

Partisan Politics and Aggregation Failure with Ignorant Voters[†]

Faruk Gul
and
Wolfgang Pesendorfer

Princeton University

March 2005

Abstract

We analyze how voter ignorance effects competing candidates' policy choices and election outcomes. We show that voter ignorance of policy positions has no effect in large elections provided voters know the preference distribution in the electorate. We then explore a model where voters are ignorant of policy positions and of the preference distribution. In that case limit equilibria (as the number of voters gets large) yield *partisan politics* (i.e., candidates may not adopt the median favored policy) and *aggregation failure* (i.e., voters may reject the median preferred alternative). These non-Downsian conclusions hold even when candidates have weak policy preferences and mostly care about winning the election.

[†] Financial support from the National Science Foundation is gratefully acknowledged.

1. Introduction

Surveys routinely find that the American electorate is poorly informed.¹ Delli-Carpini and Keeter (1993) cite a 1990-91 National Election Study survey² indicating that only 57% of voters could correctly identify relative ideological positions of the Republican and Democratic parties³ and only 45% of voters could correctly identify the parties' relative position on federal spending.⁴ The same survey shows that voters are no better informed of the electorate than they are of party positions. For example, in the 1990-91 survey only 47% of voters correctly identified the party that holds the majority in the Senate.⁵ (See Delli-Carpini and Keeter (1993), Table 2).

We study the effect of voter ignorance on candidate competition and policy outcomes. We show that vote ignorance allows candidates to choose their favored policy even if this policy is not median preferred. Furthermore, we show that a small asymmetry between candidates – for example, a small voter preference for the personality or the appearance of one candidate – can have a large impact on equilibrium policy selection.

We consider a very simple and stylized candidate competition model. There are two candidates; the *committed candidate* a is committed to a fixed moderate policy m , while the *opportunistic candidate* b must choose between a partisan policy l and the moderate policy m . Voters prefer the moderate policy m to the partisan policy l . In case both candidates choose the moderate policy, voters have a personality preference that leads them to prefer either a or b . We assume that the personality preference is less important than policy preference; that is, no voter prefers candidate b if he chooses the partisan policy.

In our model, the committed candidate chooses the median preferred policy and therefore offers the toughest possible competition for the opportunistic candidate. Nevertheless, we show that the opportunistic candidate is able to exploit voter ignorance to implement the partisan policy in some states of the world.

The standard model of party competition (Downs 1957) considers two candidates who maximize the probability of getting elected. Both candidates choose policies prior to the

¹ For an early reference, see Berelson, Lazarsfeld and McPhee (1954)

² The National Election Study survey is conducted by the Survey Research Center, University of Michigan. The sample size was 449 and consisted of US citizen of voting age.

³ 25% of the answers were “incorrect or incomplete” and 18% answered “don’t know.”

⁴ 26% of the answers were “incorrect or incomplete” and 29% answered “don’t know.”

⁵ 17% of answers were incorrect or incomplete and 36% answered “don’t know.”

election. Voters observe party policies and choose the candidate who offers the more attractive policy. The model predicts that the median preferred policy will be implemented. Candidate competition is similar to Bertrand competition in oligopoly models: if a candidate chooses a policy other than the median voter's favorite he will be "under-cut" by his opponent. As a result, the Downsian prediction of median preferred outcomes holds even when the candidates have policy preferences.

Our model differs from the standard Downsian model in three ways: first, we assume that some voters are ignorant of the opportunistic candidate's policy choice. We incorporate this ignorance into a strategic model by assuming that each voter observes the realized policy choice with a probability between 0 and 1.

Second, we assume that voters are ignorant of other voters' preferences. To model this, we introduce a state variable that determines the distribution of personality preferences and assume that voters do not observe the realized state.

Third, the (opportunistic) candidate learns the state (the distribution of personality preferences) before selecting his policy. Our assumptions are motivated by the fact that candidates often take (secret) opinion polls measuring how their personality is perceived by voters. These opinion polls may provide precise information.

