Which Structural Parameters Are "Structural"? Identifying the Sources of Instabilities in Structural Models

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Abstract

The objective of this paper is to identify which parameters of a structural model are stable over time. Existing works only test whether a given subset of the parameters is stable over time, but cannot be used to find which subset of parameters is stable; our procedure instead is informative regarding the nature of instabilities affecting macroeconomic models, and sheds light on the economic interpretation and causes of such instabilities. It provides clear guidelines on which parts of the model are reliable for policy analysis and which are possibly misspecified. Our empirical findings mainly suggest that instabilities are concentrated in Euler and IS equations and monetary policy reaction functions, but that the Phillips curve is stable. Such results offer important insights to guide the future theoretical development of macroeconomic models.

PRELIMINARY AND INCOMPLETE

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1 Introduction

The objective of this paper is to identify which parameters of a structural model are stable over time. This is an important question, as one of the main advantages of structural models is to offer a framework to qualitatively evaluate the effects of economic policies without being subject to the Lucas' critique. However, such experiments make sense only if the parameters of the model are constant over time: parameter instabilities are a signal of possible model misspecification, which remains a concern for researchers estimating Dynamic Stochastic General Equilibrium (DSGE) models (see Del Negro, Schorfheide, Smets and Wouters, 2004), and, if neglected, render the structural model untrustworthy to evaluate the consequences of alternative policies. But, among the components of the rich structure of DSGE models, which are the components that are stable and which are unstable?[1]

This paper considers a representative New Keynesian model that has the basic features of the models now becoming popular in central banks and academia (Ireland, 2007), and asks the substantive question: "Which parameters are stable?". An answer to this question provides important information for both empirical and theoretical researchers regarding which parts of the model rely on stable parameters and which parts don't. The former are exactly the features of the model that policy-makers can rely upon when doing policy evaluation, and the latter are those that could possibly be mis-specified and therefore require further theoretical modeling efforts. Our empirical results strikingly show that the parameters of the standard Phillips curve equation are structurally stable, whereas the parameters in the Euler equation for consumption and those in the monetary policy reaction function are not. While the fact that the parameters in the monetary policy reaction function are unstable is well-known, the finding that, in a prototypical structural New Keynesian model parameter instability also affects the parameters in the Euler and IS equations (and only those) is new. This provides important and useful guidelines for future research dealing with the issue of modeling structural macroeconomic models.

Since there are no appropriate econometric techniques to address this issue, another substantial contribution of this paper is to propose a new methodology for identifying the subset of structural parameters of a model (or of other aspects of the data) that are stable among the set of model's parameters. Our method has the advantage of identifying which "blocks" of the model contain parameters that are "structurally invariant", and which are

[1] This question was raised by both Sims (2001) and Stock (2001) in their comment to the Cogley and Sargent (2001) paper, and eventually addressed by Cogley and Sargent (2007a,b).

not, and therefore provide directions as to which parts of the structural model should be modified in order to build a structural model whose deep parameter are time-invariant. Two such techniques are discussed. One to construct confidence sets of stable parameters, which we call the CIS ("Confidence Interval for the set of Stable parameters") procedure, and one to construct a set of parameters that contains the stable ones with probability one, which we call the ICS procedure ("Information Criterion for the set of Stable parameters"). The advantage of the CIS procedure relative to the ICS procedure is that it provides researchers with a set of stable parameters that contains the true set of stable parameters with a pre-specified probability level. However, with some probability (that is controlled by the size of the test) it is possible that a parameter is stable and it is not included in the confidence set of stable parameters. There is nothing worrisome in this, as it is exactly what would happen in a standard hypothesis testing procedure, where with some pre-specified probability the null is rejected even if true. We recognize, though, that in some situation this may be undesirable, and we offer the ICS procedure, which identifies the set of stable parameters with probability approaching one asymptotically. However, by construction, the ICS procedure is less powerful than the CIS procedure.

Many researchers have realized the importance of developing techniques to identify with parameters are time-varying among a set of possible parameters. There is a considerable interest estimating structural macroeconomic models with time-varying parameters (see Clarida, Gali, and Gertler (2000), Owyang and Ramey (2004), Cogley and Sargent (2005), Primiceri (2005), Fernandez-Villaverde and Rubio-Ramirez (2007), and Justiniano and Primiceri (2007)), in testing for structural breaks in macroeconomic data (see among others Gurkaynak et al. (2005), Fernald (2007) and Ireland (2001)), and in interpreting the causes of the time variation in macroeconomic aggregates (for example, the Great Moderation phenomenon) by relating it to parameter changes in the structural model (Stock and Watson (2002, 2003), and Cogley and Sargent (2001, 2005a,b)). In particular, Cogley and Sbordone (2005) investigate the stability of the estimated parameters of a Phillips curve relationship in the face of changes elsewhere in the economy, and Ireland (2001) attempts to identify which parameters have been subject to breaks by applying standard structural break tests to each of the parameters separately. However, when used repeatedly to test structural change in more than one subset of parameters, such tests lead to size distortions in the overall procedure. Fernandez-Villaverde and Rubio-Ramirez (2007) also question whether the parameters of a representative New Keynesian model are invariant to shifts in momentary policy by allowing some of the parameters to shift. Again, as pointed out by Cogley (2007), their results are difficult to interpret because they evaluate which parameters are unstable by looking at subsets of such parameters changing "one-at-a-time".[2] Similarly, in the effort of shedding light on the causes of the Great Moderation, Stock and Watson (2002) allow either the parameters in the monetary policy reaction function and/or the variance of the shocks to change over time, while keeping constant the slope of the Phillips curve and the slope of the IS equation. But what if the latter had changed as well? Estrella and Fuhrer (1999) also attempt to interpret results of structural breaks in a joint system of New Keynesian equations versus results of structural breaks in only the monetary policy block by using the "one-at-a-time" approach criticized by Cogley (2007). The advantage of our procedure is to provide a tool that can be used by researchers in exactly those situations and does not rely on a "one-at-a-time", ad-hoc approach.[3]

The technique proposed in this paper is related to recent contributions in the structural break test literature. Andrews (1993), Andrews and Ploberger (1994) and Nyblom (1989) propose tests for structural breaks in the parameters, but their tests are for a specific null hypothesis on a subset of the parameters. Our paper instead allows the researcher to determine the subset of parameters that do not have structural breaks. Our paper is also very different from Bai and Perron (1998), who consider sequential tests for determining the number and location of structural breaks, because their procedure is applied to identify possibly multiple breaks in a specific parameter and it is not a quest of which parameters are subject to breaks. Our procedure is also more distantly related to the literature on sequential model selection and hypotheses testing, in particular the works by Hansen et al. (2005) and Pantula (1989).[4]

The paper is organized as follows. The next section provides an overview of the new techniques that we propose, Sections 3 and 4 present the main empirical results of the paper, and Section 5 concludes.

[2] Fernandez-Villaverde and Rubio-Ramirez (2007) allowed only a subset of the parameters at a time to be time-varying, and Cogley's (2007) criticism is that it is difficult to interpret their results, as we don't know which of the parameters really changed. However, if one tried to allow all the parameters to be time-varying and repeatedly use structural break tests to identify which parameters are time-varying, one would incur into size distortions, as discussed above.

[3] A similar question to the one addressed in this paper is considered by Del Negro and Schorfheide (2007). They quantify the degree of misspecification of macroeconomic models due to time-varying parameters, and provide a diagnostic tool that allows researchers to parameterize the discrepancies between theory and data.

[4] Our procedure is also different from the approach in Berger (1982), who proposes doing separate t-tests on each parameter in isolation: in our case, under the alternative that another parameter might be time-varying, the t-test on a single parameter is not consistent.

2 The econometric procedures

This section presents our econometric procedure to construct a confidence set for the stable parameters, and compares it with a naive procedure based on discarding parameters when their individual tests for parameter stability reject the null. We will show that our testing procedure controls size, is consistent and produces a confidence set of stable parameters with a pre-specified coverage, whereas the naive procedure leads to size distortions.

