Models of Bargaining and Price Determination of Residential Real Estate, with and without Real Estate Agents

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Abstract

Residential real estate accounts for a large share of wealth (e.g. 33% in U.S. in 2005 according to FRB balance sheet data), and GDP (about 11% in the U.S. in 2005 according to the BEA). Yet surprisingly, there are few models available to analyze price determination and "equilibrium" in this market. This research will contribute to our understanding of the housing market in two main areas: 1) via collection and analysis of new, previously unavailable high frequency micro data on individual housing transactions, with details on how list prices are revised over time, and information on each visit by buyers, and outcomes of bargaining (including the sequence of offers and whether or not seller rejected or accepted each offer) for a large sample of homes over a significant period of time, and 2) via development of a computationally tractable empirical model of "temporary equilibrium" in particular housing markets. We use the term "temporary equilibrium" to denote the fact that many housing markets may have an imbalance between supply and demand at any point in time, as reflected in the often used adjectives "seller's markets" and "buyer's markets". Even though there may be a temporary imbalance in supply and demand, we believe prices adjust relatively rapidly to clear the market. Our model describes a process of endogenous adjustment in prices (and in seller and buyer beliefs) so that at any given point in time, agents in these markets can be described as having "approximately rational expectations" even though there may be a longer run continuing imbalance between the number of homes being sold and the number buyers looking to buy them in any particular housing market.

We will build, empirically estimate and test a dynamic model of the housing market with three types of agents: 1) sellers, 2) buyers, and 3) real estate agents. Sellers decide whether to list their home with a real estate agency, and subsequently how to revise their listing price over time, and whether or not to accept offers from buyers if/when they arrive. Buyers search among the set of available homes in the particular housing market under consideration, and decide whether or not to make an offer on houses they visit, and if so, whether their initial offer should match or be above or below the seller's list price. If a seller rejects their initial offer, buyers decide (in n additional "bargaining rounds") how much to increase their previous offer, or whether they should "walk" and search for other homes for sale. Real estate agents are modeled as having access to a technology and data, the *multiple listing service,* that can result in a higher rate of arrival of buyers and possibly better match of potential buyers (i.e. buyers who are willing to pay more for the home than from the general population of searchers).

1 Introduction

Residential real estate accounts for a large share of wealth (e.g. 33% in U.S. in 2005 according to FRB balance sheet data), and GDP in modern economies (about 11% in the U.S. in 2005 according to the BEA). Furthermore, a vast majority of households own their home (about 70% in the U.S.), and the sale and/or the purchase of a home are often the largest financial transactions households engage in. In the year 2005 alone, more than 8 million homes were bought and sold in the U.S. Yet surprisingly, there are few models available to analyze the housing transaction process.

Our proposed research will contribute to our understanding of the housing market in two main areas. The first contribution is the collection and analysis of new, high frequency micro data on individual housing transactions, with details on how list prices are revised over time, and information on each visit by buyers, and outcomes of bargaining (including the sequence of offers and whether or not seller rejected or accepted each offer) for a large sample of homes over a significant period of time. The second contribution is to develop computationally tractable models of the behavior of buyers, sellers, and intermediaries in the housing market, and to estimate these models using our data.

This research will allow us to address the following important questions: How do sellers choose their listing price? How to they revise it? How do sellers decide whether or not to accept an offer? What information about their own transaction history and the overall market environment matters for their listing price and offer acceptance strategies? How do buyers choose their first offer, and if rejected, their next offer(s)? Do institutional differences in the negotiation process over housing transactions affect market outcomes and market efficiency? What is the value of the realtors' contribution to the housing transaction process?

We believe there are potential public policy benefits resulting from better analytical models of the residential real estate market. The U.S. Department of Justice is currently investigating the U.S. National Association of Realtors to determine whether it has created unfair barriers to entry, particularly in restricting access to the *multiple listing service*, a large online database of homes for sale, in order to maintain large real estate commissions, which are typically 6% in the United States. Although our initial focus will be to understand bargaining and price determination under the *status quo* (i.e. assuming that houses are sold via a real estate agent), we believe our model can be extended to include real estate intermediaries and an endogenous choice of whether to sell via a real estate agency, or to "sell by owner". With this extension in place, it will be possible to study the IO issues connected with real estate agents, possibly even including endogenous determination of real estate contracts and commissions.

A unique aspect of our research is that we have access to detailed micro data from a foreign country, England, in addition to data from the U.S. Real estate laws and institutions are significantly different in England compared to the U.S. and we believe these differences will help shed light into the characteristics of institutions and into the relative efficiency of different forms of organization of the housing market.

To date, the lack of adequate data has limited the scope of empirical research on housing transactions. Existing data sets typically include property characteristics, time to sale, initial listing price, and sale price. They do not contain information on the buyer's side of the transaction (e.g., the timing and terms of offers made by potential buyers), or on the seller's behavior between the initial listing and the sale of a property (e.g., the seller's decision to reject an offer or to revise the listing price). This explains why most of the empirical literature on housing transactions has either focused on the determinants of the sale price or on the role of the initial listing price and its effect on the time to sale (e.g., Horowitz (1992), Miller and Sklarz (1987), and Zuehlke (1987)).

Recent attempts to overcome some of the data limitations by supplementing conventional data sets with additional information have generated valuable insights. For example, Genesove and Mayer (1997) build a data set for the Boston condominium market where they are able to uncover the financial position of each seller. They find that sellers with high loan-to-value ratio tend to set a higher initial listing price, have a lower probability of sale but, if and when they sell, obtain a higher price. Glower et al. (1998) conduct a phone survey to obtain information on each seller's motivation (e.g., whether or not they have a planned moving date), for a real estate transaction data set for Columbus, Ohio. The evidence suggests that sellers convey information about their willingness to sell (i.e., their reservation value), through the listing price. Similar evidence is also reported, in Anglin et al. (2003), Genesove and Mayer (2001), Knight et al. (1998), and Knight (2002).

In addition to contributing to the empirical literature on housing transactions, our proposed research also contributes to the theoretical literature on the strategic interactions between buyers and sellers in the housing market (e.g., Arnold (1999), Chen and Rosenthal (1996ab), Coles (1998), Horowitz (1992), Krainer (2001), Taylor (1999), Yavaş (1992), and Yavaş and Yang (1995)). Our research highlights the importance of accounting for incomplete information in the matching and bargaining environment where buyers and sellers interact, possibly through intermediaries.

2 Data

Our research will use several newly-collected data sets on residential real estate transactions, building on prior work by Merlo and Ortalo-Magné (2004) that analyzed a new data set of individual residential property transactions in England. The main novelty of the data is the record of all listing price changes and all offers made between initial listing and sale agreement. This study characterized a number of key stylized facts pertaining to the sequence of events that occur within individual property transaction histories, assessed the limitations of existing theories in explaining the data, and suggested new theoretical frameworks for the study of the strategic interactions between buyers and sellers that is the primary focus of the new research to be described below.

To motivate our theoretical models, it is useful to describe the data and summarize some of the key empirical findings. In England, most residential properties are marketed under sole agency agreement. This means that a property is listed with a single real estate agency that coordinates all market related activities concerning that property from the time it is listed until it either sells or is withdrawn. Agencies represent the seller only. Listing a property with an agency entails publishing a sheet of property characteristics and a listing price. Although not legally binding, the listing price is generally understood as a price the seller is committed to accept.

The listing price may be revised at any time at the discretion of the seller. Potential buyers search by visiting local real estate agencies and viewing properties. A match between the seller and a potential buyer occurs when the potential buyer makes an offer. Within a match, the general practice is for the seller to either accept or reject offers. In the event the seller rejects an offer, the potential buyer either makes another offer or walks away. If agreement occurs, both parties engage the administrative procedure leading to the exchange of contracts and the completion of the transaction. This procedure typically lasts three to eight weeks. During this period, among other things, the buyer applies for mortgage and has the property surveyed. Each party may cancel the sale agreement up to the exchange of contracts.