Our focus is on large elections. Therefore, we study limit equilibria as the number of voters goes to infinity. We normalize the opportunistic candidate's utility function so that his utility of winning the election with the partisan policy is 1, his utility of losing the election is 0, and his utility of winning with the moderate policy is $\mu \in (0, 1)$. Hence, μ close to 0 describes a candidate who derives utility from winning only if he can implement his favored policy while μ close to 1 describes a candidate who is motivated primarily by winning the election. If μ is close to 1, we refer to the candidate as an *office seeker*. Since we find this to be the more descriptive case, many of our results assume that the opportunistic candidate is an office seeker.

Our main results establish the following departures from the standard Downsian model:

- (1) *Personality Matters*: The distribution of personality preferences affects the policy choice. If the opportunistic candidate has a substantial personality advantage he chooses the partisan policy. We refer to this phenomenon as *partisan politics*.

- (2) *Non-median Election Outcomes*: The candidate who offers the median preferred policy may lose the election. We refer to this phenomenon as *aggregation failure*.
- (3) *Partisan politics and aggregation failure when the opportunistic candidate is an office seeker*: An office seeker wins the election whenever the majority prefers his personality. The probability that the office seeker implements the partisan policy is positive and decreasing in δ , the fraction of informed voters. If δ is close to 0, the office seeker implements the partisan policy with probability close to 1 whenever he is elected.

Our results imply that small asymmetries between candidates can have large effects on election outcomes. There is evidence that voters react to superficial differences in candidates.⁶ Our model suggests that such behavior can be rationalized as equilibrium behavior when voters are poorly informed and candidates care mostly about winning. In that case, the naive strategy of simply voting for the candidate who looks better or has the nicer personality is equilibrium behavior even though voters place little weight on such traits.

Voters behave in this seemingly naive way because they must condition on the event that a vote is pivotal. An office seeker (μ close to 1) will choose the moderate policy when he expects the election to be close. Anticipating this behavior, voters conclude that conditional on a vote being pivotal the opportunistic candidate is likely to choose the moderate policy. As a result, uninformed voters who face an office seeker behave *as if* the opportunistic candidate always chooses the moderate policy and vote according to their personality preference.

A high probability of choosing the moderate policy when the election is close does not translate into a high unconditional probability of choosing the moderate policy. Because an office seeker receives a large share of the uninformed vote he can choose the partisan policy over a wide range of states without risk of losing the election. Conditioning on being pivotal creates a wedge between voting behavior and ex ante policy choices. The size of this wedge depends on the fraction of informed voters. If the fraction of informed voters

⁶ For example, Redlawsk and Lau (2003) conduct experiments in which subjects participate in mock elections where an unattractive candidate was pitted against an attractive opponent. Voters were presented with pictures and descriptions of the candidates' personality. The experimental results suggest a significant effect of attractiveness on voter behavior.

is relatively small then this wedge becomes large. As the fraction of informed voters goes to 0, the opportunistic office seeker will choose the partisan policy and win the election whenever the majority prefers his personality.

To isolate the effects of our three main assumptions, we consider two alternative versions of our main model. First, we study a benchmark model in which voters are ignorant of the candidate’s policy choice but both the voters and the candidate know the distribution of personality preferences. In that model, Downsian predictions are attained: in large elections, the moderate policy is implemented with probability 1 and therefore neither partisan politics nor aggregation failures occurs. The candidate whose personality is preferred by the majority wins the election. Hence, in the benchmark case personality preferences have no effect on the policy outcome. Despite the fact that only a fraction of voters are informed, the outcome is *as if* all voters are know the policy choices.

Unlike the voters, the opportunistic candidate observes the state (i.e., the distribution from which voter preferences are drawn). To isolate the effect of this latter assumption we consider a model in which neither the voters nor the opportunistic candidate observe the state. We show that if the opportunistic candidate is an office seeker, the election outcome is as in the benchmark case above: the moderate policy is implemented in all states.

It might be argued that political competition forces candidates to inform the voters of their positions. Hence, the evidence of voter ignorance may be considered puzzling. In section 5, we investigate this hypothesis. We find that giving the candidate the opportunity to increase the proportion of informed voters has no effect when the candidate is an office seeker. Hence, permitting voluntary disclosure does not mitigate partisan politics or aggregation failure. This is true even though informing voters is costless. However, our analysis suggests that informing voters about the opponent’s position, provided such information can be revealed credibly, may be an effective remedy for partisan politics and aggregation failure. Hence, we find a role for “negative campaigning.”

