and for player 2 $r_2 = 2$ dominates all other strategies. Therefore an equilibrium point is found for the strategy pair ($r_1 = 3$, $r_2 = 2$) with the payoff pair (63, 100). If we treat this game as infinite, we get the equilibrium point $\left(r_1 = \sqrt{8},\ r_2 = \sqrt{\frac{5}{2}}\right)$.

3. Size of new equipment as a parameter of action

The problem we are dealing with here is described by Andersen and Edwards[22]. For the new product, new equipment, and possibly one or more plants, will have to be constructed. The firm will incur losses if the demand cannot be satisfied or if there is idle capacity. As the demand (or as Andersen and Edwards put it, the market penetration) cannot be predicted accurately, this situation is similar to a game against nature.

Andersen and Edwards treat this problem in the following way (we formalize their more descriptive exposition): If we denote the market penetration by d and the size of the new plant by s, we can write the profits as a function of d and s, $e(d, s)$. We can then define a maximal regret as a function of the plant size:

$$m(s) = \max_d \left[\max_s e(d, s) - e(d, s)\right], \tag{II.3.1}$$

where $d' \leqq d \leqq d''$. We find then an optimal plant size s^*, which minimizes the maximal regret: $m(s^*) = \min_s m(s)$. It is obvious that the authors have applied the Savage criterion for games against Nature[23],

[22] S. L. Andersen and G. D. Edwards, jr, op. cit.

[23] J. Milnor: Games against Nature. In: Decision Processes. Edited by R. M. Thrall, C. M. Coombs, and R. L. Davis. New York: 1954, p. 49.

which is quite justified for this problem. But we should be aware that the values of $m(s)$ are strongly dependent on the choice of d' and d'', i. e., on the boundaries of the conceivable degrees of market penetration, and that we have very little information as to where these boundaries may lie. It is possible that d' and d'' may be chosen rather arbitrarily, but the outcome is influenced by this choice.

We will treat this problem in a different way, describing a situation in which several firms simultaneously introduce a similar[24] product. The only parameter of action for each firm will be the choice of the size of the new plant. But the unpredictable demand will also play a role, so a game against Nature is involved.

It is characteristic of this situation that the single firm lacks not only information about the future demand but also does not know how large the output-capacity of the competitors will be when it is planning its own plant. We assume, in other words, that the competitors start the construction of a new plant for the new product at the same time.

Let x_i denote the output-capacity of the i-th plant and y be the market price. We can write $y\,(\sum_{i=1}^{n} x_i)$. This price function expresses the demand. As we are only interested in the incremental-earnings and incremental-costs applying to this situation, we need to know only the construction costs of the plant $E(x_i)$ and the production cost of the good $F(x_i)$. Let us first consider the case in which each player can sell all that he can produce. Then the payoff will be:

$$U_i = x_i\, y\,(\sum_{i=1}^{n} x_i) - E(x_i) - F(x_i), \tag{II.3.2}$$

where we assume y to be monotonically decreasing in $\sum_{i=1}^{n} x_i$ and E and F monotonically increasing in x_i. We have to assume that no player knows the demand-price function y. Let d denote a set of parameters in the function y, which are unknown to the players. We can then write

$$U_i\,(x_1, x_2, \ldots, x_n, d). \tag{II.3.3}$$

There are several ways of handling this function in order to separate the game against Nature from the ordinary game, according to the several proposals for games against Nature (Milnor[25]). If we want to apply the Laplace criterion, we replace U_i by the arithmetic mean of its values over all possible states of d. The Wald criterion can be used in the following way: We define

$$V_i\,(x_1, x_2, \ldots, x_n) = \min_d U_i\,(x_1, x_2, \ldots, x_n, d). \tag{II.3.4}$$

The players will then try to maximize V_i, taking the pessimistic view

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[24] Similar in the sense described on pp. 47 f.

[25] J. Milnor, op. cit.

implied in the Wald criterion. We do not consider the Savage and the Hurwicz criteria. The V_i of formula (II.3.4) is not necessarily a utility; therefore the resulting game cannot be treated with mixed strategies. We have to restrict ourselves to treatments with pure strategies. It is conceivable that one could determine experimentally the utility functions associated with the values V_i, but these functions could not be expected to be linear in the values of V_i.

The previous considerations may be illustrated by a simple numerical example for the two-person case: Let y represent the demand-price function by the following linear expression:

$$y = 20 - d\,(x_1 + x_2). \tag{II.3.5}$$

x_1 and x_2 each have either the value 4 or 5. d is either 1 or 1.5, representing two possible states of Nature. We consider only one cost function which includes production and plant construction costs. The cost functions for the two players are, respectively:

$$E_1 = 10 + 2\,x_1. \tag{II.3.6}$$
$$E_2 = \;\;8 + 3\,x_2.$$

We then get the following two matrices for the two possible states of Nature:

Table 2

		Player 2: 4	Player 2: 5	
Player 1	4	30 (28)	26 (35)	$d = 1$
	5	35 (21)	30 (27)	

Table 3

		Player 2: 4	Player 2: 5	
Player 1	4	14 (12)	8 (9.5)	$d = 1.5$
	5	12.5 (6)	5 (2)	

These matrices both have a clear dominance in the strategy-pairs (5,5) and (4,4) respectively, and therefore they represent equilibrium points. Games of this form may be uninteresting. But in this example we are focusing our attention on a game against Nature rather than on a game "among people".

There are several treatments for games against Nature. If we follow the idea of Laplace, we have to assume that the two states $d = 1$ and $d = 1.5$ will each occur with a probability of $\frac{1}{2}$. We can therefore compute an expected value for each strategy-pair (x_1, x_2) by taking the arithmetic mean between the two corresponding entries in the two matrices. We then get:

Table 4

		Player 2: 4	Player 2: 5	
Player 1	4	22 (20)	17 (22.25)	Laplace
	5	23.75 (13.5)	17.5 (14.5)	

In this case, too, we get a clear dominance for both players, and accordingly an equilibrium pair (5,5).

When we apply the Wald criterion, we always have to consider the smaller entry of the corresponding entries in the two matrices. This leads to the matrix for $d = 1.5$ (Tab. 3) and we will get as an equilibrium point the pair (4,4).

It is quite conceivable that the players evaluate their game situation against Nature in different ways. We may then have to face a situation as follows:

Table 5

		Player 2: 4	Player 2: 5
Player 1	4	22 (12)	17 (9,5)
	5	23.75 (6)	17.5 (2)

Player 1 applies the Laplace criterion, whereas Player 2 prefers the Wald criterion.

Of course the functions $y\,(\sum_{i=1}^{n} x_i)$, $E_i\,(x_i)$ and $F_i\,(x_i)$ may be such that we do not get clear dominance, thus yielding a more interesting game "among people". We shall not investigate these possibilities further here. The capacities of the new plants may not be fully used or the whole demand for the new good may not be satisfied (at least temporarily). For the first case, which is rather characteristic of oligopolistic markets, we have to assume that there is a lower boundary for the price under which the firms are not willing to sell. We may think of this as the case in which the price will not go below the average cost of producing a unit of the good. Similarly, we must assume for the second case that the price will not go up beyond a certain ceiling. Let us call these two limit prices y' and y'', respectively; hence $y' \leqq y \leqq y''$.

The penalty for being too optimistic is the relatively high construction cost for a large plant, which is not used to its full capacity. We have not made any assumption about the sales organizations of the firms. If all the firms have equally efficient sales departments, then we can expect that the firms with relatively large overcapacities will get a smaller share of the market than is their proportion of the total capacity, for it will always be easier to sell the first units than the last ones of the new good. If we write x' for the total amount of goods sold at the lower limit price y', we may formalize these reflections as follows: Let the market share of the i-th player be:

$$\left(x_i - \left(\sum_{j=1}^{n} x_j - x' \right) \frac{x_i^2}{\sum_{j=1}^{n} x_j^2} \right). \tag{II.3.7}$$

It is easily seen that if we sum up these expressions for $i = 1, 2, \ldots, n$, we get exactly x'. Thus the disadvantage of the total overcapacity is distributed according to the squares of the individual firms' capacities.

If we drop the assumption that all the firms have equally well operating sales organizations, we must consider advertising costs. No other treatment would be appropriate in the case of over-capacity. We will deal with the variation of advertising costs in a later section of this paper.

Let us now analyze the other extreme situation in which the firms are unable to satisfy the demand for the new good at the ceiling price y''. In this case the disadvantage to an individual firms is that it cannot fill the orders of some of its customers. This can happen even in the case of a new product because a firm may have well-established business connections from its selling of other goods. The costs which occur here are often called "shortage costs"; they are difficult to express numerically, as they consist of a loss of an intangible — goodwill. The most natural way to formalize these costs would be to write them as a monotonically increasing function H of $c_i - x_i$, where c_i denotes the demand for the good at the price y'' directed to the i-th firm and stemming from its former business relations. A more complicated case is that in which the sum of the firms' capacities exceeds the demand, but for an individual firm k, $x_k < c_k$. In this case some firms will have overcapacity costs while others will incur shortage costs.