2.1 Notation and definitions

Consider a general parametric model with parameters $\theta_t = (\beta_t, \delta) \in \Theta \subseteq \mathbb{R}^{p+q}$ for $t = 1, 2, ...,$ where $\beta_t \in B \subseteq \mathbb{R}^p$ and $\delta \in D \subseteq \mathbb{R}^q$. Let the parameters β_t be time-varying, and the parameters δ be stable. To formalize the problem, let $s \in \{0,1\}^{p+q}$ denote a parameter selection vector and $\theta(s)$ denotes a subset of θ selected by the selection vector s, where s_i denotes the i-th element of such vector.[5] We also let s^* denote the population selection vector that selects only the constant parameters: $\theta(s^*) = \delta$. Note that it is possible that s^* is the vector of ones, in which case all parameters belong to the stable confidence set, or s^* is the vector of zeros, in which case none of the parameters belongs to the stable confidence set. The problem considered in this paper is that it is not known which parameters are time-varying and which are stable. In other words, s^* is unknown. We will propose a sequential procedure that uses sample information to estimate s^* by an estimator $\widehat{s} \in \{0,1\}^{p+q}$. With our procedure, the estimator $\widehat{s}$ will be equal to s^* with a pre-specified probability level.

Let the observed sample be $W = \{W_t : 1 \leq t \leq T\}$ and $\mathcal{T}_T(W; s)$ be a consistent test statistic for testing the null hypothesis that the parameters $\theta(s)$ are constant over time: $H_0(s) : \theta_t(s) = \theta(s)$ versus the alternative that the parameters $\theta(s)$ are time-varying; for example, in the case of a one-time structural break at an unknown fraction of the sample size $[\pi T]$: $H_A(s) : \theta_t(s) = \theta_1(s) \cdot 1(t \leq \pi T) + \theta_2(s) \cdot 1(t > \pi T)$, where $\pi \in \Pi \subset (0,1)$.[6] For notational simplicity, we will omit the dependence of the test statistic on the observed sample, and use $\mathcal{T}_T(s)$. For example, let e_i be the $(p+q) \times 1$ vector whose i-th element is one and the other elements are zero, $\mathbf{1}_{(p+q)\times 1}$ be the $(p+q) \times 1$ vector of ones, and $\mathbf{0}_{(p+q)\times 1}$

[5] For example, when $p + q = 3$, $s_0 = (0,1,0)$ indicates that the second element of θ is not time-varying and the first and third elements are time-varying in population.

[6] Our procedure could also be used in the presence of multiple breaks, along the lines of Bai and Perron (1998), and in the presence of structural breaks of the random walk type (as we do in the empirical section below).

be the $(p+q) \times 1$ vector of zeros. Then, $\mathcal{T}_T(e_i)$ will denote the individual test for testing the null hypothesis that the i-th parameter in θ, $\theta(e_i)$, is constant over time, and $\mathcal{T}_T\left(1_{(p+q)\times 1}\right)$ will denote the joint test for testing the null hypothesis that the all the parameters in θ are constant. Also, let $k_\alpha(s)$ denote the critical value of $\mathcal{T}_T(s)$, and $pv(s)$ denotes its p-value. For example, in the aforementioned case of a one-time structural break at an unknown time, when using Andrews' (1993) QLR test statistic, $k_\alpha(s)$ is the critical value of such test at level α for testing a number of restrictions equal to the number of nonzero elements in s.

In what follows, we will propose two procedures: the first is to construct a confidence set for the stable parameters that contains the true set of stable parameters with a pre-specified probability level, which we call the "CIS procedure", and the second is a method to consistently estimate the set of stable parameters, which we call the "ICS procedure".

2.2 Confidence sets for stable parameters: the CIS procedure

We propose the following recursive procedure: first, test the joint null hypothesis that all parameters are stable. If the test does not reject, then all the parameters belong to the confidence set of stable parameters. If it does, calculate the p-values of the individual test statistics for testing whether each of the parameters are stable. Start by eliminating from the confidence interval the parameter with the lowest p-value, then test whether the remaining parameters are jointly stable.[7] If they are, then the confidence set of stable parameters includes such parameters; otherwise, eliminate the parameter with the second lowest p-value from the set, and continue this procedure until the joint test on the remaining parameters does not reject stability: this will identify the set of constant parameters. We formalize the CIS ("Confidence Interval for Stable parameters") procedure in the following Algorithm:

Algorithm 1 (The CIS procedure) *Step 0. Initially, let* $s_0 = \mathbf{1}_{(p+q)\times 1}$. *Test* $H_0^{(0)}(s_0)$ *against* $H_A^{(0)}(s_0)$ *by using the test* $\mathcal{T}_T(s_0)$. *If the test does not reject, let* $\widehat{s}_{CIS} = s_0$. *If the test rejects, calculate the vector of test statistics* $\mathcal{T}_T(e_i)$ *for* $i = 1, ..., p+q$, *and order them such that to their p-values are increasing:* $pv(e_1) \leq pv(e_2) \leq ... \leq pv(e_{p+q})$. *Without loss of generality, let* e_1 *identify the parameter with the smallest p-value.*[8] *Continue to step 1.*

[7] Alternative procedures could involve calculating the F-test for every subset of parameters and use that to choose which parameters to eliminate in the sequential procedure. However, this procedure is more computationally burdensome, especially when applied to the estimation of DSGE models, so we will not consider it here.

[8] Actually, one does not even need to compute p-values: one may simply pick the estimated test statistic

Step 1. Without loss of generality, let $s_1 = [0, \mathbf{1}_{1\times(p+q-1)}]'$. *Test* $H_0^{(1)}(s_1)$ *against* $H_A^{(1)}(s_1)$ *by using* $\mathcal{T}_T(s_1)$. *If the test does not reject, let* $\widehat{s}_{CIS} = s_1$. *If the test rejects, let* e_2 *identify the parameter with the second smallest p-value, and continue to step 2.*

...

Step j. Without loss of generality,, let $s_j = [\mathbf{0}_{1\times j}, \mathbf{1}_{1\times(p+q-j)}]'$. *Test* $H_0^{(j)}(s_j)$ *against* $H_A^{(j)}(s_j)$ *by using* $\mathcal{T}_T(s_j)$. *If the test does not reject, let* $\widehat{s}_{CIS} = s_j$. *If the test rejects, let* e_j *identify the parameter with the j-th smallest p-value, and continue to step (j+1).*

...

Step (p+q-1). Without loss of generality, let $s_{p+q-1} = [\mathbf{0}_{1\times(p+q-1)}, 1]'$. *Test* $H_0^{(p+q-1)}(s_{p+q-1})$ *against* $H_A^{(p+q-1)}(s_{p+q-1})$ *by using* $\mathcal{T}_T(s_{p+q-1})$. *If the test does not reject, let* $\widehat{s}_{CIS} = s_{p+q-1}$. *If the test rejects, let* $\widehat{s}_{CIS} = \mathbf{0}_{(p+q)\times 1}$.

Appendix A shows that the algorithm defines a confidence set of stable parameters with coverage $(1-\alpha)$. Importantly, note that there are size distortions in existing tests for structural breaks when used repeatedly to test structural change in more than one subsets of parameters, in the sense that such tests would find a structural break eventually in one of the parameters with probability approaching one. Proposition (4) in Appendix A shows that.

2.3 Consistent methods for estimating the set of stable parameters: the ICS procedure

We also consider a procedure that consistently selects the stable parameters (rather than providing a confidence interval). While an information criterion does not suffer from asymptotic size distortions and can be used to consistently estimate the set of stable parameters, it would be computationally demanding. For example, when there are $(p+q)$ structural parameters, the standard information criterion procedure requires that the model be estimated $2^{(p+q)}$ times. Instead we propose a practical procedure to estimate the set of stable parameters consistently. The idea is to replace the critical values in the CI procedure by diverging ones. By doing so, it will be more conservative but it will estimate the set of stable parameters consistently.