1.1 Related Literature

Several authors have examined the robustness of Downs’ results by introducing policy motivated candidates and uncertainty about median voter preferences. For example,

Wittman (1977) and Calvert (1985) consider a model with two candidates, uncertain distribution of voter preferences but no asymmetric information. In Bernhard, Duggan and Squintani (2003) and Chan (2001) candidates have asymmetric information. In all these models, candidates typically choose distinct policy positions. Because the median’s policy preference is not known, candidates trade-off the probability of losing the election against winning with a less desired policy. However, when candidates are office seekers and mostly care about winning they converge to the same policy position. In contrast, in our model the tendency to choose partisan positions is most pronounced when a candidate is an office seeker.

Ansola-behere and Snyder (2000), Groseclose (2001), Aragoes and Palfrey (2002, 2004) examine models of Downsian competition with uncertain median preferences where one candidate has a “valence” advantage formally equivalent to the personality advantage analyzed here. Aragoes and Palfrey (2002) show that equilibrium entails mixed strategies and hence candidates typically choose distinct policy positions.

In all related studies of Downsian competition, aggregation failure cannot occur since voters know the policy choices of the candidates. Candidates choose partisan positions in the hope that it will be favored by the realized median preference. Hence, distinct policy positions benefit the median voter in some states of the world. In our model, candidates choose partisan positions even though they know the median prefers the moderate policy in all states of the world. Candidates benefit from this behavior because uninformed voters cannot detect the partisan choice.

Austen Smith and Banks (1995) and Feddersen and Pesendorfer (1996, 1997) study models with asymmetrically informed voters. The Feddersen and Pesendorfer papers show that large elections effectively aggregate information if policy positions are fixed and voters are uncertain about the “quality” of the candidates’ policies. Our model has both asymmetrically informed voters and candidate competition. In our context, the Feddersen and Pesendorfer result would correspond to a situation where the opportunistic candidate’s policy is exogenously (and randomly) chosen. The difference here is that the candidate’s policy choice is a strategic variable. Our benchmark model (i.e., when both the voters and the candidate know the distribution of preferences) extends the information aggregation results to the case where candidates choose policies strategically.

In a series of papers, McKelvey and Ordeshook (1985, 1986) argue that even if voters are ignorant of policy choices they may still infer which candidate offers the preferred policy from polling data, endorsements, and other public information. In other words, McKelvey and Ordeshook argue that ignorance about policy choices alone may not lead to non-median outcomes. Our conclusion is similar: if voters learn the preference distribution through opinion polls then we are in the benchmark case where information is aggregated and voter ignorance about policy choice is irrelevant. However, if some fraction of voters remains uninformed about preference distribution then the election cannot aggregate information and non-median outcomes will result.

2. The Model

First, we describe the preferences of all agents and the distribution of voter preferences.

Candidates: Two candidates stand for election. Candidate a is committed to the moderate policy m , while candidate b must choose between m , and a partisan policy l . Candidate b 's payoff is 1 if he is elected and implements l , $\mu \in (0, 1)$ if he is elected and implements m , and 0 if he is not elected.

Voters: There are $2n+1$ voters. A voter's payoff depends on the implemented policy and on the identity of the winning candidate. Since policy l is implemented only if candidate b is elected, there are three possible election outcomes, denoted l, m_a, m_b , where m_j means j wins the election and implements policy m . A voter's type is a pair (λ, ν) where $\lambda \in [0, 1]$ and $\nu \in \{a, b\}$. Every type (λ, ν) receives the payoff 0 if the outcome is l . Type (λ, b) 's payoff is $\lambda + \epsilon$ if the outcome is m_b and λ if the outcome is m_a . Type (λ, a) 's payoff is $\lambda + \epsilon$ if the outcome is m_a and λ if the outcome is m_b . The parameter ϵ is strictly positive.

Our specification of voter preferences implies that all voters prefer m to l ; if both candidates choose m then each voter has a preference for one of the candidates. We interpret this as a "personality preference". Voters are differentiated by the weight $\frac{\lambda}{\lambda+\epsilon}$ they place on policy versus personality. A higher λ means more weight on policy and less on personality. Lowering ϵ decreases the fraction of types for whom $\frac{\lambda}{\lambda+\epsilon}$ is above any given threshold. Hence, fixing the distribution of λ and increasing the parameter ϵ increases the importance of personality for the electorate as a whole.