The payoff function will now be the following:

$$U_i = x_i\, y\,(\sum_{j=1}^{n} x_j) - E_i(x_i) - F_i(x_i) - H_i(c_i - x_i), \qquad \text{(II.3.8)}$$

$$y' \leqq y \leqq y'' \text{ and for } \sum_{j=1}^{n} x_j \geqq x'.$$

$$U_i = \left(x_i - (\sum_{j=1}^{n} x_j - x') \frac{x_i^2}{\sum_{j=1}^{n} x_j^2} \right) y' - E_i(x_j) - F_i(x_i) \qquad \text{(II.3.9)}$$

$$\text{for } \sum_{j=1}^{n} x_j \leqq x'.$$

$$H_i(\pi) = 0, \ \pi \leqq 0.$$

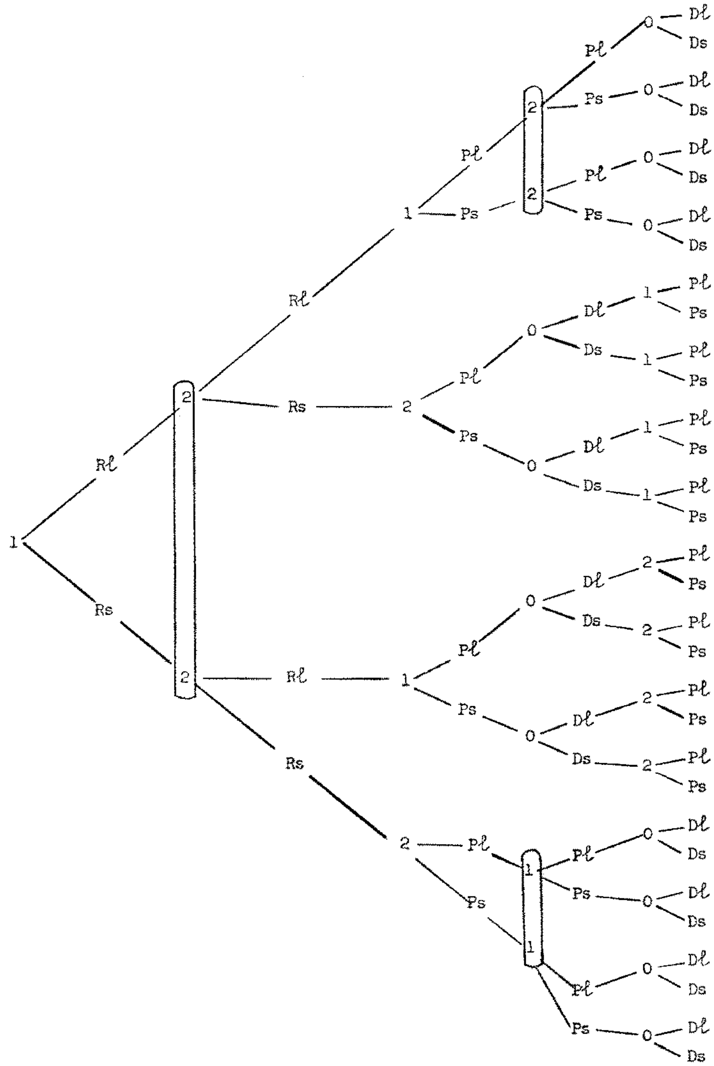
In this case we do not explicitly express the fine differences between the occurrence of the several costs, but we can assume that these differences are already built into the functions. By using the discount factor, we could construct other more complicated models. Of course, the above game can be treated in the same way as the previous simpler model, especially for the consideration of an unpredictable demand function which may be implied in y.

4. Research time and the size of new equipment as the parameters of action

This is a very interesting situation. The individual firm determines by the appropriate choice of starting time and research costs the time at which it will be ready to produce the new product. It then has to determine the size of its new plant. Let us briefly outline the advantages of entering the market early and of entering it late. The "first-comer" has the advantage of a longer selling period and he will gain prestige and goodwill as he is able to serve his customers first. The "late-comer" has the advantage of smaller total research costs; and since he has more information about the size of the demand, he will be able to adjust his plant size more accurately to the needs of the market and hence will save overcapacity or shortage costs. A late-comer may also learn technically from the mistakes which were made by an early entrant.

This situation may be formalized typically as a game with two moves for each player (length of research time and size of plant) in which the state of information plays a role; therefore, it may best be represented in the extensive form. Let us consider the simplest possible case in which either a short, *Rs*, or long, *Rl*, research time is chosen and in which the new plant may either be large, *Pl*, or small, *Ps*. Also, the chance-mechanism (i. e., the unknown demand) may have two moves — a large demand, *Dl*, or small one, *Ds*. We then get the following tree (see p. 58). One may assign payoffs to the 32 endpoints according to the weight of the several costs. Of course, in real situations we will have more complications. There will not only be two possible moves at each point for the players, and the unpredictable demand especially will vary over a larger number

Table 6



of values. Furthermore, in reality a manager is not simply confronted either with no information at all or with complete information about the move of the chance mechanism (i. e., real demand); he will usually have incomplete information about demand. A firm may start its research work after another firm but before the other firm has finished its research work.

Thus we get a series of staggered market entrances. However, the amount of information available to a firm will increase (i. e., tend toward completeness) the later a firm enters into the market.

5. Advertising costs as the parameter of action

We will study the effects of advertising here under the assumption that all other data are given, thus demonstrating some typical properties of advertising effects. Gillman[26] has given a game model of an advertising struggle which is directed to one potential buyer and in which the choice of the distribution of the advertisements over a given period of time is the strategic variable. In our model, we assume the presence of a sufficiently large number of potential buyers such that the market can be split according to the different advertising efforts, and the amount spent for advertising at a given time is the strategic variable.

We assume that the price y of the new product will be a function of the total amount of goods sold s, and the total output capacity of the new industrial branch $\sum_{i=1}^{n} x_i$, which we denote by x. Normally, the quantity is considered as a function of the price. Here we write the function in the other direction similarly to section 3.

(II.5.1) We can write y as a function $y\ (s, x)$. The advertising expenditure for the i-th firm is written as A_i. We abbreviate $\sum_{i=1}^{n} A_i = A$. Now, the total quantity sold s will be a monotonically increasing function of the total advertising costs A, but the rate of increase for s will get smaller and smaller, the higher the absolute value of A. This is reasonable since a market can be over-saturated with advertising and further increases of advertising efforts will induce less and less sales. This can be expressed mathematically as follows:

$$s = s\ (A), \quad \frac{ds}{dA} > 0, \quad \frac{d^2 s}{dA^2} < 0. \qquad \text{(II.5.2)}$$

Let us now consider the effort of a single firm: If we denote the quantity sold by the i-th firm by s_i, we can write $s_i\ (A_i)$. We now assume that the shares of the market s_i $(i = 1, 2, \ldots, n)$ will be in the same proportion as the advertising expenditures. Therefore $s_i\ (A_i)$ is completely determined by the two following conditions:

$$\frac{s_i}{s_j} = \frac{A_i}{A_j} \text{ for all } i, j \qquad \text{and} \qquad \sum_{i=1}^{n} s_i = s\ (\sum_{i=1}^{n} A_i). \qquad \text{(II.5.3)}$$

[26] L. Gillman: Operations Analysis and the Theory of Games. An Advertising Example. Journal of the American Statistical Association, *45* (1950), p. 541.

As in all former considerations, we are interested only in the incremental earnings and incremental costs which result from the given situation. Consequently, we need to know only the earnings, advertising costs and production costs F_i for the quantity s_i. One could argue that there is a certain time-lag between the appearance of advertising and its effects on the sales. If we assume that this time lag is just one period of time and make use of the discount factor and the formulae (I.1), we can write a payoff function as follows:

$$U_i = v\,\psi\, s_i\, y\,(A) - \psi\, A_i - \psi\, F_i\,(s_i). \qquad \text{(II.5.4)}$$

We can put U_i into a form in which it is a function of the several $A_j{}'s$, thus:

$$U_i\,(A_1, A_2, \ldots, A_n). \qquad \text{(II.5.5)}$$

So far we have described a situation which is by no means characteristic for new products, but rather for well established goods. Higher advertising costs will be necessary in the beginning when a new product has to conquer a new market than later as the product becomes more widely known and accepted. We can say, therefore, that correspondingly smaller expenditures on advertising are necessary to stimulate the same demand until a certain stable situation is reached. Let superscripts denote the moments of time to which the given expression refers. If we keep $A = \sum_{i=1}^{n} A_i$ fixed for a moment over all t, we expect that $s^{(t)}\,(A)$ will be a monotonically increasing function of t, i. e., the same total advertising effort will create an increasing demand. But for a fixed t and variable A, we will still have the properties (II.5.2) and (II.5.3).