Let $|s|$ denote the number of parameters selected by the selection vector s: $|s| = \Sigma_{i=1}^{p+q} s_i$. Let ν_T denote a sequence such that $\nu_T \to \infty$ as $T \to \infty$ and $\nu_T = o(T)$. This will be the

which is the largest. Because the degrees of freedom for testing each parameter individually are the same, and therefore the critical values are, the largest test statistic has the smallest p-value.

basis of our penalty function. Common choices are: BIC-type penalty (for which $\nu_T = \ln T$) and Hannan-Quinn-type penalty (for which $\nu_T = \varsigma \ln\ln T$ for $\varsigma > 2$).[9]

Algorithm 2 (The ICS procedure) *Step 0. Initially, let* $s_0 = \mathbf{1}_{(p+q)\times 1}$. *Test* $H_0^{(0)}(s_0)$ *against* $H_A^{(0)}(s_0)$ *by using the test* $\mathcal{T}_T(s_0)$ *with critical value* $|s_0|\nu_T$. *If the test does not reject, let* $\widehat{s}_{ICS} = s_0$. *If the test rejects, calculate the vector of test statistics* $\mathcal{T}_T(e_i)$ *for* $i = 1, ..., p+q$, *and order them such that their p-values are increasing. Without loss of generality, let* e_1 *identify the parameter with the smallest p-value. Continue to step 1.*
Step 1. Let $s_1 = [0, \mathbf{1}_{1\times(p+q-1)}]'$. *Test* $H_0^{(1)}(s_1)$ *against* $H_A^{(1)}(s_1)$ *by using* $\mathcal{T}_T(s_1)$ *with critical value* $|s_1|\nu_T$. *If the test does not reject, let* $\widehat{s}_{ICS} = s_1$. *If the test rejects, let* e_2 *identify the parameter with the smallest p-value among the parameters associated with* s_1 *and continue to step 2.*
...
Step j. Let $s_j = [\mathbf{0}_{1\times j}, \mathbf{1}_{1\times(p+q-j)}]'$. *Test* $H_0^{(j)}(s_j)$ *against* $H_A^{(j)}(s_j)$ *by using* $\mathcal{T}_T(s_j)$ *with critical value* $|s_j|\nu_T$. *If the test does not reject, let* $\widehat{s}_{ICS} = s_j$. *If the test rejects, let* e_j *identify the parameter with the smallest p-value among the parameters associated with* s_j *and continue to step (j+1).*
...
Step (p+q-1). Let $s_{p+q-1} = [\mathbf{0}_{1\times(p+q-1)}, 1]'$. *Test* $H_0^{(p+q-1)}(s_{p+q-1})$ *against* $H_A^{(p+q-1)}(s_{p+q-1})$ *by using* $\mathcal{T}_T(s_{p+q-1})$ *with critical value* ν_T. *If the test does not reject, let* $\widehat{s}_{ICS} = s_{p+q-1}$. *If the test rejects, let* $\widehat{s}_{ICS} = \mathbf{0}_{(p+q)\times 1}$.

In words, $\widehat{s}$ identified by Algorithm (1) is the greatest set of parameters for which the test does not reject the null hypothesis of parameter stability. See Appendix A for a proof of the consistency of the ICS procedure.

3 Time-variation in a representative New Keynesian model

We consider a New Keynesian model with the basic features of many recent representative New Keynesian models. The model is developed in Ireland (2007), and includes a generalized Taylor rule for monetary policy that allows the central bank's inflation target to adjust in

[9]Note that the AIC-type penalty ($\nu_T = 2$) would result in an inconsistent selection criterion and is ruled out by our assumptions on ν_T.

response to other shocks that hit the economy. This feature is particularly appealing for our purposes, as it provides an additional way of allowing for time-variation in monetary policy: since our objective is to construct a confidence set containing the stable parameters, which may include not only monetary policy parameters but also preference and technology parameters, we would like our conclusions to be robust to possible misspecification of the monetary policy rule, including possible time variation in the inflation target of the Central Bank. The model also allows for a variety of features that have been found to be important to match the empirical data, namely habit formation, forward-looking price setting, and adjustment costs. We will refer to this (general) model as the "endogenous inflation target" model. As special cases, the model includes the "exogenous inflation target" (the case considered by Clarida, Gali and Gertler (2000), among others, where the Central Bank inflation target is constant over time) and the "backward looking price setting" model (where firms set their prices according to a backward looking rule). The log-linearized model is directly from Ireland (2007) and it is included in Appendix A for reference. The data are quarterly time series of per-capita GDP in real terms, the GDP price deflator, and the three month U.S. Treasury bill interest rate from 1959:1 through 2004:2.

Our first analysis focuses on the situation in which there is a single, unanticipated, and once-and-for-all shift in some of the parameters of the structural model at an unknown time, and in which there is an immediate convergence to a rational-expectations equilibrium after the regime change. If the time of the change were known and if we had a strong suspicion about which parameters could possibly have been affected by the change, we could re-estimate the model in the two sub-samples and test whether there was a break. The problem arises because we don't know which parameters could have been affected by the break: we have a variety of potential candidates (including monetary policy, preference, and Phillips curve parameters) and we don't know which ones were affected by the break. Although single and once-and-for-all shifts in parameters are an important modeling device, they are only one way to model time-variation. Another possibility that we will consider later is a framework in which the parameters change continuously over time, captured by a random walk type behavior, as considered by Cogley and Sargent (2005), Cogley and Sbordone (2005), and Primiceri (2005).

We use our procedure to address the long-standing question whether structural parameters are structural. As a representative test for structural break, we use Andrews' (1993) QLR test.[10] The joint test on all the parameters strongly rejects the null hypothesis of

[10] Note that his LR-like statistics, eq. (4.5), simply becomes the likelihood ratio test calculated as the

parameter stability. The estimated time of the break (given by the date associated with the highest value of the QLR test statistic) is 1980:4. Table 1 shows p-values for the t-tests for structural breaks on individual parameters. According to the individual tests, for example, in the "Endogenous inflation target" model both α and δ_z are constant. However, as shown in Proposition (4), such procedure is invalid. Therefore, we proceed by considering our recursive procedure.

Table 1 also shows results for Algorithm (1). For computational simplicity, in the algorithm we fixed the break date to be the estimated one, and we therefore used a standard Chow test for structural break.[11] In order to give an economic interpretation of the sources of time variation, we divide the parameters in three groups: (i) those influencing the Euler equation $(\gamma, z, \rho_a, \sigma_z, \sigma_a)$; (ii) those influencing the Phillips curve either in the standard Phillips curve relationship (α) or measuring the persistence and standard deviation of the cost-push shock (ρ_e, σ_e); and (iii) those influencing monetary policy (either the usual output gap and inflation aversion parameters (ρ_{gy}, ρ_π) or the long-term inflation target parameters $(\sigma_\pi, \delta_z, \delta_e)$, or the serial correlation and standard deviation of the transitory monetary policy shock, ρ_v, σ_v). In the table, the parameters are ordered according to the value of the Chow test for structural break using the estimated break data from the ones where the evidence of time variation is the strongest to the ones where the evidence is the weakest (labeled "QLR test statistic"). P-values are also reported.

The results are striking. According to our algorithm (1), the set of stable parameters is $\{\alpha\}$. The strongest evidence on time variation comes from the parameters in the monetary policy reaction function of the Central Bank, a fact first noticed by Clarida, Gali and Gertler (2000). The table shows that such instabilities affect not only the parameters in the standard monetary policy reaction function $(\rho_\pi, \rho_{gy},\sigma_\pi, \delta_z, \delta_e)$, but also the parameters governing the long-term inflation target of the Central Bank. The most remarkable result is that time variation afflicts not only the parameters in the monetary policy reaction function, but also most of the "structural" parameters in the Euler and IS equations. There is a sense in which, therefore, such parameters are not "structural". The weakest evidence of time variation is in the parameter of the Phillips curve, α, which belongs to the set of stable parameters.

difference between the constrained model (that is, the loglikelihood estimated over the full sample) and the unconstrained one (that is, the weighted average of the loglikelihood estimated separately over all possible two sub-samples, weighted according to the percentage of observations in each of the two sub-samples).

[11] Since we evaluate the Chow test statistic at the estimated break date, the p-values are calculated using Andrews' (2003) critical values with 1 degree of freedom.

The latter result reinforces that in Cogley and Sbordone (2005), who also claim that the estimated parameters of the Phillips curve are stable in the face of changes elsewhere in the economy.[12] However, our results are stronger than theirs, in the sense that: (i) we allowed all the parameters to be possibly time-varying and chose the set of stable parameters according to statistical criteria; (ii) we do not have to make a maintained assumption as to the nature of the time variation (except that our tests are admissible for one-time structural changes, but have power against random walk time-varying parameters); (iii) we do not make maintained assumption as to the VAR underlying the data.