Distribution of Types: The two dimensions of a voter’s type are drawn independently; λ has probability distribution F with support $[0, 1]$. The variable ν is a binary random variable and $s \in [0, 1]$ denotes the probability that ν takes on the value a .

Assumption: F admits a continuous, strictly positive density f on its support $[0, 1]$.

We refer to s as the state of the electorate. The parameter s specifies which candidate is more likely to have a personality advantage. For $s < 1/2$ each voter is more likely to prefer candidate b ’s personality. We say that b has a personality advantage if $s < 1/2$ and that a has a personality advantage if $s > 1/2$. We write $F \times s$ for the probability distribution used to assign types in state s .

3. The Benchmark Case

We consider a benchmark case with a fixed state s . Hence, the distribution of personality preferences is fixed and common knowledge among voters and candidates. Proposition 1 extends the Downsian prediction of median preferred outcomes to the case where only a (small) fraction of voters know the candidate’s policy choice.

In the next section, we will assume that voters are uncertain about the distribution of voter preferences (i.e., the state s) and about the candidate’s policy choice. In that case, equilibrium outcomes will not be Downsian.

This section analyzes the following voting game:

- (i) Nature draws types for the voters according to $F \times s$ and voters learn their own type.
- (ii) Candidate b chooses a policy. Let $\sigma^b \in [0, 1]$ denote candidate b ’s strategy where σ^b is the probability b chooses m .
- (iii) Candidate b policy choice is realized. Each voter is independently informed of the realized policy with probability $\delta \in (0, 1)$.
- (iv) Each voter casts a vote for a or b . An uninformed voter’s action depends on his type (λ, ν) . An informed voter’s action depends on the his type and on b ’s realized policy.
- (v) The candidate who receives the most votes ($n + 1$ or more) wins the election and implements his policy.

We analyze symmetric Nash equilibria in weakly undominated strategies. We call such equilibria *voting equilibria*. Informed voters and voters who prefer a 's personality (type (\cdot, a) voters) have a simple dominant strategy. Type (\cdot, a) voters always vote for a . Type (\cdot, b) voters who are informed vote for b if b chooses m and vote for a otherwise. Only uninformed type (\cdot, b) voters have a non-trivial decision problem. The lemma below shows that a best response for those voters is characterized by a cutoff.

Lemma: Fix $\sigma^b \in [0, 1]$ and consider a symmetric and weakly undominated best response by voters. There is a cutoff $\sigma^v \in [0, 1]$ such that an uninformed type (λ, b) votes for b if $\lambda < \sigma^v$ and for a if $\lambda > \sigma^v$.

Proof: If $\sigma^b = 0$ then the unique symmetric and weakly undominated best reply for voters is to vote for a irrespective of type. Hence, the cutoff is $\sigma^v = 0$. If $\sigma^b = 1$ the unique symmetric and weakly undominated best reply for all type (\cdot, b) voters is to vote for b irrespective of λ . In that case, the cutoff is $\sigma^v = 1$. If $1 > \sigma^b > 0$, there is a positive probability that an uninformed type (\cdot, b) voter is pivotal. This follows because informed type (\cdot, b) voters either all vote for a (if b chooses l) or all vote for b (if b chooses m). In at least one of these cases there is a strictly positive probability that a vote is pivotal. Let θ denote the probability that b chooses m conditional on the event that an uninformed type (\cdot, b) is pivotal. Note that θ is independent of λ . The optimal action for an uninformed voter type (λ, b) is to vote for b if

$$\lambda < \epsilon \cdot \frac{\theta}{1 - \theta}$$

and to vote for a if this inequality is reversed. This proves the Lemma. \square

Since F is continuous, the cutoff σ^v suffices as a description of the strategy of uninformed type (\cdot, b) voters. In the following, we represent the behavior of voters in a voting equilibrium with a cutoff σ^v and let $\sigma = (\sigma^b, \sigma^v)$ denote an equilibrium profile. It is understood that the cutoff σ^v describes the behavior of uninformed type (\cdot, b) voters and that all other voters choose their dominant strategy.