We then get the following payoffs:

$$U_i = v\, y \sum_{t=0}^{t'} v^t\, s_i{}^{(t)} - \sum_{t=0}^{t'} v^t\, A_i{}^{(t)} - \sum_{t=0}^{t'} v^t\, F_i\,(s_i{}^{(t)}), \qquad \text{(II.5.6)}$$

where we assume a fixed price y and no change in the production functions F_j over the time. t' denotes the number of time periods which are being considered.

$s_i{}^{(t)}$ must be treated not only as a function of $A_i{}^{(t)}$ and $A^{(t)}$, but also of the individual and the total advertising efforts of the previous periods. Hence we have:

$$s_i{}^{(t)}\,(A_i{}^{(0)}, A_i{}^{(1)}, \ldots, A_i{}^{(t)};\; A^{(0)}, A^{(1)}, \ldots, A^{(t)}). \qquad \text{(II.5.7)}$$

The greater the total amount of the previous advertising efforts, the greater will be the success of the present advertising, Thus, a typical game situation arises. A firm may hold back its advertising in the beginning when advertising does not yet pay off well, thus leaving the burden of the pioneering efforts in the market to the other firms and profiting from their earlier efforts. But if its own early advertising efforts are too small, the market as a whole and its share of the market will also be smaller, and therefore the profits will decrease. One may then expect to find an optimal strategy, consisting of a certain sequence of advertising costs over time which takes into account the previous efforts of the competitors in this field.

To conclude this section, let us give an example of a function which satisfies the conditions (II.5.2) and (II.5.3). We put:

$$s = \gamma \sqrt{A}. \tag{II.5.8}$$

From (II.5.3) we get

$$s_i = \frac{\gamma A_i}{\sqrt{A}}. \tag{II.5.9}$$

If we assume the cost functions to be linear in s_i, the payoff is an expression of the following form:

$$U_i = A_i \left(\frac{\alpha_i}{\sqrt{A}} - \beta_i \right). \tag{II.5.10}$$

Differentiating, we get:

$$\frac{dU_i}{dA_i} = -\beta_i + \frac{\alpha_i}{\sqrt{A}} - \frac{\alpha_i A_i}{2A^{3/2}}, \tag{II.5.11}$$

$$\frac{d^2 U_i}{dA_i^2} = \frac{\alpha_i}{A^{3/2}} \left(\frac{3}{4} \frac{A_i}{A} - 1 \right). \tag{II.5.12}$$

The second derivative is negative for $A_j \geqq 0$, $j = 1, 2, \ldots, n$ and $A_i > 0$. Therefore, it is possible to obtain a maximum by putting

$$\frac{dU_i}{dA_i} = 0. \tag{II.5.13}$$

This gives a cubic equation. Thus, optimal values for A_j, $j = 1, 2, \ldots, n$ can be defined and once again, the case of a Nash equilibrium for non-cooperative games applies.

A numerical example for $n = 2$ illustrates this type of function. For (II.5.10) we write:

$$U_i = A_i \left(\frac{3}{\sqrt{A_1 + A_2}} - 1 \right), \quad (\mathrm{i} = 1, 2). \tag{II.5.14}$$

Let the possible choices for A_i or A_2 be: 1, 2, 3, 4, which gives the following matrix:

Table 7

A_1 \ A_2	1	2	3	4
1	1.121 (1.121)	0.732 (1.464)	0.500 (1.500)	0.342 (1.368)
2	1.464 (0.732)	1.000 (1.000)	0.684 (1.026)	0.450 (0.900)
3	1.500 (0.500)	1.026 (0.684)	0.675 (0.675)	0.401 (0.535)
4	1.368 (0.342)	0.900 (0.450)	0.535 (0.401)	0.243 (0.243)

An equilibrium point is obtained here in the pure strategy pairs $(A_1, A_2) = (3, 2)$ and $(A_1, A_2) = (2, 3)$.

In the case of continuous strategies, we apply condition (II.5.13), which yields the following cubic equation for A_1 as the unknown:

$$4\,A_1^3 + (12\,A_2 - 3)\,A_1^2 + 12\,(A_2^2 - A_2)\,A_1 + 4\,A_2^3 - 12\,A_2^2 = 0. \quad \text{(II.5.15)}$$

By interchanging A_1 and A_2, the corresponding equation for A_2 results. Their solutions yield an equilibrium point for non-cooperative games in the sense of Nash.

6. Size of the new plant and advertising as parameters of action

All that was said in section 3 and 5 above applies also to this case. Therefore we will discuss only what is specific to the case of a simultaneous variation in plant size and advertising costs.

In the case in which the capacity of the new plants can be fully used but where no unsatiated demand is left, we only have to treat the price y as a function of the sum of all capacities and the sum of all the advertising costs. Thus, we have $y\,(x, A)$ with y monotonically decreasing in x and monotonically increasing in A.

Somewhat greater sophistication is needed to treat the cases of overcapacity or shortage. Let us consider only the simplest cases: If at the lower price limit y', the full capacity still cannot be exhausted, we have the situation as described in formula (II.3.8). But now the share of the total overcapacity which has to be carried by a player will depend on his advertising expenditure. Thus we write the market share s_i as

$$s_i = [x_i - (x - x')\,\lambda_j],\ \sum_{j=1}^{n} \lambda_j = 1,\ x > x'.\quad x = \sum_{j=1}^{n} x_j. \qquad \text{(II.6.1)}$$

Of course, λ_i is a function of the several advertising costs. If we impose the condition:

$$\frac{\lambda_i}{\lambda_j} = \frac{A_j}{A_i},\ \text{for all } i, j, \qquad \text{(II.6.2)}$$

we get for λ_i

$$\lambda_i = \frac{\prod\limits_{\substack{j=1 \\ j \neq i}}^{n} A_j}{\sum\limits_{j=1}^{n} \prod\limits_{\substack{k=1 \\ k \neq j}}^{n} A_k}. \qquad \text{(II.6.3)}$$

It is easily seen that this function also satisfies $\sum\limits_{j=1}^{n} \lambda_j = 1$.

Using this definition of λ_i we rewrite formula (II.3.10) as follows:

$$U_i (A_1, A_2, \ldots, A_n;\ x_1, x_2, \ldots, x_n) = \qquad \text{(II.6.4)}$$
$$= v\,[x_i - (x - x')\,\lambda_i]\,y' - E_i(x_i) - F_i(x_i) - A_i,$$

where the time difference can be expressed in the functions E_i and F_i.

The shortage costs, which are in fact a loss of goodwill because of unfulfilled orders, are harder to determine and to describe than the overcapacity costs. It would be rather artificial to define shortage costs as a function of advertising expenditure. However, one special case is of particular interest, namely the case in which one firm has an overcapacity whereas another firm faces a shortage. Of course, in this situation a part of the demand which was originally directed toward the latter firm will shift to the former. But the entire unsatiated demand will not necessarily shift. This depends on the loyalty of the customers and on the imperfection of the market phenomena which go back ultimately to the advertising efforts of the firms. One might therefore define for such cases a shift-factor as a function of the previous advertising expenditures. But in this situation, a price difference will also occur which will complicate such a shift function. It is also possible that the demand which shifts from one firm to another remains partly or totally unsatiated, because the second firm's capacity is too small to satisfy the new and unexpected consumers. We will not go further into this situation.

7. Price policy as a domain of action

In this section, we will make use of the new approach to demand theory as first outlined by Morgenstern[27]; in this treatment the demand curve will not be the same after some purchases have taken place as it was before.

Let us first discuss the case of a monopolist. We construct an aggregate demand function. The consumers are assumed to be willing to buy only one unit of the good each, and will be ordered according to the maximal price each one is ready to pay. If we assume furthermore that the number of the potential buyers is large enough, we can smooth the

[27] O. Morgenstern, op. cit.

resulting curve, which is expressed by the price-quantity function $y\ (x)$ or $x\ (y)$.

Now, it is characteristic for a market of a new product that the demand increases during the first periods of time. We assume a linear increase as follows: In each period the same number of consumers representing the same aggregate demand function enters the market and joins those consumers left from the previous periods with an unsatiated demand. Furthermore we assume that it takes a sufficiently long period of time until a replacement of a previously purchased good is necessary so that the consumers who buy definitely leave the market for the period under consideration. We do not consider successive entrances into the market, but assume that the whole group of new potential buyers assigned to a specific time interval joins the market at once. Every consumer whose price bid equals or exceeds the prevailing market price is served immediately. The last assumptions are that the monopolist will either keep the price constant or lower it, but will never increase it, that the whole demand is always satiated, and that price changes occur at the same moment as the entrance of new consumers into the market. The only condition imposed on the price-quantity function is that $y\ (x)$ is monotonically decreasing with x and that $x\ (y)$ is monotonically decreasing with y.