For each property it represents, the agency keeps a file containing a detailed description of the property, its listing price, and a record of listing price changes, offers, and terms of the sale agreement, as required by law. The information contained in each individual file is also recorded on the accounting register that is used by each agency to report to the head office. Although all visits of a property by potential buyers are arranged by the listing agency, recording viewings is not required either by the head office or by law. However, individual agencies may require their agents to collect this information for internal management purposes.

The first data set we will use in our research was obtained from the sales records of four real estate agencies in England. These agencies are all part of Halifax Estate Agencies Limited, one of the largest network of real estate agents in England. Three of these agencies operate in the Greater London metropolitan area, one in South Yorkshire. Our sample consists of 780 complete transaction histories of properties listed and sold between June 1995 and April 1998 under sole agency agreement. Each entry in our data was validated by checking the consistency of the records in the accounting register and in the individual files.

Each observation contains the property's characteristics as shown on the information sheet published by the agency at the time of initial listing, the listing price and the date of the listing. If any listing price change occurs, we observe its date and the new price. Each match is described by the date of the first offer by a potential buyer and the sequence of buyer's offers within the match. When a match is successful, we observe the sale agreed price and the date of agreement which terminate the history. In addition, for the properties listed with one of our Greater London agencies (which account for about a fourth of the observations in our sample), we observe the complete history of viewings. Since events are typically recorded by agents within the week of their occurrence, we use the week as our unit of measure of time. Our data spans two geographic areas with different local economic conditions and two different phases of the cycle in the housing market. While the local economy in Greater London has been experiencing a prolonged period of sustained growth, this has not been the case in South Yorkshire. Furthermore, from June 1995 to April 1998, the housing market in the Greater London metropolitan area went from a slow recovery to a boom. While this transition occurred gradually, for ease of exposition we refer to 1995-96 as the recovery and to 1997-98 as the boom.

This data set was the one analyzed by Merlo and Ortalo-Magné (2004), and their main findings can be summarized as follows. First, listing price reductions are fairly infrequent; when they occur they are typically large. Listing price revisions appear to be triggered by a lack of offers. The size of the reduction in the listing price is larger the longer a property has been on the market. Second, the level of a first offer relative to the listing price at the time the offer is made is lower the longer the property has been on the market, the more the property is currently over-priced, and if there has been no revision of the listing price. Negotiations typically entail several offers. About a third of all negotiations are unsuccessful (i.e., they end in a separation rather than a sale). The probability of success of a negotiation decreases with the number of previous unsuccessful negotiations. Third, in the vast majority of cases, a property is sold to the first potential buyer who makes an offer on the property (i.e., within the first negotiation), although not necessarily at the first offer. The vast majority of sellers whose first negotiation is unsuccessful end up selling at a higher price, but a few end up accepting a lower offer. The higher the number of negotiations between initial listing and sale agreement, the higher the sale price.

In addition to this data, our analysis will also rely on a new data set for England that we recently collected from a real estate agency in the city of Reading. This agency operates a paperless office where all realtors work cooperatively on all the properties listed. This implies that every real estate agent records every details of any action on every property carefully. The quality of the data is exceptional. We have details about every property that was handled within the agency between January 2000 when they started implementing the paperless office and June 2004 (almost 2500 properties). In particular, unlike the first data set, we have information on all the properties that were listed, regardless of whether they sold or were withdrawn. For every listing, we know the same information as in the first data set. In addition, we know details about all the visits to the property. We know how many hits the property got on a weekly basis on the agency's website. We know if and when the property was advertised through the press and via mailings.

The agency also maintains files for each potential buyer. In particular, we know the date of their first enquiry in the agency, whether they are first-time buyer, buying to move in or to rent the property out, and the price range within which they are looking. We obviously have all the characteristics of the property that are posted by the agency on its website. In addition, we also know the valuation done by the agency's own appraiser. We have also mapped a small subset of the properties with a GIS system to obtain precise information on the area and shape of the parcel of land that supports each property and details about each location. We plan to implement this procedure for all the properties in our sample.

Finally, we have started to collect new data for the market of Madison, Wisconsin and are investigating opportunities to collect data in Chicago. Madison is a particularly interesting case study because it has a very well organized for-sale-by-owner market that is largely dominated by `fsbomadison.com.` The owners of this service have been very cooperative in providing us with data. We are also working closely with various realtors in Madison and their association to assemble data sets that are comparable to the ones we put together for England.

3 Models: Overview

This section describes the overall model of a "temporary equilibrium" in the housing market. Subsequent sections will describe various key "submodels" such as a model of the seller's determination of listing prices and offer acceptance strategies, the buyers' search and offer decisions, a model of bargaining between buyers and sellers, and models of real estate intermediaries. The models presented here are intended to provide a basis for a structural econometric framework that might explain the stylized facts of bargaining over housing prices described in Merlo and Ortalo-Magné (2004). We also plan to generalize these models to accommodate the additional features of the other data sets we will use in our analysis that we described in the previous section.

We define a concept of "temporary equilibrium" that is consistent with a temporary imbalance in supply and demand for housing, reflecting the possibility that a housing market could either be a "buyer's market" (i.e. where there are relatively more properties for sale relative to the number of buyers, and housing prices are trending downward over time) or a "seller's market" (i.e. where there are relatively more buyers than properties for sale, and transaction prices are trending upwards). The main requirement for a temporary equilibrium is that beliefs of sellers and buyers are approximately correct. Buyers have beliefs about the distribution of the qualities and prices of the houses that are currently available for sale. Similarly, sellers have beliefs about the arrival rate and bidding propensities of buyers (i.e. the likelihood a buyer will make a offers and the probability distribution of the size of the offers). These beliefs are approximately correct if they are nearly the same as the actual probabilities and distributions that are observed in the housing market.

The motivation behind this "approximate equilibrium" approach is to avoid the problems involved in attempting a "direct solution" of the bargaining and equilibrium problem, especially in terms of attempting to circumvent the "curse of dimensionality" involved in a direct Bayesian Nash equilibrium formulation of general specifications of the "negotiation subgame". Under the latter approach, since each negotiation between a buyer and seller in the housing market is naturally modeled as a game of two-sided incomplete information, we would have to specify a Bayesian updating process mapping priors of the buyer and seller about each other's valuation of the property in question. Thus one of the relevant "state variables" for the buyer and seller during a negotiation is the posterior probability of the valuation of their opponent, conditional on the history of the bargaining process so far. Since it is unlikely that these posterior beliefs would be a member of any natural conjugate-prior family (which could be summarized by a finite-dimensional vector of parameters), the posterior in most realistic cases would not be describable by a finite-dimensional vector, and therefore at least one important state variable for the problem would be infinite-dimensional.

We propose a pragmatic alternative that we believe represents a good approximation of how actual agents are likely to think about and behave in negotiations over the sale of a house, and assume that the buyer and seller have beliefs about the probabilities their opponents will take various actions during the negotiations. For example, we assume that the seller has beliefs about the rate of arrival of buyers, the conditional probability that a buyer who views the property will make an offer, the conditional probability density of the offer given that one is made, and the probability the buyer will continue to negotiate and provide another offer if the seller rejects the buyer's first offer, and so forth. These probability distributions reflect the fact that the seller is unaware of the buyer's underlying valuation of the property, as well as other types of unobserved heterogeneity in the preferences of the buyer (i.e. the buyer's search costs and level of impatience to find a house, etc.). Thus, unlike a full Bayesian Nash equilibrium formulation, we do not assume that the seller explicitly updates beliefs via Bayes rule after every possible negotiation history, although the sequence of conditional probabilities might possibly be consistent with such updating.