In the recent macroeconomics literature the reduction in volatility of the GDP growth has received significant attention (see Stock and Watson, 2002). We will consider changes in the standard deviation of five structural shocks in Ireland's (2007) model: the inflation target shock, technology shock, preference shock, cost-push shock and transitory monetary shock. Following Stock and Watson (2002) we impose the break date of 1984:Q1. Table 2 reports structural parameter estimates and standard deviations where the unstable parameters are selected by our procedure. Based on these estimates we obtain the standard error of the structural shocks in Ireland's (2007) model. The last column of Table 3 shows the ratio of the standard deviation in the second sample period over the one in the first time period. The inflation target shock has experienced the largest reduction and the reduction is larger than any of the reductions in the standard deviations in Table 8 of Stock and Watson (2002). However, when we calculate the relative contribution of each shock to the total reduction in the variance of GDP by using the DSGE model, we find that the technology shock seems to be explaining most of such decrease, although both the monetary policy and the preference shocks played a role.

Figure 1 plots impulse response functions estimated by allowing the relevant parameters to have a structural break. Panel A shows the impulse responses before the break and Panel B shows the impulse responses after the break. The magnitude of the impulse responses of each of the variables to the preference shock change considerably before and after the break, as well as the responses to the inflation target shock. Interestingly, in the case of the responses to the cost-push shock, not only the magnitude of the impulse responses change, but also their shape. This suggests that the transmission mechanism in the U.S. economy has changed starting the early Eighties, and that this might have played a role in the Great

[12] The sample used in Cogley and Sbordone (2005), 1960:1-2003:4, is similar to the one used here, although their definition of interest rate is different (they use the Fed Funds rate) and they also include data for real marginal cost.

Moderation.

4 Time variation in a VAR with drifting parameters

We also consider estimating a VAR with drifting coefficients and volatilities. Models of this type have been estimating by Primiceri (2005) and Cogley and Sargent (2005), among others. We use our procedure to shed light on which parameters are time varying: whether those in the conditional mean or the volatilities, and in which equations the instabilities the are concentrated. This provides an answer to the question of whether time series have responded with a time-invariant impulse responses to possibly time-varying shocks or whether the impulse responses have themselves changed over time. First, we consider a reduced form VAR with GDP, inflation, and the interest rate. Then, we identify the for the subset of parameters that are evolving over time, and we plot their estimated time path. Finally, we consider a structural VAR in the spirit of Stock and Watson (2001), where the shocks are identified according to a Cholesky decomposition, and plot the time path of the identified structural shocks.

We consider the following reduced-form VAR, where y_t is per-capita GDP in real terms, π_t is the GDP price deflator, and r_t is the three month U.S. Treasury bill interest rate minus the inflation rate (that is, the real interest rate).[13] The lag length is chosen according to the BIC and is equal to one.

$$\begin{aligned}
\begin{pmatrix} r_t \\ y_t \\ \pi_t \end{pmatrix} &= \underbrace{\begin{pmatrix} k_{11} \\ k_{22} \\ k_{33} \end{pmatrix}}_{K} + \underbrace{\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}}_{A} \begin{pmatrix} r_{t-1} \\ y_{t-1} \\ \pi_{t-1} \end{pmatrix} + \underbrace{\begin{pmatrix} u_{r,t} \\ u_{y,t} \\ u_{\pi,t} \end{pmatrix}}_{u_t} \\
\text{where } V(u_t) &= \Omega = \begin{pmatrix} \omega_{11} & \omega_{12} & \omega_{13} \\ \omega_{21} & \omega_{22} & \omega_{23} \\ \omega_{31} & \omega_{32} & \omega_{33} \end{pmatrix}.
\end{aligned}$$

We let both parameters in the conditional mean (K, A) and covariance matrix (Ω) to change over time according to a random walk:

[13] For comparability, the time series of the variables are the same as in the previous section. That is, they are calculated in deviations from their steady state levels.

$$
\begin{aligned}
K_t &= K_{t-1} + v_{K,t} \\
A_t &= A_{t-1} + v_{A,t} \\
\Omega_t &= \Omega_{t-1} + v_{\Omega,t}
\end{aligned}
$$

Panel A in Table 4 shows the results. Nyblom's (1989) test for the joint hypothesis of stability of all parameters rejects the null of stability. Interestingly, the same test applied to testing the hypothesis of stability of the parameters either in the conditional mean ($k's$ and $\alpha's$) or in the variance ($\omega \in \Omega$) shows that only the latter rejects the hypothesis of stability. Therefore, as pointed out in Cogley and Sargent (2005), instabilities seem to be concentrated in the parameters governing the variance. However, we cannot really rely on such tests, as they repeatedly test hypotheses without taking into account the previous step. Furthermore, such tests do not identify which variances are time-varying. We therefore apply our testing procedure, and show that the biggest evidence of parameter instability comes from the variance of GDP.

Finally, we attempt to give our results a structural interpretation by identifying the shocks according to a recursive VAR identification used, among others, by Stock and Watson (2001) and Primiceri (2005) for a similar VAR. The Cholesky decomposition follows the order inflation, output and the interest rate:

$$
\underbrace{\begin{pmatrix} u_{r,t} \\ u_{y,t} \\ u_{\pi,t} \end{pmatrix}}_{u_t} = \underbrace{\begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ 0 & \sigma_{22} & \sigma_{23} \\ 0 & 0 & \sigma_{33} \end{pmatrix}}_{\Sigma} \underbrace{\begin{pmatrix} \eta_{r,t} \\ \eta_{y,t} \\ \eta_{\pi,t} \end{pmatrix}}_{\eta_t} \tag{1}
$$

where $\eta_t \sim iid\,(0, I)$ are the structural shocks and:

$$
\begin{aligned}
K_t &= K_{t-1} + v_{K,t} \\
A_t &= A_{t-1} + v_{A,t} \\
\Sigma_t &= \Sigma_{t-1} + v_{\Sigma,t}
\end{aligned}
$$

The interpretation of (1) is as follows: the first equation represents the monetary policy rule, the second equation is the IS equation and the third is a Phillips curve.

Panel A in Table 5 shows the results and Table 6 reports parameter estimates obtained over the full sample. Again, a joint test on all parameters rejects the null of parameter stability, and simple joint tests on all $\sigma's$ rejects the null of stability whereas a simple joint test on all the parameters in the conditional mean does not reject stability. However, the structural VAR analysis uncovers the very interesting result that the evidence of time variation is concentrated in the transmission mechanism rather than in the impulse: a 95% confidence set of the parameter only excludes σ_{13}. In other words, the variances of the structural shocks are constant (i.e. all σ_{ii} are constant) and the instability is concentrated in the transmission mechanism, and more specifically in the monetary policy reaction function. The instability appears to be crucially concentrated in the monetary policy response of the interest rate to inflation.

For robustness, we also consider one-time breaks in both the reduced-form and the structural VARs. The break is identified to be in 1985:2 in the reduced form VAR, and we use such a date in our analysis. The results are again basically similar. Panel B in Table 4 shows that the instability in the reduced form VAR is concentrated on the variances: a 95% joint confidence set for the stable parameters *excludes* all the variances $\{\sigma_{11}, \sigma_{22}, \sigma_{33}\}$, including the variance of GDP growth. However, when we look at the structural shocks (see Panel B in Table 5), interestingly we again find that instabilities are concentrated in the monetary policy (as well as the IS equation), as this time the confidence set for the stable parameters *excludes* ω_{33} only.[14]

5 Conclusions

This paper investigates which of the "structural" parameters of a representative DSGE model are stable over time. We do so by developing new econometric tools that allow researchers to identify the set of stable parameters of a structural economic model. Our conclusions are that the empirical evidence is strongly in favor of stability in the parameters of the Phillips curve, whereas instabilities are mostly a concern for the monetary policy reaction function and the Euler equation for consumption. Since parameter instabilities are indicative of model misspecification, our results validate the empirical success of models of the Phillips curve. Our contribution therefore sheds light on the economic interpretation and cause of such instabilities. It provides clear guidelines on which parts of the model are reliable for policy analysis and which are possibly mis-specified. Such results offer important insights to

[14] In the analysis of Panel B in Table 5 we used a recursive F-test for structural breaks to order the parameters rather than the individual p-values, as most individual p-values were equal to one. For simplicity, we focused on searching over permutations over the variances.

guide the future theoretical development of macroeconomic models.