Let superscripts denote the moments of time t. The first group of consumers enters the market at $t = 1$. For the quantity which is really sold at time t, we write $\tilde{x}^{(t)}$, whereas $x^{(t)}$ denotes the "theoretical" quantity $x\ (y^{(t)})$. We then get

$$\tilde{x}^{(t)} = x^{(t-1)} + t\ (x^{(t)} - x^{(t-1)}). \tag{II.7.1}$$

The following figure illustrates this formula (Fig. 1).

The curve AB represents the consumer group which enters the market at $t = 1$. The quantity $x^{(t)} = \tilde{x}^{(1)}$ is sold. At $t = 2$, a consumer group with the same curve appears. The part AC of the first curve is now simply replaced, but below C the new consumers are added to those whose demand has been left unsatiated at $t = 1$. Therefore, below C the distances between the line $Cx^{(1)}$ and the original curve AB have doubled and we get the new demand curve ACD. For $t = 3$ the same argument can be applied, but now the distances below E between $Ex^{(2)}$ and AB have trebled. Thus we arrive at the curve AEF.

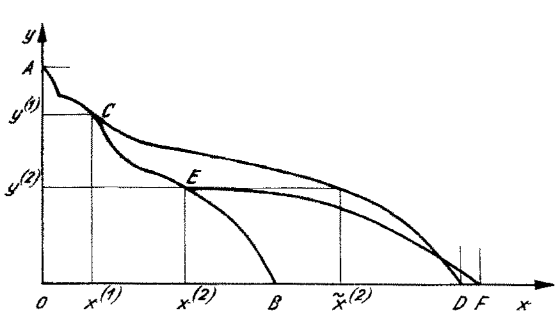


Fig. 1

In figure 2 we show the same procedure for a linear demand function: From this figure it is more easily seen that in the lower part the demand becomes more and more elastic. Therefore, there will be a strong incentive for price-cutting in markets of new products.

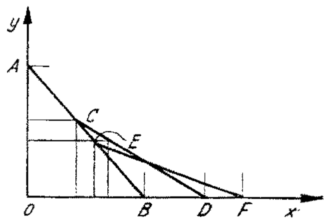


Fig. 2

It is important to note here that this conclusion remains valid if we introduce far more complicated models. It is easily seen that instead of a constant increase of the number of potential buyers by a fixed consumer group, we could have the addition of any conceivable group to the hitherto unsatiated demand in quite irregular sequences, and still the same argument would hold. The only conditions which are needed are:

1) Among a consumer group newly entering the market there are some people whose maximal price bids are below the market price prevailing at the moment of market entrance.

2) The consumers whose demands are unsatiated do not systematically raise their maximal price bids.

The first condition is quite realistic. It simply says that there are people who wish to have a good which they cannot afford to buy. The second condition may be justified by the fact that the maximal price bids are primarily determined by the consumers' incomes. The increase in the consumers' incomes is by far not fast enough to upset the effect described above in an expanding market of a new product.

If one does not use Morgenstern's approach to demand theory, this phenomenon cannot be explained in a natural way. How artificial the explanations are without this approach may be illustrated by a quotation from Lever: "Certain products – especially new ones – may have small and inelastic demands until advertising is used to build up public acceptance. When this has been achieved price reductions may increase the demand considerably and thus advertising has had the effect of increasing the elasticity. Examples are motor cars and radio receivers."[28] This passage was written before the appearance of Morgenstern's paper. It is unbelievable that since then so many economists are still working with the oldfashioned demand curves.

The extra-profit for the monopolist will now be:

$$U = \sum_{t=1}^{t'} v^t y^{(t)} \tilde{x}^{(t)} - F(\tilde{x}^{(t)}). \qquad \text{(II.7.2)}$$

In this case, the monopolist is confronted with a decision problem under certainty. One could also assume a lack of information about the demand function $x(y^{(t)})$ and therefore consider this problem as being one of

[28] E. A. Lever: op. cit., p. 87.

decision making under risk or uncertainty (according to whether we assume known probabilities of the several states of demand or no information at all).

Let us now study the duopoly case. We do not follow the treatment as given by Shubik[29], as we consider the changes of demand as a dynamic process. We assume a linearly expanding market as above. Although the figures are drawn for linear demand functions in order to make the essential facts more visible, we assume a more general price-quantity relation, as above.

Fig. 3 shows the demand ABO at $t = 1$. The first firm fixes a price $y_1^{(1)}$, but is only willing to satiate the part Y_1D of the whole demand Y_1E. The group of consumers CB will be left out because their price-bids

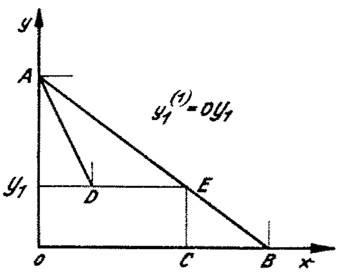


Fig. 3

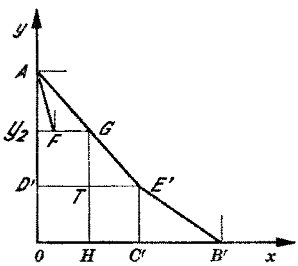


Fig. 4

are smaller than $y_1^{(1)}$. Fig. 4 shows the situation after the first firm's consumers have left the market. As the consumers are chosen at random for every price above $y_1^{(1)}$, the number of consumers decreases by the same percentage. As DE in Fig. 3 equals $D'E'$ in Fig. 4 and as the triangle EBC in Fig. 3 shifts to the left to $E'B'C'$ in Fig. 4, the new demand is now represented by the curve $AE'B'$. The second firm's price is OY_2, but it will only satiate the part Y_2F of the whole demand Y_2G. Fig. 5 shows the situation at $t = 2$. The demand ABO of Fig. 3 has now been replaced by the demand of another consumer group. To ABO of Fig. 5, we add the left over unsatiated demand. FG in Fig. 4 equals $F'G'$ in Fig. 5. As the consumer group $AG'F'$ also buys at lower prices than $y_2^{(1)}$, we draw $G'G''$ parallel to $F'B$. Now the consumer group $GE'T$ is added. $T'E''$ of Fig. 5 equals TE' in Fig. 4 and $E''E'''$ is

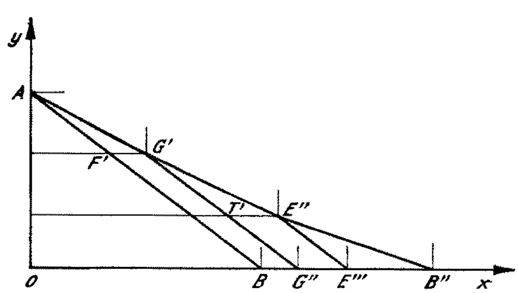


Fig. 5

[29] M. Shubik: Strategy and Market Structure. New York: 1959, pp. 88–91.

parallel to $T'G''$. Finally, the last consumer group joins the demand curve: $E''' B''$ in Fig. 5 equals $C' B'$ in Fig. 4. Thus, the kinked line $AG' E'' B''$ represents the demand at $t = 2$ before any purchases are made.

This treatment applies to the case in which both firms offer a homogeneous good and where we have a perfectly transparent market. Under these conditions we can assume that all consumers who can pay the price $y_1^{(1)}$ try to buy from player 1 until this player is sold out. Then the remaining consumers will go to the second firm. We call the firm with the lower price at $t = 1$ "player 1" and the other firm "player 2". To the demand as shown in Fig. 5 we can apply the same argument as above. We will get as a result a series of price-quantity pairs for both firms as the strategy-spaces in a two-person non-constant sum game with the payoff functions as in (II.7.2).

If we use the notations $\tilde{x}_1^{(1)}$, $\tilde{x}_2^{(1)}$ for the offered quantities, the demand for the second player at $t = 1$ is:

$$x_2^{(1)} (y_1^{(1)}, y_2^{(1)}) = x (y_2^{(1)}) \frac{x_1^{(1)} - \tilde{x}_1^{(1)}}{x_1^{(1)}} \text{ for } y_2^{(1)} \geqq y_1^{(1)}. \tag{II.7.3}$$

For the player whose price is lower at $t = 2$, we use a point as subscript. (This, of course, is not necessarily player 1.) The demand then for this player is according to Fig. 5:

$$y_.^{(2)} \geqq y_2^{(1)} \Rightarrow x_.^{(2)} (y_1^{(1)}, y_2^{(1)}, y_.^{(2)}) = x (y_.^{(2)}) \left(2 - \frac{x_1^{(1)} . \tilde{x}_2^{(1)}}{x (y_2^{(1)}) (x_1^{(1)} - \tilde{x}_1^{(1)})}\right). \tag{II.7.4}$$

$$y_2^{(1)} \geqq y_.^{(2)} \geqq y_1^{(1)} \Rightarrow x_.^{(2)} (y_1^{(1)}, y_2^{(1)}, y_.^{(2)}) = \tag{II.7.5}$$

$$= x (y_.^{(2)}) \left(1 + \frac{x_1^{(1)} - \tilde{x}_1^{(1)}}{x_1^{(1)}}\right) + x (y_2^{(1)}) \frac{x_1^{(1)} - \tilde{x}_1^{(1)}}{x_1^{(1)}} - \tilde{x}_2^{(1)}.$$

$$y_1^{(1)} \geqq y_.^{(2)} \Rightarrow x_.^{(2)} (y_1^{(1)}, y_2^{(1)}, y_.^{(2)}) = \tag{II.7.6}$$

$$= 2 x (y_.^{(2)}) - \tilde{x}_1^{(1)} - \tilde{x}_2^{(1)} + x (y_2^{(1)}) \frac{x_1^{(1)} - \tilde{x}_1^{(1)}}{x_1^{(1)}}.$$

Similarly, the formulae for the later stages of this development of the demand could be given. If we include the case of replacement demand we will get a more complicated model which, however, still can be treated.