At the same time, we do not allow buyers and sellers in a negotiation process to have arbitrary beliefs about the possible actions of their opponents. Instead, we assume that the buyers and sellers have approximate "rational expectations" about their opponents in a housing negotiation. In other words, the beliefs we use are required to be approximately "self-confirming" in the sense that buyers' and sellers' beliefs are approximately equal to the actual conditional probability distributions that are in effect over a given period of time. The "given period of time" can be relatively short (i.e. a period of "disequilibrium" in the local housing market when excess supply or demand might be changing), but it must be long enough for it to be plausible that buyers and sellers have well-formed beliefs (i.e. accurate beliefs) of the rate of arrival of buyers, of the number of properties coming on the market, and of realized housing transaction prices. To be more concrete, we might expect that our equilibrium is relevant for periods as short as 3 to 6 months or so, or about the average duration of time to sell or buy a house in most housing markets (if there is a severe imbalance of supply and demand, such as a period of severe excess supply where few transactions are taking place, longer periods might be necessary to enable collection of sufficient data to enable sufficiently accurate estimation of the "behavioral probabilities" necessary to determine equilibrium beliefs). We would expect that real-estate agents would be constantly monitoring the local housing market and could convey their observations of the local housing market conditions to the buyers and sellers that they advise. This information (as well as other information in newspapers, trade magazines, and word of mouth) could be viewed as the practical sources of the "beliefs" of buyers and sellers that we will model. These beliefs may not be exactly correct (in the sense of exactly coinciding with the actual distributions that would be realized over the period in question), but at least approximately correct, in the sense that beliefs of buyers and sellers should not differ in obvious and easily correctable ways from the realized distributions, which could be estimated via standard statistical methods.

Thus, our model of temporary equilibrium in the housing market is consistent with a view that buyers and sellers learn relatively little from the outcome of individual housing negotiations. Instead, their beliefs are formed mostly via communications with real estate agents, discussions with friends, reading the newspaper, etc. Our model, however, does allow buyers and sellers to revise their beliefs over the course of bargaining over a specific property. The belief revision process is based on estimated probabilities formed from a large number of "similar" observations of bargaining outcomes, according to the approximate self-confirming beliefs described above. For example, if a buyer were to make an offer on a property and the seller were to reject this offer, the seller would have revised beliefs about the likelihood the buyer would "walk" or would submit a revised offer. The seller also has beliefs about the magnitude of a revised offer if the buyer were to submit one. In general the generic beliefs that we posit reflect our view that given the degree of heterogeneity and "noise" in the housing market, the gains to a buyer or seller from using an explicit Bayesian updating process to reflect the additional information from the idiosyncratic characteristics of the particular property in question, and the revision of beliefs during the course of an individual negotiation would be small.

The main "trick" involved in modeling temporary equilibrium is to define appropriate "sufficient statistics" that enable us to capture the idiosyncratic features of individual properties in a parsimonious fashion, in such a way that it is possible, via observation of a limited number of housing transactions, to form accurate estimates of the behavioral probabilities (i.e. the conditional probabilities governing buyer and seller behavior). At the same time we seek flexible specifications of these beliefs so that the iterative "learning process" described below will converge to a temporary equilibrium configuration.

The temporary equilibrium is computed by solving the sellers' optimal pricing and sales strategies and buyers' optimal search and offer strategies under trial values for sellers' and buyers' beliefs. Then actual transaction prices are computed by simulating a heterogeneous population of buyers and sellers for a large housing tract (i.e. a reasonably contiguous and homogenous yet sufficiently large number of houses in an area of a city, such as the London housing market described in Merlo and Ortalo-Magné). This simulation is conducted many times for a reasonably short period of time, with enough simulated observations drawn in order to estimate the conditional probability distributions comprising sellers' and buyers' beliefs. With these revised beliefs the buyers' and sellers' problems are re-solved and new simulated data are generated. This process continues until convergence, i.e. until the input beliefs of sellers and buyers are approximately equal to the estimated probability distributions from the simulations of the model. Thus, the temporary equilibrium can be regarded as an approximate "rational expectations equilibrium" in the housing market, but allowing for the possibility of temporary imbalances between supply and demand of housing.

Although via the use of stochastic simulations it is possible for us to generate arbitrarily large simulated samples of housing transactions to enable accurate non-parametric estimation of the conditional belief probabilities, in reality, buyers and sellers may not have this capability. They must form beliefs based on a single realization of housing transaction outcomes. Real estate agents, on the other hand, have the ability to aggregate over the transactions in which they are directly involved, and to exchange information with other real estate agents. They may therefore play a key role as providers of valuable information to buyers and sellers.

4 The Seller Model

In this section, we formulate the seller's problem of how to set and revise list prices sequentially over time and how to bargain with arriving buyers as a discrete time finite-horizon dynamic programming problem. We take the decision to sell a house (via a real estate agency) as a given, and consider only the decision of which price to list the house at initially, how to revise this price over time, whether or not to accept offers that are made, and whether to withdraw the house.

We assume a 2-year horizon, so that if a house is not sold after 2 years, we assume that the house is withdrawn from sale and the seller obtains an exogenously specified "continuation value" representing the use value of owning (or renting) their home over a longer horizon beyond the 2 year decision horizon in this model. This continuation value may or may not equal the seller's belief about the "financial value" of his/her home, i.e. their expectation of what their house will sell for on the market. We summarize the seller's beliefs about the financial value of their home by the value F_t where t denotes the time period (i.e. current week in the 2 year selling horizon). We consider a formulation that allows the seller's beliefs about the financial value of their home to evolve over time in response to outcomes (rates of arrivals of offers, magnitudes of offers, etc.) from listing their home with a real estate agency. This valuation will be represented as a function of observable "hedonic" characteristics of the home (whose values we assume is commonly agreed upon by all sellers) and an additional idiosyncratic component that reflects both unobserved (to the econometrician) characteristics of a given house and also idiosyncratic variations among sellers in their personal evaluations of the financial value of their homes. Thus, we write F_t as

$$F_t = \exp\{X\beta + \nu_t\} \tag{1}$$

where X is a vector of time-invariant (at least relative to the period of time for which the house is being sold) observed characteristics of the house (number of bedrooms, baths, square feet, location/school district, etc.), β represents the commonly agreed "weighting factors" for how these various characteristics affect the financial value of a home, and ν_t is the idiosyncratic component to the seller's belief about the financial value of his/her home as well as embodying the net effect of unobserved (to the econometrician) attributes of the home and other idiosyncratic aspects of a particular house. Thus, $\exp\{X\beta\}$ represents the results of a standard "hedonic" regression that provides a basic component of the valuation of the home, and $\exp\{\nu_t\}$ represents idiosyncratic factors affecting the value of the home, causing it to be valued (or priced) below or above its "hedonic value". In our initial formulations, we will adopt a proportional specification in which in every relevant occurrence in the model, the hedonic component $\exp\{X\beta\}$ appears as a proportional factor. This will imply that the hedonic value can be "factored out." In particular, all list prices, reservation values and so forth for the seller in this model will be proportional to the hedonic value, and our model can be interpreted as providing a valuation and characterizing the optimal selling strategy for the "idiosyncratic deviations" of the value of the home from its hedonic value. This is a useful computational simplification, since it implies that it is not necessary to solve separate dynamic programs for each specific home that is listed for sale. Instead a single dynamic program can be solved for characterizing selling value and optimal listing and bargaining strategies in terms of percentage deviations from the hedonic value (whatever value it might happen to be), and then by appropriate proportional adjustment (i.e. by multiplying by the hedonic values) we can obtain prices and selling strategies for a heterogeneous set of homes. Of course, the proportionality assumption may be a strong and empirically untenable one. Until we have more direct evidence against it, the computational benefits are sufficiently compelling that we have decided to adopt it as a point of departure for our work. However the discussion below is valid whether or not we assume that the hedonic value $\exp\{X\beta\}$ has been factored out of the financial values, list prices, reservation prices, seller's optimal value function, etc.