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6 Appendix A - Propositions and proofs

6.1 The CIS procedure

For any two vectors a and b, let $\max(a,b)$ denote the vector whose i-th element is the maximum of $a(i)$ and $b(i)$, where $a(i)$ denotes the i-th element of a. We make the following assumption:

Assumption 1. For all $s^* \in \{0,1\}^{p+q}$ such that $s^* \neq 0_{1\times(p+q)}$, $\mathcal{T}_T(s) \underset{d}{\Rightarrow} D(s)$ if $s = s^*$ and $\mathcal{T}_T(s) \underset{p}{\rightarrow} \infty$ if $\max(s, s^*) > |s^*|$, where $|s|$ denotes the number of components in s that are different from zero.

Remarks. Assumption 1 requires that $\mathcal{T}_T(s)$ has a well-defined asymptotic distribution under the null hypothesis, and diverges to positive infinity when testing includes at least a parameter that is unstable under the alternative hypothesis of parameter instability. This assumption, for example, is satisfied in the Andrews' (1993) QLR test, the Andrews and Ploberger's (1993) Exp-W and Mean-W tests, and Elliott and Muller's (2005) qLL test.[15]

The following Proposition shows that, by selecting the parameters associated with $\widehat{s}$ identified in Algorithm (1) one obtains a confidence set of the stable parameters that has coverage $(1-\alpha)$.

Proposition 3 *Let Assumption 1 hold, and let $\widehat{s}$ be estimated as described by Algorithm (1). Then:*

$$\lim_{T\to\infty} \Pr\{\hat{s} = s^*\} \; = \; 1-\alpha \tag{2}$$

for any $s^* \in \{0,1\}^{p+q}$ such that $s^* \neq 0_{1\times(p+q)}$,

$$\lim_{T\to\infty} \Pr\{\hat{s} = s^*\} \; = \; 1 \tag{3}$$

for $s^* = 0_{1\times(p+q)}$, and

$$\lim_{T\to\infty} \Pr\{\hat{s} \neq s^* \text{ and } \hat{s} \geq s^*\} = 0. \tag{4}$$

for any $s^* \in \{0,1\}^{p+q}$.

Proof of Proposition (3). Let $k_\alpha(s)$ denote the critical value of $\mathcal{T}_T(s)$ of the null distribution $D(s)$ at the level of significance, α. Recall that $p = p + q - |s^*|$ is the number

[15] Notice however that the Elliott and Muller's (2005) statistics rejects for *small* values of the statistics, whereas all other statistics reject for large values.

of unstable parameters. First, suppose that $p = 0$. Then $\underset{T\to\infty}{lim} Pr\left(\mathcal{T}_T\left(s_0\right) < k_\alpha\left(s_0\right)\right) = \underset{T\to\infty}{lim} Pr\left(\mathcal{T}_T\left(s^*\right) < k_\alpha\left(s^*\right)\right) = 1 - \alpha$, thus proving (2) under $H_0^{(0)}\left(s_0\right)$. When $p = 0$, (3) does not apply and (4) trivially holds. Next, suppose that $p > 0$ and $q > 0$. Note that $|\max(s_j, s^*)| > |s^*| = q$ for any s_j and $j = 0, 1, 2..., p-1$. By the consistency of $\mathcal{T}_T(s_j)$ for s_j such that $|\max(s_j, s^*)| > |s^*|$, the null hypotheses in steps $0, 1, 2, ..., p-1$ are all rejected and each of the p unstable parameters is selected in these p steps with probability approaching one. Therefore the null model in step p, s_p, converges in probability to s^* and (4) holds. Because $\underset{T\to\infty}{lim} Pr\left(\mathcal{T}_T\left(s_p\right) < k_\alpha\left(s_p\right)\right) = \underset{T\to\infty}{lim} Pr\left(\mathcal{T}_T\left(s^*\right) < k_\alpha\left(s^*\right)\right) = 1 - \alpha$, (2) holds. When $p > 0$ and $q > 0$, (4) does not apply. Lastly, suppose that $q = 0$. Then the null hypotheses in steps $0, 1, 2, ..., p$ are all rejected and each of the p unstable parameters is selected in these steps with probability approaching one. Therefore $\hat{s}$ converges in probability to s^*, and (3) and (4) hold. ■

Proposition 4 *Let the naive testing procedure be as follows:* $\widetilde{s} = s$, *where the i-th component of* s, $\widetilde{s}\left(i\right)$, *is such that:*

$$\widetilde{s}\left(i\right) = \begin{cases} 1 \; if & \mathcal{T}_T\left(e_i\right) < k_\alpha\left(e_i\right) \\ 0 & otherwise \end{cases}$$

Then $\lim_{T\to\infty} \Pr\left(\widetilde{s} \neq s^* | s_i = s^*\right) > \alpha$ *for every* $s^* \in \{0,1\}^{p+q}$ *provided* $p + q - i > 1$.

Proof of Proposition (4). Without loss of generality, consider the case $s^* = [\mathbf{0}_{p\times 1}, \mathbf{1}_{q\times 1}]'$. Suppose that $p + q - j = 2$. Then:

$$\begin{aligned} Pr\left(\widetilde{s} \neq s^* |\, s_j = s^*\right) &= \Pr\left(\mathcal{T}_T\left(e_{j+1}\right) > k_\alpha\left(e_{j+1}\right) \text{ or } \mathcal{T}_T\left(e_{j+2}\right) > k_\alpha\left(e_{j+2}\right)\right) \\ &= \Pr\left(\mathcal{T}_T\left(e_{j+1}\right) > k_\alpha\left(e_{j+1}\right)\right) + \Pr\left(\mathcal{T}_T\left(e_{j+2}\right) > k_\alpha\left(e_{j+2}\right)\right) \\ &\quad - \Pr\left(\mathcal{T}_T\left(e_{j+1}\right) > k_\alpha\left(e_{j+1}\right) \text{ and } \mathcal{T}_T\left(e_{j+2}\right) > k_\alpha\left(e_{j+2}\right)\right) \\ &= 2\alpha - \Pr\left(\mathcal{T}_T\left(e_{j+1}\right) > k_\alpha\left(e_{j+1}\right) \text{ and } \mathcal{T}_T\left(e_{j+2}\right) > k_\alpha\left(e_{j+2}\right)\right) \\ &> \alpha \end{aligned}$$

where the last inequality follows since $\Pr\left(\mathcal{T}_T\left(e_{j+1}\right) > k_\alpha\left(e_{j+1}\right) \text{ and } \mathcal{T}_T\left(e_{j+2}\right) > k_\alpha\left(e_{j+2}\right)\right) < \alpha$ provided that the joint distribution is non-singular. The proof for cases in which $p+q-j > 2$ is analogous although it is notationally more complicated. ■

6.2 The ICS procedure

We make the following assumption:

Assumption 2.

(a) Let ν_T be a sequence such that $\nu_T \to \infty$ and $\nu_T = o(T)$.

(b) *For all* $s^* \in \{0,1\}^{p+q}$ *such that* $s^* \neq 0_{1\times(p+q)}$, $\mathcal{T}_T(s) \underset{d}{\Rightarrow} D(s)$ *if* $s = s^*$ *and* $\mathcal{T}_T(s) \underset{p}{\to} \infty$ *if* $\max(s, s^*) > |s^*|$, *where* $|s|$ *denotes the number of components in* s *that are different from zero, and* $c(s) > 0$ *is some positive constant.*

Remarks. Assumption 2(a) is a slight modification of Assumption 1 and is satisfied by most structural break tests, including Andrews (1993) and Andrews and Ploberger (1994). Basically, it requires that the moment conditions converge in probability to some limiting nonzero value when evaluated when including parameters that have a break: since the parameters will converge to some pseudo-true parameter value different from the true, time-varying parameter, the expected value of such moment conditions will not be zero. Such limiting value will be zero when the moment conditions are evaluated only at stable parameters, otherwise will be a positive number. Note that when $\max(s, s^*) > |s^*|$ there will be at least one moment condition that is in expectation different from zero, and given that the test statistic is asymptotically equivalent to a quadratic form of such moment conditions, it will be positive.

Assumption 2(b) defines the properties required for the penalty function. The penalty function is necessary to offset the increase in the value of the test statistic $\mathcal{T}_T(s)$ that typically occurs when testing instabilities on a larger number of parameters even if the additional parameters are stable. For example: a BIC-type penalty involves $\nu_T = \ln(T)$, a Hannan-Quinn-type penalty involves $\nu_T = \varsigma \ln\ln(T)$ for some $\varsigma > 2$. An AIC-type penalty would involve $\nu_T = 2$ but, as well known, AIC-type penalties does not result in a consistent selection criterion and in fact it does not satisfy our Assumption 2(b).