Another problem is posed by the case in which the firms offer a greater quantity than can be absorbed by the market for all prices. Probably the only adequate treatment of this case would be the one including advertising efforts, which, however, violates the condition of a homogeneous good.

8. Price policy and research time as parameters of action

We will give only the descriptive exposition of the essential property of this situation without a mathematical formulation. In the previous section we have seen that in the case of an expanding market, excluding replacement demand, it will be advantageous to lower the price (for a monopolist as well as for a duopolist), as the consumers with low price-bids remain in the market from the previous period of time. Here there is an advantage to enter the market early with the new product, because it is then possible to sell at a high price in the beginning. This is especially important if the machinery used for the production of the new good is introduced gradually and if therefore the supply is so small in the beginning that it corresponds to a high price level which will be undercut considerably at a later time. On the other hand, concentrating research efforts into a short period of time will mean increasing total research costs. This situation can be treated as a decision problem for the monopolist as well as for a typical oligopolistic case.

9. Advertising and price policy as parameters of action

If we have advertising efforts, we no longer can speak of a homogeneous good, because it is the objective of advertising to establish an image of one firm's good as different from all other similar goods. It is not necessary in this case to speak of several markets and to introduce cross-elasticities. If the competing goods fulfill the same needs, the same people are potential buyers for all of these goods.

Let us present here a simple model which illustrates this situation. In the case of two firms with similar goods, every potential buyer may have two maximal price-bids, one for each of the two goods. If we call these bids b_1 and b_2 and the given prices y_1 and y_2, a plausible but not cogent rule for the consumer's decision would be to expect that he prefers good 2, if $\frac{y_1}{y_2} > \frac{b_1}{b_2}$ and vice versa. This problem is an outstanding case in economics which could be analyzed by experimental investigations.

We assume for our model that $\frac{b_1(k)}{b_2(k)} = \frac{A_1}{A_2}$ for all k, where k denotes the several consumers. We then get two demand functions as follows (Fig. 6).

In Fig. 7, the same situation is shown for linear demand functions. As $\frac{OY_1}{OY_2} < \frac{OA'}{OA}$, good 1 is preferred, but only Y_1C is supplied. This supply is assigned at random to the consumers OH. In order to construct the remaining demand for supplier 2, we have to find the point F on the second demand curve AB perpendicularly above D. On the horizontal line through F, KG equals CD. Therefore the line segment AG represents the remaining demand for the second supplier. At the price Y_2 the quantity Y_2T can be sold.

Here we have two demand functions x_1 and x_2. The segment CD can be written as $x_1(y_1) - \tilde{x}_1$. Now the remaining demand for the second player is:

$$x_2(y_1, y_2; x_1) = x_2(y_2)\,\frac{x_1(y_1) - \tilde{x}_1}{x_1(y_1)}. \tag{II.9.1}$$

Starting with such a model, we could introduce an increasing demand. The incremental demand now would be an increasing function of the sum

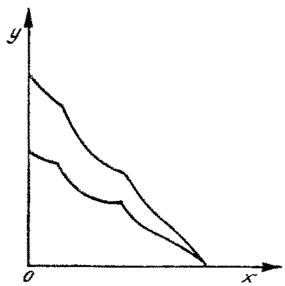


Fig. 6

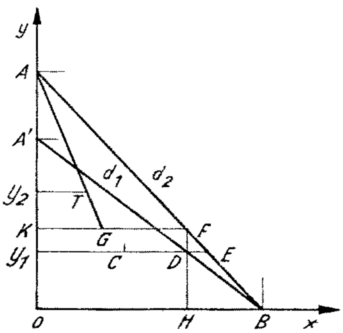


Fig. 7

of the advertising costs. A series of advertising costs, prices and quantities would then represent the strategy space of a firm.

III. Cooperative Situations for the Introduction of New Products

1. Forms of cooperation and side payments

In oligopolistic markets cooperation among the firms is possible by determining the values which have to be taken for one or more parameters of action by the members of a coalition. Thus, all the parameters of action which were considered in Part II of this paper could be subject to joint fixing. Furthermore, there are other kinds of possible settlements such as those concerning patents, as well as conventions about the geographical areas into which the several firms will expand their business. Many of these procedures are outlawed. Price-fixing, e. g., is illegal in most industrialized countries. We will confine ourselves in this paper to the domain which is always within the framework of the law, namely to the settlements concerning patents.

It should be obvious that the release of patents is always an act of cooperation between the firms involved. Such a transaction will be made only if it is to the advantage of both partners. (In fact, every economic action with this property — barter, trade, purchase, etc. — is cooperative behavior in the game theory sense.)[30] This kind of transaction does not

[30] See J. v. Neumann and O. Morgenstern: Theory of Games and Economic Behavior. 3rd revised ed., Princeton (N. J.): 1953, pp. 555 ff.

exclude the possibility of competition between the partners. It is possible that firm A as well as firm B is better off when firm A sells a patent to firm B, although they continue to compete afterwards. One can say then that they agree to carry out their competition on a level which is favorable to both. For example, firm A may *expect* that firm B will produce a similar new good with a different patent which will involve both sides in a costly advertising struggle. If the new products are based on the same patent, they will look much less different and therefore there will be less opportunity for competitive advertising campaigns. The cooperative aspect of patent transaction is even more evident in the case of patent exchanges or patent pooling.

The price paid for the release of a patent can be considered a side payment in a cooperative game. The firm which is willing to purchase a patent will expect incremental profits from the adoption of the other firm's invention. If the patent price exceeds the value of these expected incremental profits, it will be unacceptable. On the other hand, the firm possessing the patent may have smaller profits if it sells the patent than if it does not. Therefore, the price the firm receives for releasing the patent must be at least equal to this difference in the profits. There may be prestige considerations which also play a role in such patent transactions. A firm may reject the adoption of another firm's invention and try instead to create a similar invention rather than admit publicly that the competitor is leading in this field. (This may have been the reason why R. C. A. rejected in 1948 the adoption of the longplaying record invented by Columbia.)

From our previous considerations, it should be clear that the field of patent transactions contains so many complications that it would be difficult to formalize them in the sense of an economic theory. We will confine ourselves to two examples of two-person game situations.

2. Cooperation and starting time of research as parameters of action

In the event that a new technical principle (an "invention" in the sense of a basic insight without an already developed application) is known by two firms, each firm has to decide whether or not and when it will engage in development research in order to obtain a marketable product from this new principle. This decision will be influenced to a great extent by the expectations about the future behavior of the other firm. If the other firm is first in obtaining a marketable product and if our firm adopts the other firm's patent, then our own previous development research represents a waste of research investment. On the other hand, if we do not engage in development research and the other firm refuses to sell its patent later on, we are worse off than we would have been if we had made at least some effort in development research.

This dilemma may be formalized by the means of game theory: Each player has two strategies concerning negotiations in the patent field: $N =$ non-cooperative behavior, $C =$ cooperative behavior. Furthermore, each player has a choice from among 3 different starting times for development research: e = early, m = intermediate, l = late. We assume that the time difference between "late" and "early" is equal to the period of time required for developing the new marketable good. Thus, if player 1 chooses "e" and player 2 "l", the former is in possession of a new patent at a point in time when the latter has not yet started his research. We assume, furthermore, that the negotiations about the patent release take place as soon as one firm has finished its development research.

The following numerical example of a 6×6 matrix reflects all the properties of this situation as described above. These payoffs may be thought of as evaluations in terms of utilities of the situation after side payments have been made, i. e., after the price for the patent release has been paid. Of course, one could easily write the payoffs before side payments are made, as is usually the case in the literature. But we assume that the side payment lies between the limits which we have described above. We are not interested in the bargaining aspect of the cooperative game. Of course, this game can only be treated non-cooperatively, as the decision about cooperation or non-cooperation is built in as a strategy choice.