Buyers may or may not agree with the seller's (privately held) belief about the financial value of their home. Thus, we will shortly describe "offer distributions" for the value of offers to buy the home (if made) which will depend on F_t and also on the current listing price P_t, but which will stochastically diverge from F_t and P_t reflecting the buyer's own idiosyncratic valuation of the house as well as strategic considerations about the buyer's optimal search and purchasing strategy. We will attempt to specify these arrival probabilities and offer distributions in a flexible way so that they can be regarded as "reduced forms" that are consistent with the solution to the underlying "buyer's problem" that we outline in the next section of the proposal.

Since we do not model the default option of not selling one's house beyond the relative short 2 year time interval considered in this analysis, we will simply invoke a flexible specification of the "continuation value" $V_t(F_t, \tau)$ which represents the discounted expected utility (in monetary equivalent units) of owning a home and not selling it within the 2 year window considered in this model. We assume that if the seller has not sold by the terminal period T they obtain the continuation value $V_T(F_T,\tau)$ which depends on — but does not necessarily equal — the seller's terminal belief about the financial value of their home, F_T^s. Since F_s^T reflects both observed and unobserved attributes of the home, it is convenient to allow the continuation value to depend on F_T so it also reflects those attributes, but we also allow the continuation value to depend on utility function parameters τ and some or all of the τ parameters can be treated as unobserved heterogeneity. For example, a simple specification for the continuation might be $V_t(F_t,\tau) = h_t(\tau)F_t$ where $h(\tau)$ is some function of the utility function parameters. If $h_t(\tau) > 1$, the seller's continuation value for not selling their home is greater than their estimate of its financial value, and if $h_t(\tau) < 1$, the seller's continuation value is less than their estimate of its financial value. In general we expect $V_t(F_t,\tau) < F_t$, but it is possible that a seller would still decide to sell their house even if the opposite inequality holds. This is because of randomness and skewness in buyer valuations, which might create a sufficiently high option value that makes it worthwhile for the seller to try to sell their home in hopes that there might be a buyer willing to pay much more than F_t for the home even though they believe that "on average" the (maximum) amount they would be willing to pay to own the home, $W_t(V_t,\tau)$ exceeds their estimate of what they could sell it to a typical buyer, F_t.

We allow a seller to decide to withdraw their home from being listed, which we assume is the same as deciding not to sell it, since we do not at this point allow a "for sale by owner" decision, or any choice about selling through different real-estate agents. Although for our first dataset described in Section 3 we do not observe withdrawals, we do in our second data set.

Thus, the seller will have 3 main decisions: 1) whether or not to withdraw the property, 2) if sell, how much to set the listing price P_t at each period, and 3) if an offer arrives at price O_t, whether or not to accept it. We assume that the first two decisions are made at the start of each week and the seller precommitts to them for the rest of the week. Within the week, if one or more offers arrive, the seller can engage in bargaining with the prospective buyer(s). The state variables in the model are 1) the listing price set in the previous week, P_t, 2) the history of offers H_t up to start of week t (this is a vector that can include a number of different pieces of information, but we have in mind at least a) the duration since last offer, and b) the highest offer received so far), and 3) the seller's current estimate of the financial value of their home, F_t. Let $S_t(P_t,F_t,H_t)$ denote the maximum expected present discounted value of an optimal selling strategy. We have

$$S_t(P_t,F_t,H_t) = \max\left[V_t(F_t,\tau), \max_P\left[u_t(\tau,P,F_t,H_t) + \beta ES_{t+1}(P,F_t,H_t)\right]\right] \quad (2)$$

The Bellman equation says that at each week t, the optimal selling strategy involves choosing the larger of 1) the continuation value of (permanently) withdrawing the home from the market, 2) or continuing to sell, choosing an optimal listing price P. The function $ES_{t+1}(P,F_t,H_t)$ is the conditional expectation of the week $t+1$ value function S_{t+1} conditional on the current state variables (F_t,H_t) and the listing prince P that is in effect during week t. The function $u_t(\tau,P,F_t,H_t)$ represents the current week "holding cost" to the seller of having their home on the market. It is the net utility (in money equivalent units) of the use value of owning the home less the "hassle costs" of having to show the house to prospective buyers.

We do not have space to go into detail to describe the equation for ES_{t+1}, but we will describe briefly the way we model the seller's beliefs about the arrival of offers from buyers, the distribution of the size of the offer, and the probability that the buyer(s) will "walk" (i.e. not make a counter offer and search for other houses) if the seller rejects the buyer(s) offer(s). These components are used to construct a formula for ES_{t+1}. Following Merlo and Ortalo-Magné (2004) we assume that the seller's only bargaining decision is to accept or reject offers made by buyers: the seller does not make a price "counter offer" if he/she rejects the buyer(s)' offer. We assume that within a given week there are at most 4 possible stages of offers and accept/reject decisions by buyers and the the seller. Below, to keep notation simpler, we write ES_{t+1} for the case where only a maximum of two offers and accept/reject decisions on the part of the seller are allowed to occur within the week.

Let $\lambda_t(n|P,V_t,H_t)$ denote the conditional probability that n offers will arrive within a week, where $n = \{0,1,2+\}$; i.e. we do not initially discriminate among the realized number of buyers who are competing in the "auction case" when more than 1 buyer makes an offer in a given week. Let O_j be the highest offer received at stage j of the bargaining process, (i.e. if more than 1 buyer makes an offer, then O_j is the highest of the offers made). Then we let $f_j(O_j|n_t,O_{j-1},P,V_t,H_t)$ denote the seller's beliefs of the conditional probability of the highest offer at stage j given the number of offers received at week t, n_t, and the highest offer at stage $j-1$, O_{j-1} (if $j=1$ then O_0 is treated as a null set, since there were no previous offers in the week prior to the first offer). So if one or more buyers make an offer, the highest offer is a draw from this conditional distribution and the seller must decide whether or not to accept it. If an offer O_j is accepted, the function $N_t(O_j)$ denotes the net sales proceeds (net of real estate commissions, taxes, and other transactions costs) received by the seller. The seller must decide whether to accept the net proceeds $N_t(O_j)$, selling the home, or reject the offer and hope the buyer(s) submit a more attractive counter offer, or hope that some better offer arrives in some future week.

If a seller rejects the offer O_j, there is a probability $\omega_j(O_j|O_{j-1},n,P,V_t,H_t)$ that the buyer(s) will "walk" and not make a counter offer as a function of the last two rejected offers, O_j and O_{j-1}, the total number of offers, n, and the current state (P,V_t,H_t). We assume that if there is more than 1 buyer who makes an offer and if the seller rejects the highest of the buyers' offer, either all buyers will stay in to make a counteroffer or all will walk, i.e. we do not consider the combinatorially more complex case where some buyers walk and other buyers stay in.

Finally, let $\pi_t(\nu_{t+1}|n_t,O_j,\nu_t,P,H_t)$ denote the transition probability for the idiosyncratic, time-varying component of the seller's financial valuation of their house. This transition probability depends on the number of offers received, n_t, the last offer received, O_j where j indexes the j^{th} offer that the seller receives from buyers (after rejecting the $j-1$ previous offers), the value of the seller's idiosyncratic "valuation shock" ν_t at the start of the week, the listing price posted at the start of the week, P, and the history H_t at the start of the week. The π_t transition probability captures the "learning effects" of actual experience in selling the home on the sequential revision of the seller's beliefs of the financial value of his/her home. In particular, it can allow us to capture a seller who is initially overly optimistic about the financial value of their home (i.e. their initial draw ν_0 is too high) and that subsequent experience with long periods of no offers or low offers can cause the seller to revise downward their estimate of the financial value of their home. Conversely, positive experience in the market could lead a more pessimistic seller to revise upward their estimate of the financial value of their home. Thus, we believe we can capture many of the features of more complicated Bayesian learning models and equilibrium bargaining models in a simpler "reduced form" approach that may nonetheless be consistent with a fully Bayesian game-theoretic formulation of the bargaining problem.