Proposition 5 (Consistency of the ICSeq procedure) *Let Assumption 2 hold, and let* $\widehat{s}_{ICSeq}$ *be estimated as described by Algorithm (2). Then,*

$$\widehat{s}_{ICSeq} \underset{p}{\to} s^* \tag{5}$$

Proof of Proposition (5). Because $\nu_T(s)$ is diverging, the tests have size zero, i.e., $\alpha = 0$. Because the test statistics diverge faster than $\nu_T(.)$ under the alternative hypothesis, the tests remain consistent. Therefore (5) follows from (2). ■

7 Appendix B - The economic model

Ireland's (2007) model loglinearized around the steady state where consumption, output, and the marginal utility of consumption grow at the rate of technological process (a random walk with drift) is as follows. Let $\widehat{y}_t$, $\widehat{\pi}_t$, $\widehat{e}_t$, $\widehat{z}_t$, $\widehat{a}_t$, $\widehat{v}_t$, $\widehat{\lambda}_t$ $\widehat{\pi}_t^*$ denote the deviation of output, inflation, the cost-push shock, technology, the preference shock, the transitory monetary policy shock, the marginal utility of consumption, and the time-varying inflation target from the their steady state levels, and the following hold:

$$
\begin{aligned}
&\widehat{g}_t^y = \widehat{y}_t - \widehat{y}_{t-1} + \widehat{z}_t, \\
&\widehat{g}_t^\pi = \widehat{\pi}_t - \widehat{\pi}_{t-1} + \widehat{\pi}_t^*, \\
&\widehat{a}_t = \rho_a \widehat{a}_{t-1} + \sigma_a \varepsilon_{at}, \\
&\widehat{e}_t = \rho_e \widehat{e}_{t-1} + \sigma_e \varepsilon_{et}, \\
&\widehat{z}_t = \sigma_z \varepsilon_{zt}, \\
&\widehat{v}_t = \rho_v \widehat{v}_{t-1} + \sigma_v \varepsilon_{vt}
\end{aligned}
$$

The model builds on a series of parameters: z (the steady state level of technology), β (the discount factor), γ (the habit formation), α (the parameter measuring the extent to which price setting is backward or forward looking – $\alpha = 0$ means purely forward looking), ψ (a function of the magnitude of the adjustment cost and of the long-run level of the cost-push shock), ρ_π (the Fed's inflation aversion), ρ_{gy} (the Fed's aversion to the output gap), σ_π (the standard deviation of the shock to the inflation target), δ_e (the reaction of the time-varying inflation target to the shock to the time varying elasticity of demand for each intermediate good), δ_z (the reaction of the time-varying inflation target to the temporary shock to aggregate technology). The parameters β, z, ψ are calibrated prior to estimation.

The core of the model is formed by the following equilibrium conditions:

(1) the IS curve:

$$(z-\gamma)(z-\beta\gamma)\widehat{\lambda}_t = \gamma z \widehat{y}_{t-1} - (z^2+\beta\gamma^2)\widehat{y}_t + \beta\gamma z E_t \widehat{y}_{t+1} + (z-\gamma)(z-\beta\gamma\rho_a)\widehat{a}_t - \gamma z \widehat{z}_t$$

(2) the Euler equation:

$$\widehat{\lambda}_t = E_t \widehat{\lambda}_{t+1} + \widehat{r}_t - E_t \widehat{\pi}_{t+1}$$

(3) the Phillips curve:

$$(1+\beta\alpha)\widehat{\pi}_t = \alpha\widehat{\pi}_{t-1} + \beta E_t \widehat{\pi}_{t+1} + \psi\left(\widehat{a}_t - \widehat{\lambda}_t\right) - \widehat{e}_t - \alpha\widehat{\pi}_t^*$$

(4) the Monetary Policy reaction function:

$$\widehat{r}_t - \widehat{r}_{t-1} = \rho_\pi \widehat{\pi}_t + \rho_{gy} \widehat{g}_t^y - \widehat{\pi}_t^* + \widehat{v}_t$$

$$\widehat{\pi}_t^* = \sigma_\pi \varepsilon_{\pi t} - \delta_e \varepsilon_{et} - \delta_z \varepsilon_{zt}$$

Ireland (2007) considers three specifications:

(i) the endogenous inflation target case (all parameters are estimated freely);

(ii) the exogenous inflation target case ($\delta_e = \delta_z = 0$);

(iii) the backward looking price setting ($\alpha = 1$).

8 Tables and Figures

Table 1. Empirical results for the one-time structural change scenario.

Models:	Endog. infl. target			Exog. infl. target			Backward-looking price	
Parameter:	Individual p-value	Recursive p-value		Individual p-value	Recursive p-value		Individual p-value	Recursive p-value
ρ_{gy}	0	0	σ_a	0	0	ρ_v	0	0
δ_e	0	0	ρ_v	0	0	ρ_π	0	0
ρ_e	0	0	ρ_e	0	0	σ_v	0	0
ρ_v	0	0	ρ_{gy}	0	0	ρ_a	0	0
σ_z	0	0	σ_π	0	0	δ_e	0	0
σ_v	0	0	γ	0	0	ρ_e	0	0
σ_π	0	0	σ_z	0	0	σ_π	0	0
γ	0	0	σ_v	0	0	σ_z	0	0
σ_e	0	0	ρ_π	0	0	σ_e	0	0
ρ_π	0	0	σ_e	0.72	0.31	γ	0	0
ρ_a	0.04	0	ρ_a	0.80	1	σ_a	0	0
σ_a	0.015	0	α	1	1	α	0	0
δ_z	0.365	0.013	- -			ρ_{gy}	0.016	0.402
α	0.999	1	- -					
	Set of stable parameters at 95% confidence level: $\mathcal{S}=\{\alpha\}$			Set of stable parameters at 95% confidence level: $\mathcal{S}=\{\alpha,\rho_a,\sigma_e\}$			Set of stable parameters at 95% confidence level: $\mathcal{S}=\{\rho_{gy}\}$	

Note to table 1. The table reports values of the QLR test statistic for testing one-time structural breaks on individual parameters, and their p-values. The tests are implemented in a Wald form, using standard errors obtained by bootstrap with 1,000 replications.

Table 2. Parameter Estimates

Parameters	1959:Q1-1984:Q4	1985:Q1-2004:Q1
γ	0.234	0.426
	(0.001)	(0.001)
α	0.000	0.000
	(0.000)	(0.000)
ρ_π	1.143	0.722
	(0.368)	(0.154)
ρ_{gy}	0.228	0.206
	(0.062)	(0.048)
ρ_a	0.895	0.939
	(0.003)	(0.003)
ρ_e	0.026	0.599
	(0.003)	(0.155)
ρ_v	0.053	0.237
	(0.001)	(0.001)
σ_a	0.028	0.028
	(0.009)	(0.009)
σ_e	0.001	0.001
	(0.000)	(0.000)
σ_z	0.016	0.011
	(0.002)	(0.002)
σ_v	0.004	0.001
	(0.001)	(0.000)
σ_π	0.000	0.000
	(0.002)	(0.000)
δ_e	0.002	0.000
	(0.000)	(0.000)
δ_z	0.000	0.000
	(0.000)	(0.000)

Note to Table 2. In parentheses are standard errors.

Table 3. Standard Deviations of Macroeconomic Shocks

	1959:Q1-1983:Q4	1984:Q1-2004:Q1	s_2/s_1	Relative Contribution to GDP Var. Reduction
Inflation Target*	0.125	0.047	0.378	-0.006
Technology Shock*	1.589	0.798	0.502	0.816
Preference Shock	6.080	7.825	1.287	0.119
Cost-Push Shock	0.081	0.195	2.395	-0.129
Transitory Monetary Shock	0.330	0.169	0.512	0.132

Notes to Table 3. The standard deviations are multiplied by 100. For the processes with asterisk (*), the variance of the disturbance term, not the variance of the process, is reported because they are unit-root processes. Results are qualitatively very similar in the case of a break in 1980:Q1.