Table 8

		Player 2					
		Ne	Nm	Nl	Ce	Cm	Cl
Player 1	Ne	60 (60)	64 (50)	68 (40)	60 (60)	64 (50)	68 (40)
	Nm	50 (64)	54 (54)	58 (44)	50 (64)	54 (54)	58 (44)
	Nl	40 (68)	44 (58)	48 (48)	40 (68)	44 (58)	48 (48)
	Ce	60 (60)	64 (50)	68 (40)	80 (80)	100 (90)	110 (100)
	Cm	50 (64)	54 (54)	58 (44)	90 (100)	75 (75)	85 (80)
	Cl	40 (68)	44 (58)	48 (48)	100 (110)	80 (85)	70 (70)

The strategy which maximizes the security level for both players is *Ce*. This means that a player is ready for all eventualities by going into research early, but it still leaves the door open for an agreement. It is clear that a strategy "C" always dominates the corresponding strategy "N". Therefore we can consider only the nine last positions of the matrix if we assume rational behavior of the players. Let us also leave out "m". The following matrix has to be treated non-cooperatively:

Table 9

		Player 2 Ce	Player 2 Cl
Player 1	Ce	80 (80)	110 (100)
	Cl	100 (110)	70 (70)

This game has 3 equilibrium points: 2 in pure strategies (*Ce*, *Cl*) and (*Cl*, *Ce*), and one in mixed strategies: (2/3 *Ce* + 1/3 *Cl*, 2/3 *Ce* + 1/3 *Cl*). This last equilibrium point yields a payoff pair (90,90). Thus each player gets less in this case than in the other two equilibrium points. It is also easily seen that in this game a player has to choose a mixed strategy when he wants to maximize his security level. This makes sense as this game has to be carried out non-cooperatively.

Obviously an equilibrium in pure strategies which yields higher payoffs for both players than one in mixed strategies can be obtained if it is possible to treat this game cooperatively[31]. This would mean that negotiations start before research work is undertaken and that the research effort is concentrated in one firm, thus avoiding the case in which simultaneous research efforts cause a waste of capital. Both cases are known in economic history. The latter case means, of course, that we would have social costs of competition.

This situation can also be formalized as an infinite game using a parameter t (starting time of research) over a continuous interval of time. We give a simple example of such a formalization, using the following notation:

R = cost per unit of time for research (III.2.1)

P = incremental earnings minus incremental production and advertising costs in the case of non-cooperation, all per unit of time

Π = the same as P in the case of cooperation

Q = side payment (price for patent release)

P and Π represent the difference between the profits (excluding research costs) in the case of the new product and the profits which could have been expected if there were no new product.

For the values P and Π we have to assume given price-policies for the firms. Furthermore, in the case of an advertising struggle, we have to think of certain possible outcomes. The public may definitely prefer one product to the other, but some sort of stalemate is also possible in which both products are equally liked. If we assume a probability distribution for all these possible outcomes and assign to each outcome a certain value for the earnings, we can calculate P as the expected value of the earnings in the case of non-cooperation.

[31] The game described above is in fact not a cooperative one, although it contains "cooperation" as a strategy.

We distinguish two fictitious players. Fictitious player 1 is the firm which first invests in research; fictitious player 2, therefore, is the firm which starts its research later. The notion "real players" is used for the two firms as acting units. If there is no information, a "real player" does not know with which of the two "fictitious players" he will be identified. We put the starting point for the research of fictitious player 1 at time zero. The research of the other player will start after t time units. This construction does not mean any loss of generality, as only the time difference between the two research programs is of strategical significance.

We use the following notation:

u = payoff for fictitious player 1 in the case of non-cooperation (III.2.2)

w = payoff for fictitious player 2 in the case of non-cooperation

U, W shall denote the respective values in the case of cooperation

As mentioned above, the utilities for this model will be identified with monetary success. We then get the following payoffs, using the notations in (I.1):

$$\begin{aligned} u &= -\varphi(r)\, R + v^r \psi P, \\ w &= -v^t \varphi(r)\, R + v^{t+r} \psi P, \\ U &= -\varphi(r)\, R + v^r \psi \Pi + v^r Q, \\ W &= -v^t \varphi^{(r-t)} R + v^r \psi \Pi - v^r Q, \end{aligned} \qquad \text{(III.2.3)}$$

where r denotes the time needed for research and $0 < t \leqq r$. (We exclude the case where both firms start their research at exactly the same time.) One could specify the values for r, R, P, Π, φ, ψ for the two firms by using subscripts.

In the following, we examine the case in which the two firms are fully informed about each other's situation and in which they discuss the advantages of cooperation. They then need a scheme for the distribution of the mutually obtained benefits from cooperation. For the determination of the side payment, i. e., the royalties, we are then guided in a natural way to the several arbitration schemes for two-person cooperative games as described, e. g., in Luce and Raiffa[32] (Chapter 6). We will compute the values for the Nash bargaining model[33], and the Shapley value for this game, which will be the same. These values depend, in part, on the amount which the second firm has already spent for its now wasted research; consequently, this cost is borne to some extent by both firms.

It might seem unrealistic to compute such values, as normally the firms will not follow such an arbitration scheme. But at the least these values are a reasonable approach to the real outcomes in the cases in which both partners have about the same power in the negotiations.

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[32] R. D. Luce and H. Raiffa, op. cit.

[33] F. J. Nash: Two Person Cooperative Games. Econometrica, *21* (1953), pp. 128–140.

For the Nash solution we consider

$$(U-u)\,(W-w)=F\,(Q). \tag{III.2.4}$$

Differentiating twice, we get

$$F''\,(Q)=-2\,\varphi^2\,(r)\,R^2; \tag{III.2.5}$$

this expression is negative for all values of Q. Therefore, we are sure that $F\,(Q)$ is at a maximum, if $F'\,(Q)=0$. From this last condition, we get

$$Q=\frac{1}{2\,\varphi\,(r)\,R}\left\{v^r\,\psi\,P\,(1-v^t)+v^t\,R\,[\varphi\,(r)-\varphi\,(r-t)]\right\}. \tag{III.2.6}$$

If we introduce this value of Q in U and W from (III.2.3), we get new values of U and W with a fixed arbitrated side payment. We denote these values by U^* and W^*.

In order to calculate the Shapley values, we have to compute for a given player the arithmetic mean of the amount which the player can definitely assure himself, and the improvement of the payoff for the coalition due to his cooperation. For fictitious player 1, for example, this is the arithmetic mean computed from u and $(U+W-w)$. We get the following two Shapley values (which we denote by the same symbols as for the values of the Nash solution, since they are the same):

$$U^*=\frac{U+W+u-w}{2}, \tag{III.2.7}$$

$$W^*=\frac{U+W-u+w}{2}. \tag{III.2.8}$$

It is easy to see that these values fulfill the condition

$$U^*+W^*=U+W. \tag{III.2.9}$$

3. Model in which the history of the negotiations has an influence on the payoffs

In this second model, the history of negotiations also has an influence on the outcome. Normally, in the theory of cooperative games, a distinction is made only between cooperation and non-cooperation, without analyzing the way in which one of the two possibilities is finally reached. In the following model, we try to formalize the prelude to cooperation or non-cooperation as a game of its own. This "pre-game" can then of course be treated only as a non-cooperative game. Before the negotiations about cooperation start, an agreement cannot exist between the players indicating that they will behave in a way which leads to cooperation, for we could then ask how they arrived at this prior agreement and build up a new pre-game. And so we could ask the same question infinitely about the previous agreements. Therefore, we assume that in this pre-game the players act absolutely independently of each other.

The game will be developed first in the extensive form as a game with a finite tree. By player 1 we denote the firm which has an advantage over the other firm concerning the new product. The competitor will be called player 2. Each player can announce two attitudes, namely N, non-cooperative, and C, cooperative. The non-cooperative attitude should symbolize that behavior in real situations which consists of a propaganda campaign which is directed towards getting his product accepted by consumers and his competitor's product rejected. The cooperative attitude symbolizes the behavior of searching negotiations with the competitor and attempting to arrive at an agreement as to which innovation should be propagated (if there are two products), and how much royalties should be paid. Player 1 makes the first move. By N_1 we denote the non-cooperative behavior of player 1 and by C_1 his cooperative behavior. In the same way are defined N_2 and C_2 for player 2.

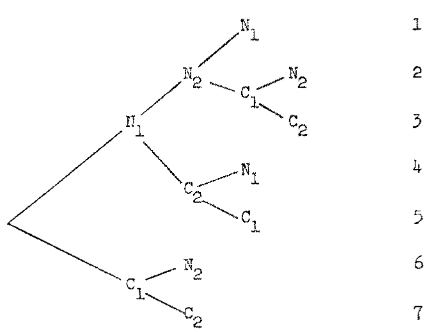
We introduce the following two rules for this game:

1) There is a branch consisting only of N's so that each player has an occasion to answer an N of the other player; thus there can be a maximum of three N's. (This rule is intuitively easy to understand. When each player has answered the N of the other with his own N, then evidently there is left only the possibility of a propaganda struggle. But after $N_1 - N_2$, this is not yet necessary, because the first player may have expected the second player to answer his N with C_2, whereas N_2 announces a stronger position than was expected by player 1. So in this case, player 1 may possibly answer on $N_1 - N_2$ with C_1.)