The last bit of notation is a law of motion for updating the history state vector, based on the bargaining outcomes within the week. We write a generic updating rule of the form

$$H_{t+1} = \Gamma(n_t, O_{j_t}, H_t) \tag{3}$$

which is appropriate in the case where H_t consists of two pieces of information, 1) the duration since last offer, and 2) the last offer received in week t, where j_t is the number of offers made and thus, O_{j_t} is the final offer. In this case the updating rule Γ is obvious: if $n_t = 0$, then the duration since last offer is incremented by 1, otherwise it is reset to 0, and if $n_t = 0$, then the maximum offer received so far is unchanged, whereas if $n_t > 0$ the maximum offer received so far is the maximum of the maximum offer received prior to week t and O_{j_t}.

The optimal selling strategy resulting from the solution to this dynamic program will consist of history dependent functions for determining the listing price at the beginning of each week, and the seller's reservation values for the within-week bargaining problem. We have programmed the seller's problem using some hypothetical assumptions about the financial value, arrival rates, seller valuations, and so forth. We do not have space to go into detail about all of these assumptions here, but it is useful to illustrate the rich types of behavior that result from simulations of the seller's model in figure 1 below.

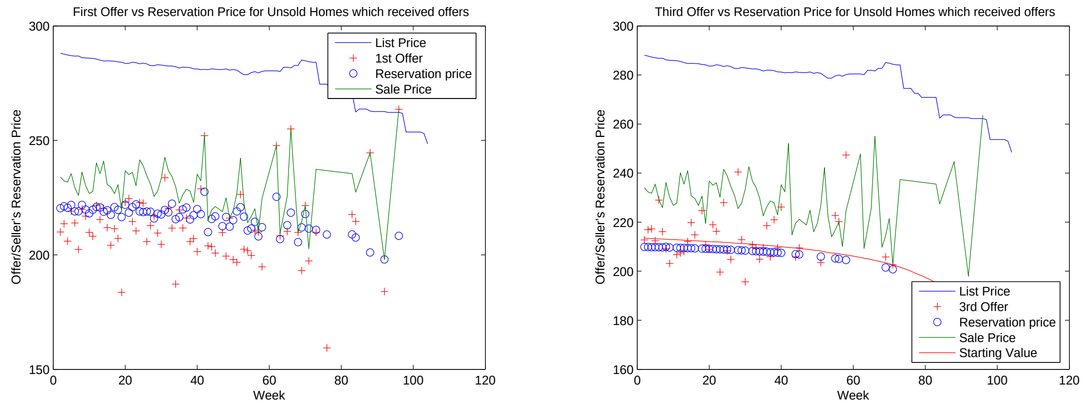

Figure 1: Simulations of Optimal Selling Strategy (500 Sellers)

These figures illustrate the results of 500 simulations of the solution to the dynamic programming for 500 *ex ante* identical sellers whose financial value for their homes is initially $F = 200$ (units are in thousands of dollars). According to our calculations, the optimal initial listing price is $P_1 = 288$. The top blue line in both panels of figure 1 is the average listing price for sellers who have not yet sold their homes. We see that over time, the mean list price decreases relatively slowly until approximately 75 weeks, at which time there is a significant drop in list prices, followed by additional large drops at 85 and 95 weeks after initial listing. By the end of the selling horizon (i.e. at 104 weeks) homes that have not been withdrawn are listed at just under $250, or about 15% less than their initial list price.

Thus, these initial simulations match a key stylized fact of the empirical analysis of list price revisions in Merlo and Ortalo-Magne (2004), i.e. that listing price reductions are fairly infrequent,but when they occur they are typically large. Our simulation results were based on a solution to seller's problem where the fixed cost of changing list prices is quite small, $15 per list price change. It is quite encouraging that this model can already match a key stylized fact in the data without resorting to high fixed "menu costs" since it is, in fact, quite inexpensive to revise listing prices at the London real estate agencies where prices are typically posted on a bulletin board rather than advertised in the newspaper. An interesting feature of the model is that "early" revisions in list prices (i.e. within the first year) are typically in response to buyers who "walk" in response to the seller's rejection of their bid. However later revisions in list prices (i.e. after the first year the home has been on the market) are typically "spontaneous", i.e. not in response to a buyer walking, but rather an attempt to lower price in response to a lack of arriving buyers.

The blue circles in the left hand panel of figure 1 represent the average reservation price of sellers for the first offer which arrives in any given week. These reservation values are significantly lower than the list price and, for the solution that we computed, the reservation fall monotonically in each round of bargaining with the week. As one would expect, reservation values are always higher in the case where 2 or more buyers are making offers (an "auction situation") compared to the case where only a single buyer is making offers. The jagged green line in both panels of figure 1 represents the average selling price of houses that sold in each week. The $+$ symbols in the left hand panel are the average first offers received. We see that a majority of first offers are below the blue reservation price circles, so sellers generally rejected the first offers in these simulations. The right hand panel plots the third offers received (once again denoted by $+$ symbols) and we see that by the 3rd round of bargaining, buyers' offers have increased and sellers' reservation values have decreased to the extent that most of the third round offers exceed the sellers' reservation values, and hence were accepted by the sellers. We see that at the third stage, the sellers' reservation values are close to their beginning of period expected value functions: in fact the final reservation value equal's the seller's expected value function at the start of the *subsequent* week.

In these simulations, the mean sales price was $231, and the mean time to sale was 20.2 weeks after the initial list. Of the 500 sellers simulated, 493 of them were successful in selling their homes with 104 weeks from initial listing. A total of 217 of the 493 transactions, or 44%, resulted from immediate acceptances, and

428 or the 493 homes sold were sold to the first buyer (or highest bidding buyer in case of multiple bidders) who made an offer on the home. These numbers are roughly consistent with the observed rates of acceptance found in the study by Merlo and Ortalo-Magne (2004).

While we do not claim that this initial solution and simulation of the model is fully realistic in all details, it does at least qualitatively reflect many of the stylized facts about bargaining over residential real estate from the analysis of Merlo and Ortalo-Magné (2004). These initial results make us optimistic that our approach to modeling bargaining and endogenous price determination in residential real estate markets is computationally feasible and is potentially capable of providing a good explanation of the behavior we observe in these markets.

5 The Buyer Model

In this section, we describe the decision problem faced by a potential buyer in the housing market. The buyer wants to buy a home if the net utility from buying exceeds the utility from the next best alternative (i.e. either renting, or searching and buying in some other market). Let the reservation value of not buying be given by A_0. We assume that this is fixed for a given buyer, but there is a distribution of such reservation values for new buyers entering the housing market.

For each property on the market, we represent the buyer's valuation B as a function of observable "hedonic" characteristics of the home (whose values we assume is commonly agreed upon by all buyers) and an additional idiosyncratic component that reflects both unobserved (to the econometrician) characteristics of a given house and also idiosyncratic variations among buyers in their personal evaluations of the value of the home. Thus, we write B as

$$B = \exp\{X\gamma + \mu\} \tag{4}$$

where X is a vector of time-invariant (at least relative to the period of time for which the house is being sold) observed characteristics of the house (number of bedrooms, baths, square feet, location/school district, etc.), γ represents the commonly agreed "weighting factors" for how these various characteristics affect the financial value of a home, and μ is the idiosyncratic component to the buyer's belief about the value of the home as well as embodying the net effect of unobserved (to the econometrician) attributes of the home and other idiosyncratic aspects of a particular house.