Let $\pi^o(\sigma^v)$ be the probability that a randomly selected voter votes for b conditional on b choosing the policy $o \in \{l, m\}$. That is;

$$\begin{aligned} \pi^l(\sigma^v) &= (1 - s)(1 - \delta)F(\sigma^v) \\ \pi^m(\sigma^v) &= (1 - s)((1 - \delta)F(\sigma^v) + \delta) \end{aligned} \tag{1}$$

For $x \in [0, 1]$, let $B_n(x)$ denote the binomial probability of at least $n + 1$ successes out of $2n + 1$ trials given that the probability of success in each trial is x . Hence,

$$B_n(x) = \sum_{k=n+1}^{2n+1} \binom{2n+1}{k} x^k (1-x)^{2n+1-k} \quad (2)$$

Then, $B_n(\pi^o(\sigma^v))$ is the probability that candidate b wins the election given that b chooses policy $o \in \{l, m\}$ and voters use strategy σ^v .

Given a profile (σ^b, σ^v) we let ϕ^o denote the probability that the outcome is $o \in \{l, m_a, m_b\}$.

$$\begin{aligned} \phi^l &= (1 - \sigma^b) B_n(\pi^m(\sigma^v)) \\ \phi^{m_b} &= \sigma^b B_n(\pi^m(\sigma^v)) \\ \phi^{m_a} &= 1 - \phi^{m_b} - \phi^l \end{aligned} \quad (3)$$

For a fixed $F, \mu, \delta, \epsilon, s$ satisfying the assumptions above, let \mathcal{E}_n^0 denote the set of all voting equilibria and let $\Phi_n(\sigma)$ denote the outcome ϕ of $\sigma \in \mathcal{E}_n^0$ as defined by equation (3). Let \mathcal{E}^0 denote the set of limit equilibria; that is, $\sigma \in \mathcal{E}^0$ if there exists a sequence $\{\sigma_n\}$ converging to σ such that $\sigma_n \in \mathcal{E}_n^0$ for all n . For any $\sigma \in \mathcal{E}^0$, $\Phi(\sigma)$ denotes the set of possible outcomes associated with the limit equilibrium σ . That is, $\phi \in \Phi(\sigma)$ if there exists $\sigma_n \in \mathcal{E}_n^0$ for all n such that σ_n converges to σ and $\Phi_n(\sigma_n)$ converges to ϕ .

Proposition 1 characterizes equilibrium outcomes for large electorates. It states that in a limit equilibrium the moderate policy m is implemented with probability 1. Hence, the fact that a (possibly large) fraction of voters are ignorant of b 's policy choice has virtually no impact on the election outcome when n is large. Moreover, candidate b is elected if and only if he has a personality advantage. That is, personality matters for who gets elected but does not influence the chosen policy.

Proposition 1: *If $\sigma \in \mathcal{E}^0$ and $\phi \in \Phi(\sigma)$ then $\phi^{m_b} = 1$ if $s < 1/2$ and $\phi^{m_a} = 1$ if $s > 1/2$.*

Proof: The result for $s > 1/2$ is an obvious consequence of the law of large numbers. So, assume $s < 1/2$.

If $\sigma_n \in \mathcal{E}_n^0$ and $\sigma_n^b = 0$ then b receives no votes and therefore a deviation to m strictly increases b 's expected payoff. Hence, for all n , $\sigma_n^b > 0$. Next, note that if $\sigma_n^b = 1$ then

$\pi^m = (1 - s) > 1/2$ and the result again follows from the law of large numbers. Hence, only the case where (along some subsequence) $\sigma_n^b \in (0, 1)$ for all n is left to consider.

Since b chooses both policies with strictly positive probability he must be indifferent between them. This indifference implies

$$\mu B_n(\pi^l(\sigma^v)) = B_n(\pi^m(\sigma^v)) \quad (4)$$

Next we show that (4) implies $\lim \pi^l(\sigma_n^v) = 1/2$ along any sequence $\sigma_n \in \mathcal{E}_n^0$. If $\pi^l(\sigma_n^v) \geq 1/2 + \eta$ along any (sub)sequence then $\lim B_n(\pi^l(\sigma_n^v))/B_n(\pi^m(\sigma_n^v)) = 1$ violating (4). Similarly, $\pi^l(\sigma_n^v) \leq 1/2 - \eta$ for all n implies $\lim B_n(\pi^l(\sigma_n^v))/B_n(\pi^m(\sigma_n^v)) = 0$, again violating (4). Since $\pi^l(\sigma_n^v)$ converges to $1/2$, equation (1) implies $\lim \sigma_n^v > 0$. As we noted when proving that voters must use a cutoff strategy, voter optimality requires that $\sigma_n^v = \eta \cdot \frac{\theta}{1-\theta}$ where θ is equal to the conditional probability that candidate b has chosen m given that the voter is pivotal. Hence,