2) As soon as a C occurs, the play must be at an end in the next move. (If after a C the opponent answers with C too, then an agreement is reached. If, on the other hand, the opponent answers with N, he has rejected the other's readiness for negotiations and it would not make any sense for this player to repeat his negotiation proposal. So the outcome in this case is also a propaganda struggle.)

These two rules yield a game with the following tree:

Table 10



On the right side we have assigned numbers 1 to 7 to the several final points of this tree. It is evident that 3, 5 and 7 represent cooperative situations, whereas 1, 2, 4 and 6 reflect non-cooperative results.

Let us now bring this game from the extensive form to the normal form. The first player chooses either N or C in the first move, and in the case of N_1 he has to have an answer on N_2 as well as on C_2. Thus this player has the following five essential strategies:

Table 11

	First move	Second and third moves
1)	N_1	$N_2 - N_1$ $C_2 - N_1$
2)	N_1	$N_2 - N_1$ $C_2 - C_1$
3)	N_1	$N_2 - C_1$ $C_2 - N_1$
4)	N_1	$N_2 - C_1$ $C_2 - C_1$
5)	C_1	—

Similarly, player 2 has two answers on N_1 as well as on C_1 in the first move, and furthermore in the case of $N_1 - N_2 - C_1$, he has two possibilities. Therefore the six strategies of the second player are as follows:

Table 12

	First and second moves	third and fourth moves
1)	$N_1 - N_2$ $C_1 - N_2$	$C_1 - N_2$
2)	$N_1 - N_2$ $C_1 - N_2$	$C_1 - C_2$
3)	$N_1 - N_2$ $C_1 - C_2$	$C_1 - N_2$
4)	$N_1 - N_2$ $C_1 - C_2$	$C_1 - C_2$
5)	$N_1 - C_2$ $C_1 - N_2$	—
6)	$N_1 - C_2$ $C_1 - C_2$	—

We can now construct the following 5×6 matrix, in which the entries $E_1, E_2, \ldots, E_7$ correspond to the 7 end points of the tree (Tab. 10) and should represent pairs of payoffs according to the several possible outcomes.

Table 13

		Player 2					
		1	2	3	4	5	6
	1	E_1	E_1	E_1	E_1	E_4	E_4
	2	E_1	E_1	E_1	E_1	E_5	E_5
Player 1	3	E_2	E_3	E_2	E_3	E_4	E_4
	4	E_2	E_3	E_2	E_3	E_5	E_5
	5	E_6	E_6	E_7	E_7	E_6	E_7

Thus, we have the game in normal form.

Let us now apply this specific negotiation game to the situation in which duopolists are trying to invent similar new products, as generally described in pages 70 to 73. The attitude N means in this case that a firm is either starting or continuing to advertise and sell its product, regardless of what the other firm does. C, on the other hand means that a firm is holding back its advertising and selling and is attempting to negotiate with its competitor. In other words, the communication between the two players is carried out by certain economic measures. Therefore, the payoffs [see (II.2.3)] will be modified according to the history of the "negotiation"[34]. For example, if non-cooperation is reached with the moves $N_1 - N_2 - C_1 - N_2$, player 1 will have a smaller value for the discounted advertising costs as well as for the expected earnings than when the non-cooperative state is reached by $N_1 - N_2 - N_1$. The C_1 in the third move of the former case announces that player 1 was holding back his selling policy during a certain period of time, which has the effect described above. It is also reasonable to make the same assumption about the royalties. Royalties will be paid in the three cooperative outcomes, E_3, E_5 and E_7. In E_3 $(N_1 - N_2 - C_1 - C_2)$, player 1 indicates his willingness to be cooperative after player 2 has played N. This weakens the position of player 1; and player 2 may play C_2 only under the condition that he has to pay relatively small royalties. The same reasoning applied to E_5 $(N_1 - C_2 - C_1)$ shows that in this case the outcome will be especially favorable to player 1. E_7 $(C_1 - C_2)$ represents a situation midway between E_3 and E_5. In this last case, the C_1 at the opening of the game means a stronger position for player 1 than does a C_1 after an N_2.

In Tab. 14 we give a survey of the modifications which occur in the payoffs (III.2.3) according to the history of the "negotiation". This survey is self-explanatory after the above discussion. A + sign denotes that the indicated term is altered in favor of the player being considered (i. e., if the term is positive, it will increase; if it is negative, it will decrease). A − sign denotes the opposite. If in a row, one sign is bracketed whereas another sign is unbracketed, the weight of the former deviation is smaller than that of the latter. The payoffs u and w in the non-cooperative case will occur in their original form in end-point E_1 $(N_1 - N_2 - N_1)$. Similarly, the original payoffs U and W for the cooperative case will appear in E_7 $(C_1 - C_2)$. All other end points will yield payoffs which deviate from the original u, w, U and W.

It is certainly realistic to suppose that the deviations described above do not exceed the differences between the cooperative and the non-cooperative payoffs. E. g., in the case $N_1 - N_2 - C_1 - N_2$, player 1 will

[34] The term "negotiations" will be used as a sort of super-term for all possible actions at the moment when one firm is ready to sell its new product as described above. Therefore, the term "negotiations" includes also the case where *no* discussion takes place between the two firms. We could then imagine that they will communicate only through their actions and counter-actions.

not hold back his advertising for a length of time such that player 2 would be better off than he would be in the case of cooperation. This

Table 14

End points	History (branch)	Player	Payoff considered	Expected earnings	Discounted advertising costs	Royalties
E_1	$N_1 - N_2 - N_1$	1	u			
		2	w			
E_2	$N_1 - N_2 - C_1 - N_2$	1	u	−	(+)	
		2	w	+		
E_3	$N_1 - N_2 - C_1 - C_2$	1	U		−	−
		2	W		(−)	+
E_4	$N_1 - C_2 - N_1$	1	u	+		
		2	w	−	(+)	
E_5	$N_1 - C_2 - C_1$	1	U	(+)	(−)	+
		2	W	(−)		−
E_6	$C_1 - N_2$	1	u	−	(+)	
		2	w	+	(−)	
E_7	$C_1 - C_2$	1	U			
		2	W			

property will also hold for non-economic applications of this negotiation game. Tab. 15 shows a game based on all the preceding considerations. We put $u = 60$, $w = 40$, $U = 100$, and $W = 80$.

Table 15

End points		Payoff for Player 1	Payoff for Player 2
E_1	$N_1 - N_2 - N_1$	60	40
E_2	$N_1 - N_2 - C_1 - N_2$	50	45
E_3	$N_1 - N_2 - C_1 - C_2$	90	85
E_4	$N_1 - C_2 - N_1$	65	35
E_5	$N_1 - C_2 - C_1$	105	70
E_6	$C_1 - N_2$	55	45
E_7	$C_1 - C_2$	100	80

The strategy situation for a player does not change if we apply a linear transformation to his payoffs. We now get the following matrix:

Table 16

		Player 2					
		1	2	3	4	5	6
	1	[60 40] 1	60 40	60 40	60 40	65 35	65 35
	2	60 40	60 40	60 40	60 40	[105 70] 5	[105 70] 5
Player 1	3	50 45	[90 85] 3	50 45	90 85	65 35	65 35
	4	50 45	[90 85] 3	50 45	90 85	105 70	105 70
	5	55 45	55 45	[100 80] 7	[100 80] 7	55 45	100 80

The pairs of payoffs in squares are equilibrium points in pure strategies; we have noted on the right side the end point from which they occur. Fortunately we find that for the first player strategy 1 is dominated by 2 and strategy 3 is dominated by 4. For the second player strategy 4 dominates 1, 2, and 3, and strategy 6 dominates 5. Therefore, we are left with the following 3×2 matrix:

Table 17

		Player 2	
		4	6
	2	60 40	[105 70]
Player 1	4	90 85	105 70
	5	[100 80]	100 80

Besides the two equilibrium points in pure strategies (in squares) there is a set of equilibrium points if the first player plays his strategy 5 and the second player uses a mixed strategy with the probability assigned to his strategy 4, namely $q \geqq 1/3$. The two equilibrium points in squares are neither interchangeable nor equivalent. Therefore, we cannot define a solution for this game[35].

The game described above, in which the history of the negotiation influences the payoffs, is a formalization of a situation which of course occurs not only in the field of economics. There are many relations between two people in which it may be disadvantageous to show one's sympathy toward the other too soon, and, on the other hand, a too reserved attitude may lead to the even greater disadvantage of complete disunion. The original 5×6 matrix (Tab. 16) also shows that the "normal" non-cooperative payoffs, $u = 60$ and $w = 40$, represent the maximal security levels for both players. If both players have in mind only the maximization of their security levels, this will lead to a non-cooperative solution. As we expect intuitively for such situations, cooperation can be reached only if at least one player is willing to undergo some risk. If a human relation is frozen, one party has to make the first step, even at the risk of being called "soft".