Table 4. Reduced form VAR

Panel A. Random walk parameters		**Panel B. One-time break**	
Parameter	Individual p-value	Parameter	Individual p-value
k_{11}	0.682	k_{11}	1
a_{11}	0.212	a_{11}	1
a_{12}	1	a_{12}	1
a_{13}	0.507	a_{13}	1
k_{22}	1	k_{22}	1
a_{21}	1	a_{21}	1
a_{22}	1	a_{22}	1
a_{23}	0.18	a_{23}	1
k_{33}	1	k_{33}	1
a_{31}	0.394	a_{31}	1
a_{32}	0.299	a_{32}	1
a_{33}	0.534	a_{33}	1
ω_{11}	0.334	ω_{11}	0.139
ω_{12}	1	ω_{12}	1
ω_{13}	0.21	ω_{13}	1
ω_{22}	0	ω_{22}	0.062
ω_{23}	1	ω_{23}	1
ω_{33}	0.088	ω_{33}	0.054
Joint test – all param:	0.021	Joint test – all param:	0
Joint test – all a, k:	0.305	Joint test – all a, k:	1
Joint test – all ω:	0	Joint test – all ω:	0
Set of stable parameters at 95% confidence level:		Set of stable parameters at 95% confidence level:	
$\mathcal{S} = \{k_{11}, a_{11}, a_{12}, a_{13}, k_{22}, a_{21}, a_{22}, a_{23},$		$\mathcal{S} = \{k_{11}, a_{11}, a_{12}, a_{13}, k_{22}, a_{21}, a_{22}, a_{23},$	
$k_{33}, a_{31}, a_{32}, a_{33}, \omega_{11}, \omega_{12}, \omega_{13}, \omega_{23}, \omega_{33}\}$		$k_{33}, a_{31}, a_{32}, a_{33}, \omega_{12}, \omega_{13}, \omega_{23}\}$	

Note to Table 4. The table reports the p-values of the Nyblom's (1989) test statistic for testing random-walk-type structural breaks on individual parameters in Panel A and the p-values of the Andrews' (1993) test statistic for testing one-time structural breaks on individual parameters in Panel B. In panel B, the break is dated 1985:2. ω_{ij} denotes the i-j-th element of the vech of the covariance matrix. The tests are implemented by using Li and Muller's (2007) procedure. Subscripts are as follows: $i = 1$ denotes the real interest rate, $i = 2$ denotes GDP, $i = 3$ denotes the inflation.

Table 5. Structural VAR

Panel A. Random walk parameter		**Panel B. One-time break**	
Parameter	Individual p-value	Parameter	Individual p-value
k_{11}	0.682	k_{11}	1
a_{11}	0.212	a_{11}	1
a_{12}	1	a_{12}	1
a_{13}	0.507	a_{13}	1
k_{22}	1	k_{22}	1
a_{21}	1	a_{21}	1
a_{22}	1	a_{22}	1
a_{23}	0.18	a_{23}	1
k_{33}	1	k_{33}	1
a_{31}	0.394	a_{31}	1
a_{32}	0.299	a_{32}	1
a_{33}	0.534	a_{33}	1
σ_{11}	0.334	σ_{11}	1
σ_{12}	1	σ_{12}	1
σ_{13}	0	σ_{13}	1
σ_{22}	1	σ_{22}	0
σ_{23}	0	σ_{23}	1
σ_{33}	0	σ_{33}	1
Joint test – all param:	0.021	Joint test – all param:	0
Joint test – all a, k:	0.305	Joint test – all a, k:	1
Joint test – all σ:	0	Joint test – all σ:	0
Set of stable parameters at 95% confidence level: $\mathcal{S} = \{k_{11}, a_{11}, a_{12}, a_{13}, k_{22}, a_{21}, a_{22}, a_{23}, k_{33}, a_{31}, a_{32}, a_{33}, \sigma_{11}, \sigma_{12}, \sigma_{22}, \sigma_{23}, \sigma_{33}\}$		Set of stable parameters at 95% confidence level: $\mathcal{S} = \{k_{11}, a_{11}, a_{12}, a_{13}, k_{22}, a_{21}, a_{22}, a_{23}, k_{33}, a_{31}, a_{32}, a_{33}, \sigma_{33}\}$	

Note to Table 5. The table reports the p-values of the Nyblom's (1989) test statistic for testing random-walk-type structural breaks on individual parameters in Panel A and the p-values of the Andrews' (1993) test statistic for testing one-time structural breaks on individual parameters in Panel B. σ_{ij} denotes the i-j-th element of the vech of the variance in Choleski factor. The tests are implemented by using Andrews' (1993) procedure. Subscripts are as follows: $i = 1$ denotes the real interest rate, $i = 2$ denotes GDP, $i = 3$ denotes the inflation.

Table 6. Structural VAR param. estimates

Parameter	Estimate	Parameter	Estimate
k_{11}	0.0006	σ_{11}	0.0032
a_{11}	0.8558	σ_{12}	0.0020
a_{12}	0.0258	σ_{13}	-0.0022
a_{13}	0.2371	σ_{22}	0.0082
k_{22}	0.0040	σ_{23}	0.0002
a_{21}	-0.1214	σ_{33}	0.0017
a_{22}	0.2734		
a_{23}	0.1033		
k_{33}	-0.0003		
a_{31}	0.0473		
a_{32}	0.0241		
a_{33}	-0.2630		

Note to Table 6. The table reports parameter values of the Structural VAR estimated over the full sample. Subscripts are as follows: $i = 1$ denotes inflation, $i = 2$ denotes GDP, $i = 3$ denotes the interest rate.