A buyer may or may not agree with the seller's (privately held) belief about the financial value of their home. Thus, for each offer the buyer may choose to make on a property we will define a probability that the seller may accept the offer. This probability will in general depend on the time the home has been on the market and the current listing price P_t, which are both observable, but will also reflect the seller's own idiosyncratic valuation of the house (which is part of the seller's private information), as well as strategic considerations about the seller's optimal selling strategy and the seller's beliefs about the buyers' side of the market. We will attempt to specify these probabilities in a flexible way so that they can be regarded as "reduced forms" that are consistent with the solution to the underlying "seller's problem" that we outlined in the previous section of the proposal.

We assume that a potential buyer instantaneously learns his idiosyncratic component of his valuation of a home μ upon visiting the home, and this value remains fixed thereafter. However the buyer knows the distribution from which μ is drawn, and this implies that the buyer faces a search problem when it comes to buying a home. The buyers problem is to decide which homes to visit in a given period, and for any of these homes, whether or not to make an offer.

At the start of week t the buyer enters the market and observes the entire set of houses available for sale, their list prices, the time each home has been on the market, and the vector of observable characteristics of each house. The expected value to the buyer from visiting a particular home depends on the buyers expectation of the outcome of the negotiation process if the buyer were to decide to make an offer on the house, and includes the "option value" of continuing to search if the offer is not accepted or if the buyer decided after observing μ not to make an offer on the house in the first place.

We assume that the buyers reservation value A_0 is smaller than the ex ante expected gain from visiting the most promising home listed for sale in the market. Also, among the buyers who are still on the market at the start of time t, we assume that they will visit the most promising home first, i.e the home for which their expected gain from visiting is the largest. When the buyer visits the home, the buyer incurs a search cost c but learns the idiosyncratic component of the value of owning the home μ. Given this valuation, if the buyer were to pay the listing price P_t, the buyers net gain from purchasing the home would be $B - P_t$. However, the buyer may not necessarily want to make an offer equal to the listing price. The buyer may decide to make a lower offer, or sometimes in very "hot" housing markets where there are many buyers competing for a limited housing stock, he may decide to make an offer that exceeds the listing price in order to beat out any competing bidders who may be simultaneously placing offers on the house. Let O denote the offer price submitted by the buyer, and let $V(O,t,P_t,B,A_0)$ denote the expected discounted payoff from the "bargaining continuation game" when the buyer submits an opening offer of O.

Thus, after visiting a house for sale and learning its idiosyncratic component of the value, μ, the buyer can either make an offer or decide to move on, and choose the next most promising home that is available for sale in the real estate listings. If the buyer decides to continue to search, the utility of searching the next most promising home for sale must exceed the reservation value A_0, otherwise the buyer will exit the market at this point as a "discouraged buyer."

Now consider the decision problem determining the buyer's expected utility from the bargaining continuation game. If the buyer submits an initial price offer O_1, the buyer believes that the seller will accept this offer with some probability $g_1(O_1|t,P_t,B)$. If this initial offer is rejected, the buyer can decide whether or not to make another offer, O_2 or walk away. If he makes a second offer, there will be a probability $g_2(O_2|O_1,t,P_t,B)$ that the seller will accept it, given that she rejected the previous offer of O_1 and so forth. Consistent with our specification of the seller's model we allow buyers to submit up to 4 offers on a given home. This situation defines a finite-horizon optimization problem whose solution generates optimal buyers' price offer sequences. On the other hand, in the case of an auction situation (i.e. where there are multiple potential buyers submitting bids on a given house), we assume that the buyers simultaneously submit their bids and there is a probability the seller will accept the highest offer submitted or allow for one more round of bidding. Initially, we assume that when a buyer makes his initial offer, he is unaware of whether other buyers may have also made an offer on the same home.

6 The Bargaining Model

In the two previous sections, we have described models of the seller's and the buyer's problems in general environments with two-sided incomplete information. We have formulated each problem separately, where each player treats the other side as parametric, although their beliefs are "approximately correct." In this section, we introduce a simple bargaining model with one-sided incomplete information where buyers' offer sequences and the sequence of a seller's listing and reservation prices are simultaneously determined in equilibrium, and beliefs are consistent. This model can also "explain" the main empirical findings of Merlo and Ortalo-Magné (2004). An important goal of our research is to estimate such alternative models of the players involved in the housing transaction process and compare their relative performance in fitting the data.

Suppose that the seller values her house $R + \sigma$, where R denotes the (known) rental value of the house and $\sigma \sim F$, $\sigma \in [0, \overline{\sigma}]$, is the seller's idiosyncratic valuation of the house in excess of its rental value (which is privately known by the seller). If she sells at price p at time $t = 0, ..., T$ (that is, within T periods of putting her house on the market), the seller's payoff is p; it is $R + \sigma$ otherwise. When she first puts her house on the market, the seller sets an initial listing price $R + p_L^0$, $p_L^0 \in [0, \overline{\sigma}]$, that she can then change over time. We let p_L^t denote the listing price at time $t = 0, ..., T$. The population of potential buyers of the house is characterized by a distribution of valuations $R + \beta$, where $\beta \sim G$, $\beta \in [0, \overline{\beta}]$, $\overline{\beta} \geq \overline{\sigma}$, denotes a potential buyer's idiosyncratic valuation of the house in excess of its rental value.

The aggregate conditions of the housing market can be in one of two possible states of the world, $H \in \{l, h\}$. In state $H = l$, the housing market is "slow" and $\overline{\sigma} = \sigma_l = \alpha_l \overline{\beta}$, $\alpha_l < 1$ (that is, it is a buyer's market), while in state $H = h$, the housing market is "booming" and $\overline{\sigma} = \sigma_h = \alpha_h \overline{\beta}$, $\alpha_l < \alpha_h < 1$ (that is, it is a seller's market). While the potential buyers know the state of the world (they have a global view of the market), the seller may be uncertain about it (she has only a partial view of the market). We let ϕ denote the probability a seller with valuation $\sigma \in \left[0, \alpha_l \overline{\beta}\right]$ does not know the state of the world and $\beta(d,q)$ denotes the prior beliefs of an uncertain seller about the probability that the state of the world is $H = h$. Let γ^t denote the seller's expected probability that the state of the world is $H = h$ given her beliefs at time t. (Note that if $\sigma \in (\alpha_l \overline{\beta}, \alpha_h \overline{\beta}]$, the seller knows that the state of the world is $H = h$).

If the state of the world is $H = l$ and the seller lists her house at a price $p_L^t > \sigma_l$, the buyers will know that the seller is lying about her valuation and will not make offers on her house (i.e., the rate of arrival of potential buyers is equal to zero). If, on the other hand, the seller lists her house at a price p_L^t that is smaller than or equal to the maximum potential seller's valuation conditional on the correct state of the world, the buyers will not know whether the seller is lying or not and in each period $t = 0, ..., T$, a potential buyer with valuation drawn from the distribution $G(\beta)$ will arrive with probability θ. If in any given period a potential buyer arrives (that is, a match occurs), the buyer and seller engage in a within-period, finite-horizon bargaining game with one-sided incomplete information and one-sided offers.

Suppose that the valuation of the potential buyer $R+b$ (where b denotes a realization of β) is public information within the match. The potential buyer, however, does not know the seller's valuation. He only knows the listing price and the (equilibrium) distribution of possible seller's valuations. The potential buyer and the seller play a finite-horizon bargaining game where the uninformed party, the buyer, makes all the offers. More specifically, when the buyer makes a first offer, p_1, the seller accepts or rejects. If the seller rejects, with probability w, the match is broken. With probability $1-w$, the buyer gets a chance to make a second offer, p_2, and so on. We let $k \geq 2$ denote the maximum number of offers in the bargaining game. If the match is successful, the buyer's payoff is $(R+b)-p$. If the match is not successful, the buyer gets a payoff of 0.