$$\theta = \frac{\sigma_n^b \binom{2n+1}{n+1} \pi^m(\sigma_n^v)^n (1 - \pi^m(\sigma_n^v))^n}{\sigma_n^b \binom{2n+1}{n+1} \pi^m(\sigma_n^v)^n (1 - \pi^m(\sigma_n^v))^n + \sigma_n^b \binom{2n+1}{n+1} \pi^l(\sigma_n^v)^n (1 - \pi^l(\sigma_n^v))^n} \quad (5)$$

Some simplification of (5) yields

$$\frac{\sigma_n^b}{1 - \sigma_n^b} \cdot \frac{\alpha_n^m}{\alpha_n^l} \cdot \epsilon = \sigma^v$$

where $\alpha^o = \pi^o(\sigma^v)^n (1 - \pi^o(\sigma^v))^n$ for $o \in \{m, l\}$. Note that α_n^m/α_n^l converges to 0 since $\pi(\sigma_n^v)$ converges to $1/2$. Therefore, $\lim \sigma_n^v > 0$ implies $\lim \sigma_n^b = 1$. Also, equation (1) yields $\lim \pi^m(\sigma_n^v) = 1/2 + (1 - s)\delta$. Hence, the probability of b winning conditional on choosing m converges to 1, as desired. \square

Proposition 1 establishes that neither partisan politics nor aggregation failure can occur in our benchmark model; candidate b chooses the median preferred policy and wins the election whenever his personality is preferred by the median voter.

We can contrast Proposition 1 with earlier information aggregation results in Feddersen and Pesendorfer (1997). Suppose that the strategy $\sigma^b \in (0, 1)$ is fixed and $s < 1/2$. If m is realized then the majority of the electorate prefers b (when n is large) whereas when l is realized the majority prefers a . The earlier information aggregation result implies that

for a *fixed* strategy σ^b and large n candidate b is elected with probability close to 1 if the realized action is m and candidate a is elected with probability close to 1 if the realized action is l .

In the model analyzed here, the strategy σ^b is not fixed but endogenous. Proposition 1 pins down both voter and candidate behavior. For $s < 1/2$ and n large, b chooses l with positive probability and conditional on choosing l wins with positive probability. Nevertheless, information aggregation is achieved because the probability that b chooses l converges to 0.

Next, we provide intuition for Proposition 1. Recall that π^o denotes the probability that a randomly chosen voter votes for b if b chooses policy $o \in \{l, m\}$. If π^l is less than $1/2$ and bounded away from $1/2$ then candidate b strictly prefers m to l when n is large. This is clear if b 's vote share is greater than $1/2$ conditional on m , which would mean that he wins for sure with m and loses for sure with l . If his vote share is less than $1/2$ in both cases, then his probability of winning goes to 0 with either policy, but it goes to 0 much faster with l than with m . Hence, in both cases b strictly prefers m to l .

The second step is to note that for large n , candidate b must mix in equilibrium. If in equilibrium b were to choose l for sure, then $\pi^l = 0$. In that case, the argument above establishes that he strictly prefers m . If in equilibrium b were to choose m for sure then the support of the uninformed voters would guarantee victory for b irrespective of the policy choice, i.e., $\pi^l > 1/2$. In that case, b strictly prefers l .

The third step is to observe that in order to maintain b 's indifference between l and m , it is necessary for π^l to converge to $1/2$. If π^l stays bounded above $1/2$ then b wins for sure with l and hence would never choose m . If π^l stays bounded below $1/2$, then the rate of convergence argument above establishes that b would strictly prefer m .

Finally, since the probability of winning with l converges to $1/2$ (and therefore the probability of winning with m converges to 1), conditional on a vote being pivotal it is much more likely that b has chosen l than m . Therefore, to maintain the incentives for uninformed voters it must be the case that b chooses l with vanishing probability as n goes to infinity. Hence, in large electorates b will choose m almost all the time and almost always wins when he has a personality advantage.