Figure 1 (Panel A). Impulse responses in Ireland's model before 1984

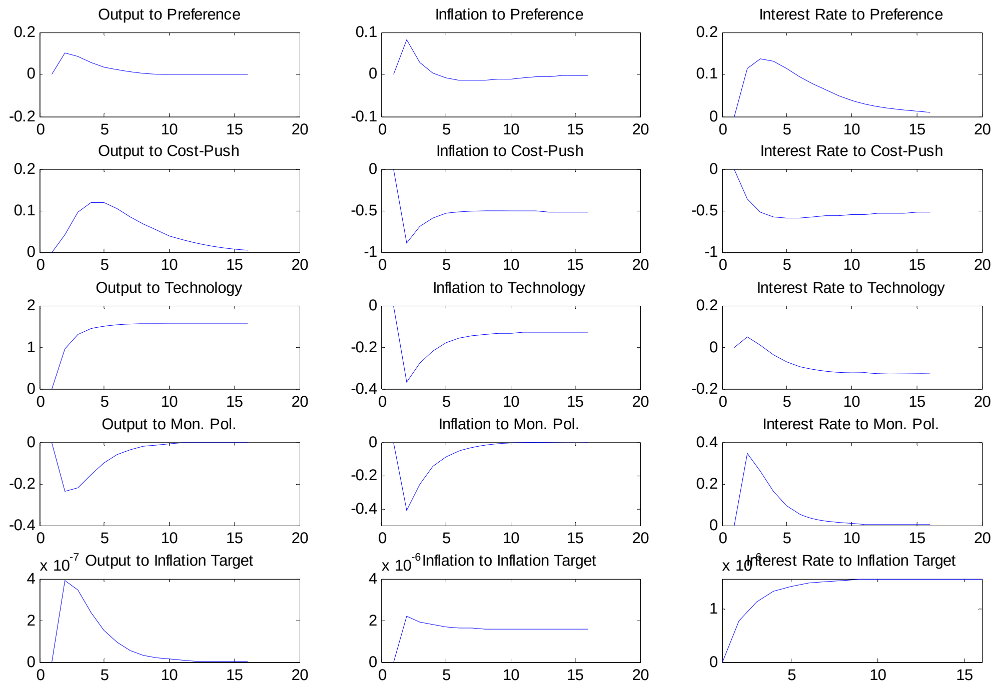


Figure 1 (Panel B). Impulse responses in Ireland's model after 1984

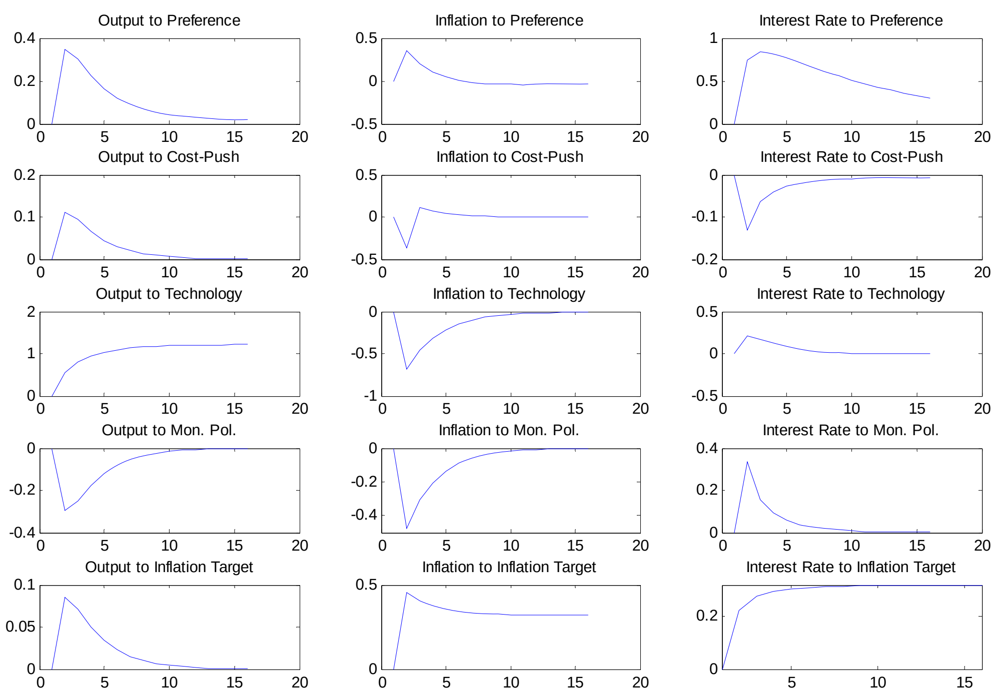


Figure 2 (Panel A). SVAR impulse responses before the Great Moderation

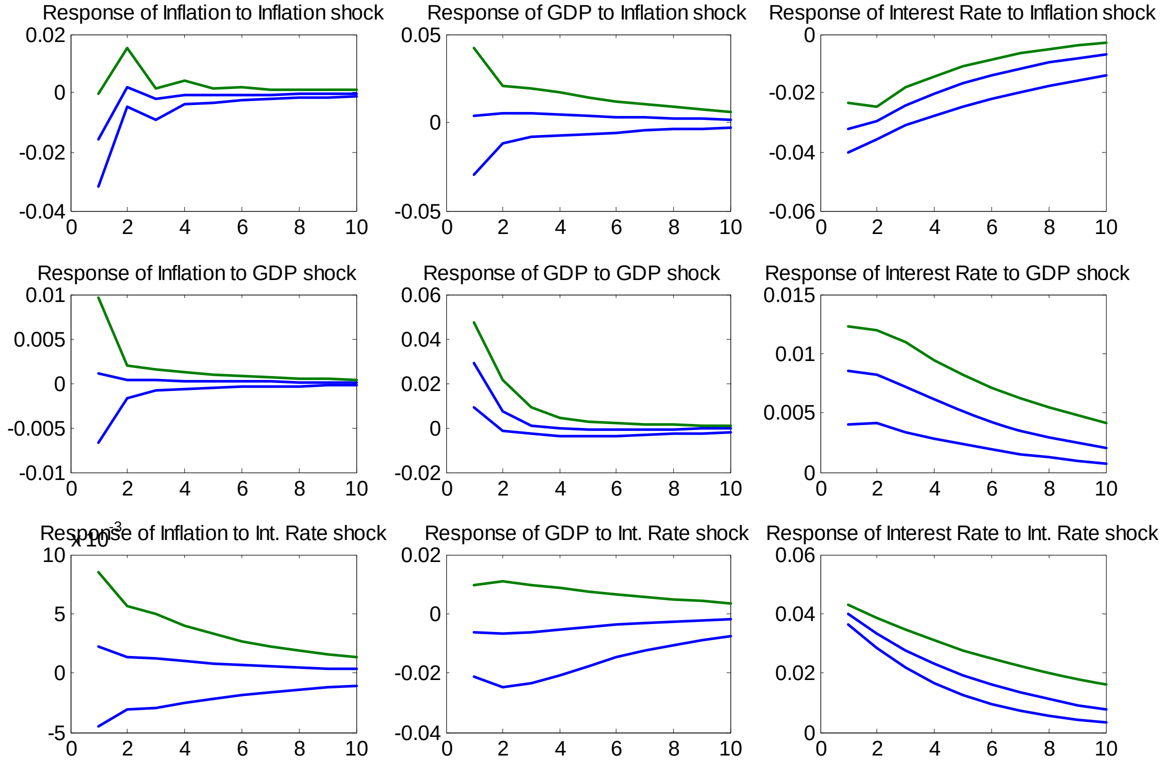
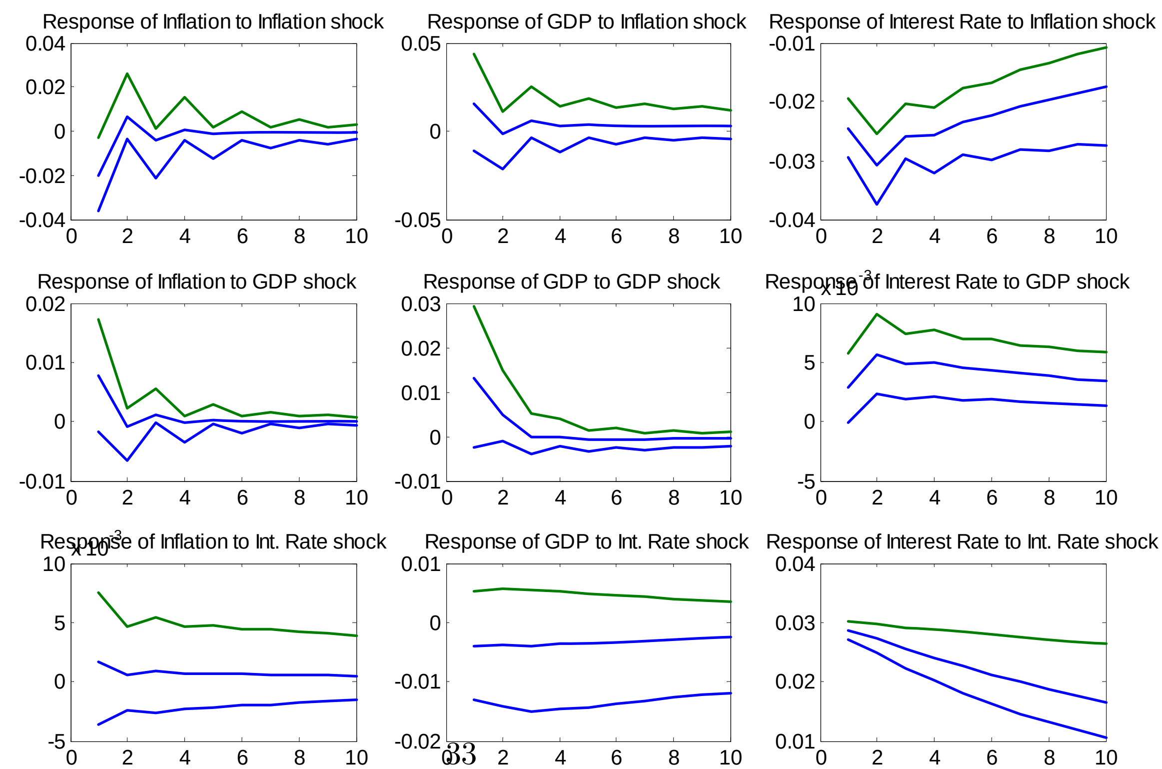


Figure 2 (Panel B). SVAR impulse responses after the Great Moderation



Notes to the figures.

Note to Figure 1. Figure 1 reports impulse responses from Ireland's (2007) unconstrained model with endogenous inflation target. Each panel shows the percentage-point response of one of the model's variables to a one-standard deviation shock. The inflation and interest rates are expressed in annualized terms. Panel A shows Impulse Responses before the estimated break, and Panel B shows Impulse Responses after the estimated break.

Note to Figure 2. Figure 2 reports impulse responses from the SVAR (1). Panel A shows Impulse Responses before the estimated break, and Panel B shows Impulse Responses after the estimated break.