Given this specification, we can normalize R to 0 (that is, we can solve the game ignoring the additive component R and then add it back at the end to the equilibrium price offers). Also, given the component of the initial listing price in excess of the rental value of the house p_L^0, we can divide σ, β, $p_1, ..., p_k$, p and p_L^t by p_L^0 and express valuations, buyer's price offers, and listing price changes as percentages of the difference between the initial listing price and the rental value of the house. Hence, $S = \sigma/p_L^0 \in [0,1]$, $B = \beta/p_L^0 \in [0, \overline{B}]$ where $\overline{B} = \overline{\beta}/p_L^0 > 1$, $P_j = p_j/p_L^0 \in [0,1]$, $j = 1, ..., k$, $P = p/p_L^0 \in [0,1]$, and $P_L^t = p_L^t/p_L^0 \in [0,1]$ where $P_L^0 = 1$.

The model described here has a unique perfect Bayesian equilibrium, which, for each period $t = 0, ..., T$, specifies the seller's choice of a listing price. Furthermore, for each period where a match occurs, and for each possible buyer's valuation, the equilibrium characterizes the sequence of offers by the buyer and the corresponding sequence of lowest seller's valuations such that the seller accepts a given offer. To solve for the equilibrium, we first characterize the equilibrium of the bargaining game in the event that a match occurs. We then characterize the optimal choice of the listing price by the seller. Since the time horizon of the seller is finite, we characterize the equilibrium by backward induction starting from the last period T.

Suppose that a buyer with valuation B arrives in period T, and the maximum number of offers within a negotiation is $k = 3$. Since T is the last period, if a sale does not occur, the seller's continuation value is equal to S, and the equilibrium of the bargaining game is characterized by critical levels of buyers' valuations, $\widetilde{B}(w)$, $\widetilde{B}'(w)$ and $\widetilde{B}''(w)$, buyers' price offer sequences, $\widetilde{P} = (\widetilde{P}_1(B,w), \widetilde{P}_2(B,w), \widetilde{P}_3(B,w))$, $\widetilde{P}' = (\widetilde{P}_1'(B,w), \widetilde{P}_2'(B,w), 1)$, $\widetilde{P}'' = (\widetilde{P}_1''(B,w), 1, 1)$, and $(1,1,1)$, and thresholds on the seller's valuation, $s(B,w)$, $s'(B,w)$, $\widehat{s}(B,w)$, $\widehat{s}'(B,w)$, and $\widehat{s}''(B,w)$ such that: If $0 \leq B < \widetilde{B}(w)$, the buyer offers the price offer sequence $\widetilde{P}$, and the seller accepts the first offer if $S \leq s(B,w)$, accepts the second offer (if the game reaches that stage) if $s(B,w) < S \leq s'(B,w)$), accepts the third offer (if the game reaches that stage) if $s'(B,w) < S \leq \widetilde{P}_3(B,w)$, and rejects all offers otherwise; If $\widetilde{B}(w) \leq B < \widetilde{B}'(w)$, the buyer offers the price offer sequence $\widetilde{P}'$, and the seller accepts the first offer if $S \leq \widehat{s}(B,w)$ accepts the second offer (if the game reaches that stage) if $\widehat{s}(B,w) < S \leq \widehat{s}'(B,w)$, and accepts the third offer (if the game reaches that stage); If $\widetilde{B}'(w) \leq B < \widetilde{B}''(w)$, the buyer offers the price offer sequence $\widetilde{P}''$, and the seller accepts the first offer if $S \leq \widehat{s}''(w)$ and accepts the second offer (if the game reaches that stage); Finally, if $B \geq \widetilde{B}''(w)$, the buyer offers the price sequence $(1,1,1)$ and the seller accepts the first offer. Recall that given our normalization, an offer of 1 corresponds to an offer at the current listing price. Hence, the equilibrium characterization yields increasing sequences of buyers' offers and seller's reservation prices, and admits the possibility of prolonged as well as unsuccessful negotiations.

Now consider the beginning of period T (or the end of period $t = T-1$), where the seller has to decide whether to revise her listing price. Note that, given the equilibrium characterization of the bargaining game, if a potential buyer arrives in the last period, the seller's expected payoff from bargaining is

$$
\begin{aligned}
& W\left(S,T,p_L^T\right) \\
= & \left\{ \int_0^{\widetilde{B}(w)} \left[\begin{array}{c} 1\{S \leq s(B,w)\}\widetilde{P}_1(B,w) + 1\{s(B,w) < S \leq s'(B,w)\}\left[(1-w)\widetilde{P}_2(B,w)+wS\right] + \\ 1\left\{s'(B,w) < S \leq \widetilde{P}_3(B,w)\right\}\left[(1-w)^2\widetilde{P}_3(B,w) + \left(1-(1-w)^2\right)S\right] + \\ 1\left\{S > \widetilde{P}_3(B,w)\right\}S \end{array} \right] dG(B) \right. \\
& + \int_{\widetilde{B}(w)}^{\widetilde{B}'(w)} \left[\begin{array}{c} 1\{S \leq \widehat{s}(B,w)\}\widetilde{P}'_1(B,w) + 1\{\widehat{s}(B,w) < S \leq \widehat{s}'(B,w)\}\left[(1-w)\widetilde{P}'_2(B,w)+wS\right] + \\ 1\{S \geq \widehat{s}'(B,w)\}\left[(1-w)^2 + \left(1-(1-w)^2\right)S\right] \end{array} \right] dG(B) \\
& \left. + \int_{\widetilde{B}'(w)}^{\widetilde{B}''(w)} \left[\begin{array}{c} 1\{S \leq \widehat{s}''(B,w)\}\widetilde{P}''_1(B,w) + \\ 1\{S \geq \widehat{s}''(B,w)\}[(1-w)+wS] \end{array} \right] dG(B) + \int_{\widetilde{B}''(w)}^{\overline{B}} [1]\, dG(B) \right\} p_L^T
\end{aligned}
$$

which is increasing in p_L^T (this is true for all $t = 0,...,T$). Hence, the seller's listing price at any time will be either $\alpha_l\overline{\beta}$ or $\alpha_h\overline{\beta}$.

If the seller knows the state of the world, she will choose $p_L^T = \alpha_l\overline{\beta}$ in state $H = l$ and $p_L^T = \alpha_h\overline{\beta}$ in state $H = h$ (that is, the seller chooses a listing price equal to the upper bound of the support of the seller's valuations in the realized state of the world). If the seller's valuation is $\sigma \in \left[0,\alpha_l\overline{\beta}\right]$ and she does not know the state of the world, her expected (continuation) payoff if she chooses $p_L^T = \alpha_l\overline{\beta}$ is $\theta W\left(S,T,\alpha_l\overline{\beta}\right) + (1-\theta)S\alpha_l\overline{\beta}$, while if she chooses $p_L^T = \alpha_h\overline{\beta}$ it is $\gamma^T\left[\theta W\left(S,T,\alpha_h\overline{\beta}\right) + (1-\theta)S\alpha_h\overline{\beta}\right] + \left(1-\gamma^T\right)S\alpha_l\overline{\beta}$. Clearly, the choice of the listing price for an uncertain seller is non trivial only in the case where the current listing price is $\alpha_h\overline{\beta}$ and the seller was never matched with a potential buyer (if a buyer arrived at any period, the seller would know that the state is $H = h$ and hence would not revise her listing price). If this is the case, if γ^T is relatively small, the seller will revise her listing price from $p_L^{T-1} = \alpha_h\overline{\beta}$ to $p_L^T = \alpha_l\overline{\beta}$. If, on the other hand, γ^T is relatively large, the seller will not revise her listing price.