4. Uncertainty about the Electorate

In this section, we relax the assumption that the state s is common knowledge. We assume that candidates are informed of the state but voters remain uncertain. Recall that the state describes the distribution of personality preferences in the electorate. Hence, we assume that candidates are better informed about the distribution of voter preferences than voters. Our main result (Proposition 3) demonstrates that in this model election outcomes do not conform to the Downsian prediction. Candidate b 's equilibrium policy choice differs from the median preferred policy (partisan politics) and the majority preferred candidate may lose the election (aggregation failure).

Recall that the random variable $\nu \in \{a, b\}$ captures voters' personality preferences and s describes the distribution of ν . We assume that candidate b observes the state s before choosing his policy. Let G be the distribution of s . Voters do not observe s , however a voter's own personality preference provides him some information about s .

Assumption: G admits a continuous, strictly positive density g on its support $[0, 1]$.

We analyze the following voting game:

- (i) Nature draws s according to G and independently assigns each voter a preference type according to $F \times s$. Voters learn their preference types.
- (ii) Candidate b observes s and chooses a policy. Candidate b 's strategy is a function from states $s \in [0, 1]$ to the probability of choosing m .
- (iii) Each voter is independently informed of candidate b 's realized policy choice with probability $\delta \in (0, 1)$. Voters do not observe the realized state s .
- (iv) Each voter casts a vote for a or b . An uninformed voter's action is a function of his type (λ, ν) . An informed voter's action also depends also on b 's realized policy.
- (v) The candidate who receives the most votes ($n + 1$ or more) wins the election and implements his policy.

As in the benchmark case of section 3, informed voters and type (\cdot, a) voters have a simple dominant strategy. Type (\cdot, a) voters always votes for a . An type (\cdot, b) voter who is informed votes for b if and only if b chooses m .

If $\delta > 1/2$ then typically a majority of voters is informed when the electorate is large. In that case, informed voters are always decisive and candidate b can win the election only if he chooses the moderate policy. Therefore, b will choose the moderate policy in equilibrium and the Downsian predictions of Proposition 1 continue to hold. We focus on the case where voter ignorance plays a role.

Assumption: *Each voter is informed of the policy choice with probability $\delta < 1/2$.*

Only voters who are uninformed and prefer the personality of b have a non-trivial decision problem. As in the benchmark case analyzed in section 3, a best response by those voters is characterized by a cutoff.

Lemma: *Fix a strategy σ^b and consider a symmetric and weakly undominated best response by voters. There is a cutoff $\sigma^v \in [0, 1]$ such that an uninformed type (λ, b) votes for b if $\lambda < \sigma^v$ and for a if $\lambda > \sigma^v$.*

Proof: Identical to the proof of the Lemma in section 3. □

As in section 3, we represent the behavior of voters in a voting equilibrium by a cutoff $\sigma^v \in [0, 1]$. As before, it is understood that the cutoff describes the behavior of uninformed type (\cdot, b) voters and all other voters choose their dominant strategy.

Given $\sigma^v \in [0, 1]$, let $\pi^o(\sigma^v, s)$ denote the probability that a randomly selected voter casts a vote for b in state s conditional on candidate b choosing policy o . That is;

$$\begin{aligned}\pi^l(\sigma^v, s) &= (1 - s)(1 - \delta)F(\sigma^v) \\ \pi^m(\sigma^v, s) &= (1 - s)[(1 - \delta)F(\sigma^v) + \delta]\end{aligned}\tag{6}$$

The probability that b wins given that he chooses policy o in state s is $B_n(\pi^o(\sigma^v, s))$. (Recall that $B_n(x)$ is the binomial probability of at least $n + 1$ successes in $2n + 1$ trials.)

Our first result establishes that every equilibrium strategy of candidate b is also described by a cutoff. That is, in any equilibrium there is a state $\sigma^b \in [0, 1]$ such that b chooses l at states $s' < s$ and m at $s' > s$ (with probability 1). We call an equilibrium in which voters and candidate b use a cutoff strategy a cutoff equilibrium. A cutoff equilibrium $\sigma = (\sigma^b, \sigma^v)$ is an element of $[0, 1] \times [0, 1]$. Since the probability of any single λ or any single s is 0, these two numbers suffice to describe a cutoff equilibrium.