IV. Simultaneous Introduction of a Large Number of New Products

1. The number of new products per unit of time as a parameter of action

There are industries which introduce a large number of different new products into the market simultaneously or in relatively short time intervals. In this case the new products do not differ in their technical function but rather in their design, style, etc. Examples of such industries are the chemical industry and the phonograph record industry. Of course, two different records may also have completely different markets.

Now, it seems to be characteristic for both these industries that economic success is concentrated strongly in only a few of these products. This success can be overwhelming that it entirely supports nonprofitable branches of the same industry. (For example, research laboratories in the case of the chemical industry and editions of very old or contemporary music in the case of the phonograph record industry may be supported because they increase the prestige of a firm. In the very long run, investments in such branches may pay off fairly well.) In the following we will focus our attention on the phonograph record industry, because in that case these properties are especially clear. We will use the terminology of this industry in the following discussion.

Let us consider a market into which several record producers introduce a certain number of new records. Their main interest will be to have a hit or even a so-called "smash" hit among their own new releases. A producer therefore might estimate a "hit-probability" for his repertoire. It would not be possible to find such a probability empirically,

[35] See the discussion in R. D. Luce and H. Raiffa: op. cit., pp. 106 ff.

even if we had all the information on the number of sales of records, because the sales of a specific record are determined not only by the musical or other properties of its tune, but also by the number and the successes of the other records on the market. A hit can be killed or at least weakened, if there is another hit already on the market or one is emerging simultaneously. Thus, if we would know the pure hit-probabilities based on the musical or entertaining qualities of the recorded music, we would have to use damping factors for the mutual weakening or annihilation of hits.

Let p be the total profits obtained from a specific record discounted back to time zero. By p' we denote the lower boundary, and by p'' the upper boundary, for the value of p. Producer 1 may introduce m new records into the market. For this group of records there may exist an average hit-probability distribution $h_{m,n}(p)\,dp$, in such a way that to each possible p there is assigned a measure $h_{m,n}$, where n denotes the total number of new records introduced by competitors into the market. The smaller is m, the more carefully can the producer select the titles, thus increasing the average hit-probability per new record. Therefore, $h_{m,n}(p)$ decreases monotonically in m for a fixed n and p. We assume that the damping effect of the rival's records is already contained in $h_{m,n}(p)$. This value therefore will decrease with the total number $m+n$ of new records. Only the number of new records is considered to have a damping effect. In reality the number of older records will also have an influence, but probably a smaller one than the most recent records. Long-lasting hits are rare and if a success from a previous period of time continues into the present one, it will no longer stir up as much attention and will therefore do less harm to a newly emerging hit than will a simultaneously successful one. We therefore neglect the number of the records introduced in preceding periods of time. It follows that $h_{m,n}(p)$ also decreases monotonically in n for a fixed m and p. We can then define a hit-expectation value:

$$H_{m,n}=\int_{p'}^{p''} p\, h_{m,n}(p)\, dp \qquad \text{(IV.1.1)}$$

for a new record on the average. The expected monetary value of the whole group of new records will be consequently $m\,H_{m,n}$.

It is clear that this is a typical game situation. The profits p may be based only on the costs of duplication of the records, but not on the fixed costs due to the use of studios, equipment and personnel for recording. If we denote the total of the latter costs by $F(m)$ (as a function of m), we get the payoff function for player 1:

$$U_1=m\,H_{m,n}-F(m). \qquad \text{(IV.1.2)}$$

Let us illustrate this game by the simplest possible two-person game.

$$H_1\, m, n = a - \frac{m}{\beta} - \frac{n}{\gamma}, \quad H_2\, m, n = a - \frac{n}{\beta} - \frac{m}{\gamma}, \quad F(m) = \delta m + \varepsilon, \quad \text{(IV.1.3)}$$

$$U_1 = m\left(\alpha - \frac{m}{\beta} - \frac{n}{\gamma}\right) - \delta m - \varepsilon, \quad \text{(IV.1.4)}$$

$$U_2 = n\left(\alpha - \frac{n}{\beta} - \frac{m}{\gamma}\right) - \delta n - \varepsilon,$$

(all constants are positive)

$$\frac{d U_1}{d m} = \alpha - \frac{2m}{\beta} - \frac{n}{\gamma} - \delta. \quad \text{(IV.1.5)}$$

$$\frac{d^2 U_1}{d m^2} = -\frac{2}{\beta}. \quad \text{(IV.1.6)}$$

This is always negative, therefore we obtain a maximum of U_1 for $\frac{d U_1}{d m} = 0$. This yields

$$m = \frac{\alpha\beta}{2} - \frac{n\beta}{2\gamma} - \frac{\delta\beta}{2}. \quad \text{(IV.1.7)}$$

As the game is symmetric, we get similarly:

$$n = \frac{\alpha\beta}{2} - \frac{m\beta}{2\gamma} - \frac{\delta\beta}{2}. \quad \text{(IV.1.8)}$$

Introducing (IV.1.8) into (IV.1.7), we get:

$$m = \frac{\beta}{2\left(1 - \frac{\beta^2}{4\gamma^2}\right)}\left(\alpha - \delta - \frac{\alpha\beta}{2\gamma} + \frac{\delta\beta}{2\gamma}\right) = \frac{\beta(\alpha - \delta)}{2\left(1 + \frac{\beta}{2\gamma}\right)} = n. \quad \text{(IV.1.9)}$$

This is an equilibrium point in pure strategies in this infinite game.

2. Model with several categories of new products

Here we consider the case in which a firm introduces a large number of new products, but we distinguish several categories of these products. Using the phonograph record industry as an example, newly released records may belong to different categories which may give rise to quite different consumption patterns. These categories may be based on different types of dance or musical forms (e. g., marches, tangos, cha-cha-chas, Dixieland jazz, Bop jazz, etc.), or on the relation to former successes (e. g., completely new compositions, imitations of the type of music hitherto "en vogue", new recordings of "evergreens", etc.).

Let us consider the following two-person case. Two record producers produce a given number of new records for a business period. We assume that each one specializes in one of the given categories. The range of possible profits is divided into a finite number of profit classes numbered from 1 to m. We call $w_1^{(i)}$ the probability that — given the choice of the categorie of the first player — a profit of the i-th class will occur, provided that the first player can act as a monopolist. Of course, we must have the condition:

$$\sum_{i=1}^{m} w_1^{(i)} = 1. \qquad \text{(IV.2.1)}$$

Similarly, we have the $w_2^{(j)}$ $(j = 1, \ldots, m)$.

In order to get from the monopolistic to the duopolistic situation, we introduce damping factors for the simultaneous emergence of hits. These damping factors depend not only on the two categories chosen by the firms, but also on the profit classes for both sides. We write the damping factors as follows:

$$\Phi_1^{(i,j)} \ (s_1, s_2), \qquad \text{(IV.2.2)}$$

where the subscript means that the damping factor is applied to the first firm's success; i and j are the two profit classes and s_1, s_2 the two choices of categories[36]. In the following, we leave out (s_1, s_2), considering these choices as given.

Let us call $p^{(i)}$ the mean value of the i-th profit class. Thus the expected profit for the first player as a monopolist will be:

$$\sum_{i=1}^{m} p^{(i)} w_1^{(i)}. \qquad \text{(IV.2.3)}$$

Now we consider the profit decrease due to the possible profits of the second player. Thus, each $p^{(i)}$ has to be multiplied by $\sum_{j=1}^{m} w_2^{(j)} (1 - w_2^{(j)} + \Phi_1^{(i,j)} w_2^{(j)})$. Consequently, the first player's expected profit as duopolist will be:

$$U_1 = \sum_{i=1}^{m} \sum_{j=1}^{m} p^{(i)} w_1^{(i)} w_2^{(j)} (1 - w_2^{(j)} + \Phi_1^{(i,j)} w_2^{(j)}). \qquad \text{(IV.2.4)}$$

This is the payoff function of a game in which the choice of a category is the strategic variable.

For estimating empirically, it might be worthwhile to choose a simple function for the relation between the Φ and i, j. Let us propose the following function

$$\Phi_1^{(i,j)} = \Phi_1 + \text{sig}\,(p^{(i)})\,(i - j)\,\Delta, \qquad \text{(IV.2.5)}$$

where Φ_1 is given for the case where both sides attain the same profit class. We use the sig $(p^{(i)})$ as a factor, because if the first player realizes a negative profit, the greater $(j - i)$ is, the greater the disadvantage to the first player. A disadvantage occurs if the negative profit is multiplied by an increasing number; thus the sign of $(i - j)$ must be reversed.

[36] We assume that the damping factor will be the same, whether a certain profit class is attained either by several small successes or just by one hit.

V. Conclusion

It was not our purpose in this paper to give a final, formal theory of the subjects treated. The objective was rather to outline a rigorous theoretical framework and to show, using a specific area within this framework, the possibilities for formal treatments. In this way, we hope that suggestions for further theoretical work in this field have been given. Thus, this paper should be considered a beginning rather than an end.