By letting $v(S,T,p_L^T)$ denote the seller's expected (continuation) payoff given her optimal choice of the listing price, we now move back one period (to $t = T-1$) and repeat the previous analysis (starting with the solution of the bargaining game conditional on a match occurring) except that we replace S with $v(S,T,p_L^T)$ and we need to derive the distribution function of $v(S,T,p_L^T)$. We then move backwards to period $t = T-2$ and so on up to the initial period $t = 0$. When doing so, note that the upper bound of the distribution of seller's valuations remains the same (normalized to 1 in the solution of the bargaining game). However, the lower bound is no longer 0 but instead is $v(0,T,p_L^T)$. This implies that buyers with $\beta < v(0,T,p_L^T)$ will not make offers on the house. In general, as we move back to earlier periods, the lower bound will be larger (the sellers are more picky and bunched closer to the listing price).

With respect to the equilibrium initial listing price, since the seller's (expected) payoff in the bargaining game is increasing in the listing price, the seller will choose either $p_L^0 = \alpha_l\overline{\beta}$ or $p_L^0 = \alpha_h\overline{\beta}$. If the seller knows the state of the world, she will choose $p_L^0 = \alpha_l\overline{\beta}$ in state $H = l$ and $p_L^0 = \alpha_h\overline{\beta}$ in state $H = h$ and never revise it. If the seller does not know the state of the world, if her beliefs about the probability that the state of the world is $H = h$ are relatively "optimistic" she will choose $p_L^0 = \alpha_h\overline{\beta}$. However, if she gets a long enough sequence of periods where no buyer arrives, she will update her beliefs downward and she may eventually switch her listing price to $p_L^t = \alpha_l\overline{\beta}$ at some $t = 1, ..., T$. If on the other hand, a buyer arrives, she will instantaneously learn that the state of the world is $H = h$ and will therefore never revise her listing price. Note that if the seller's beliefs about the probability that the state of the world is $H = h$ are relatively "pessimistic" she will choose $p_L^0 = \alpha_l\overline{\beta}$ and will never revise her listing price since learning will not occur (if $p_L^0 = \alpha_l\overline{\beta}$ the arrival rate of buyers is equal to θ regardless of what the state of the world is).

7 Modeling Real Estate Intermediaries

So far, our discussion has concentrated on providing relatively detailed models of the two key agents in the real estate market: buyers and sellers. While the models discussed so far do not yet have an explicit treatment of real estate agents, when these models are more fully developed, we believe it will be feasible to take the next step and construct explicit models of real estate intermediaries. In particular, it will be possible to extend these models to allow for an endogenous choice of whether to sell a home with the assistance of a real estate agent, versus "for sale by owner."

The first step is to identify the "service" provided by real estate agents. There are a number of possible services that real estate agents provide that have already been discussed in the formulation of our models: 1) the real estate agent has the ability to include a seller's home in a large database of homes for sale, called the *multiple listing service,* which in conjunction with other types of advertising, can help to increase the rate of arrival of potential buyers, and/or result in arrivals of buyers who are better informed and thus better "matches" for the seller's home, resulting in a higher rate of offers, or higher offers or both, 2) the real estate agent is an expert on conditions in the local housing market, and can therefore "aggregate information" and provide useful information advice to a seller about what list prices are appropriate, what arrival rates of buyers to expect, and what price the seller can expect to receive from sale of their home. From the standpoint of a buyers, real estate agents also provide valuable services via their access to the multiple listing service, by helping buyers to direct their search more efficiently to the homes in their desired location, style and price range.

We believe it is possible to quantify the value of the services provided by real estate agents in terms of the model we have already presented, i.e. to quantify the impact of a real estate agent on buyer arrival rates, on offer distributions, and so forth. If it is possible to identify the "causal effects" i.e how a real estate agent affects arrival rates, offer distributions and so forth, then it is possible to estimate the value of the real estate services using our modeling approach. In particular, since we also know the cost of a real estate agent (i.e. the real estate commission is typically 6% in the U.S., and when a sale occurs between a buyer and seller who are both represented by their own real estate agents, this commission is split 50/50 between the buying and selling agents), we can model buyers' and sellers' endogenous choice of whether to use real estate agents. In particular, a seller will list their home with a real estate agency only if the increase in the expected selling price due to higher arrival rates of buyers and/or higher offers due to improved matches and/or better advice to the seller on selling strategy, exceeds the 6% commission.

There are difficult economic issues arising from the possibility of multiple equilibria when real estate agents are introduced, and related econometric issues of how to infer "causality" given the patterns of self-selection in the decision of whether or not to use a real estate agent. In particular, until recently, the vast majority of homes have been sold via real estate agencies and the homes that are sold by owner may not be typical of the overall population of homes for sale. It might be the case that beliefs about the power of real estate agents are "self-confirming." That is, if sellers believe that they are unlikely to be successful unless they hire a real estate agent, then they will do so and most of the "good homes" will be listed with real estate agents. If buyers also believe that they will not be able to find out about good homes unless they search the multiple listing service, then the sellers' beliefs will be confirmed and the vast majority of transactions will be intermediated by real estate agents. However it could be possible that there is an alternative equilibrium where a significant share of houses are for sale by owner, and that search via avenues outside the multiple listing service (e.g. the classified ads, or independent internet posting agencies), could result in a very different equilibrium in the real estate market with a far lower share of housing transactions handled by traditional real estate agents.

We believe that this alternative equilibrium may be starting to take hold in Madison Wisconsin, where the agency `fsbomadison.com` (an agency which is currently providing us with listing and transaction data, as noted above) has already gained a significant market share and is growing rapidly at the expense of traditional real estate agencies. We are seeking comparable data from traditional real estate agencies in Madison, and believe that the entry of `fsbomadison.com` may serve an "instrument" that will allow us to infer how these alternative intermediaries affect arrival rates and offer distributions, providing a more solid grounding for making casual inferences and whether multiple equilibria is an empirically realistic possibility.

We will also study issues related to the entry and exit of real estate agents and whether there is credible evidence of entry deterrence and collusion by existing real estate agencies by virtue of their control of the multiple listing service — allegations in the recent U.S. Department of Justice suit against the American Association of Realtors. To the extent that acquiring and maintaining the listing information in the multiple listing service is a costly activity, there may be a valid argument that existing real estate agencies have property rights over this information, and that requiring that this information be provided at no cost to competitors could lead to suboptimal outcome if these intermediaries lose the incentive to acquire and disseminate this valuable information as a result of unwise court decision and/or regulation of this market. We will attempt to characterize the socially optimal configuration of the real estate market, taking into account the costs of collecting and disseminating information and use this as a benchmarket against which to evaluate the efficiency existing or *status quo* equilibrium in the housing market (including modeling presumed monopoly control of the multiple listing service by collusive real estate agents), versus alternative forms of organization of the real estate market including alternative types of contracts for providing real estate services (e.g. instead of commission based pricing, we could study markets based on "fee for service" contracts, similar to that provided by `fsbomadison.com` for listing homes on their website, or buyers paying real estate on an hourly basis for their expert assistance in searching the real estate market).

While our research to study the role of real estate agents, modeling entry and exit of these agents (and potential collusion and exclusionary tactics of existing agents), and considering alternative structures for the real estate market is necessarily highly speculative and not well developed at this point, we believe that providing a rigorous microfoundation for the real estate market by carefully studying the behavior of the two key agents in this market, buyers and sellers, will provide a solid basis on which we can build. We also believe that we have assembled some of the best possible data for which to study these issues, and as we noted above, the importance and practical relevance of having an improved understanding of the operation of residential real estate market cannot be underestimated.