Proposition 2: *There exists a voting equilibrium and every voting equilibrium is a cutoff equilibrium.*

Proof: see Appendix.

From the Lemma above we know that the optimal voter strategy is a cutoff strategy. Hence, to prove the second assertion of the proposition it is enough to show that b 's best response to any cutoff strategy is also a cutoff strategy. Note that candidate b chooses l if

$$B_n(\pi^l(\sigma^v, s)) > \mu B_n(\pi^m(\sigma^v, s))$$

and m if this inequality is reversed. To prove that b 's best response is a cutoff strategy we show that $B_n(\pi^l(\sigma^v, s))/B_n(\pi^m(\sigma^v, s))$ is decreasing in s . This is done in Lemma 2. From this it follows that if m is optimal at s then it is the only optimal action at $s' > s$. Hence, the best response must be a cutoff strategy. We use a fixed-point argument to establish the existence of a cutoff strategy equilibrium.

The probability of outcomes $o \in \{l, m_a, m_b\}$ given the strategy profile σ and the state s is denoted by $\phi^o(s)$ and given by equation (7).

$$\begin{aligned} \phi^l(s) &= \begin{cases} B_n(\pi^l(\sigma^v, s)) & \text{if } s < \sigma^b \\ 0 & \text{if } s > \sigma^b \end{cases} \\ \phi^{m_b}(s) &= \begin{cases} B_n(\pi^m(\sigma^v, s)) & \text{if } s > \sigma^b \\ 0 & \text{if } s < \sigma^b \end{cases} \\ \phi^{m_a}(s) &= 1 - \phi^{m_b}(s) - \phi^l(s) \end{aligned} \tag{7}$$

For a fixed $F, G, \epsilon, \mu, \delta$ satisfying the assumptions above, let \mathcal{E}_n denote the set of equilibria. For $\sigma \in \mathcal{E}_n$, let $\Phi_n(\sigma)$ denote the corresponding outcome. Let \mathcal{E} denote the set of limit equilibria; that is \mathcal{E} is the set of σ such that $\sigma = \lim_{n \rightarrow \infty} \sigma_n$ for $\sigma_n \in \mathcal{E}_n$ for all n . Then, define $\Phi(\sigma)$ as the set of all ϕ such that for some sequence σ_n converging to σ such that $\sigma_n \in \mathcal{E}_n$ for all n , $\Phi_n(\sigma_n)$ converges to ϕ .

The lemma below characterizes the set of limit equilibria \mathcal{E} . The states $s \in [0, 1]$ in which half of the electorate is expected to vote for either candidate play an important role in this characterization. We refer to those states as *critical states*. Let $s^o(\sigma^v)$ be the state s that satisfies $\pi^o(\sigma^v, s) = 1/2$. Hence, $s^o(\sigma^v)$ is the state at which a randomly drawn voter

chooses b with probability $1/2$ if b chooses policy o and the voter cutpoint is σ^v . We refer to $s^o(\sigma^v)$ as o 's critical state at σ^v . Let $\underline{\sigma}^v$ be defined as

$$(1 - \delta)F(\underline{\sigma}^v) = 1/2$$

Policy l 's critical state at $\underline{\sigma}^v$ is 0 (i.e., $s^l(\underline{\sigma}^v) = 0$). For $\sigma^v < \underline{\sigma}^v$, policy l 's critical state is not well defined. For $\sigma^v \in [\underline{\sigma}^v, 1]$, we have:

$$\begin{aligned} s^l(\sigma^v) &= 1 - \frac{1}{2(1 - \delta)F(\sigma^v)} \\ s^m(\sigma^v) &= 1 - \frac{1}{2((1 - \delta)F(\sigma^v) + \delta)} \end{aligned} \tag{8}$$

Clearly both s^l and s^m are increasing functions of σ^v . Moreover,

$$\frac{1}{2} \geq s^m(\sigma^v) > s^l(\sigma^v)$$

For $\sigma \in [\underline{\sigma}^v, 1]$, we define $\ell(\sigma^v)$ as follows:

$$\ell(\sigma^v) := \frac{g(s^m(\sigma^v))}{g(s^l(\sigma^v))} \cdot \left(\frac{(1 - \delta)F(\sigma^v)}{(1 - \delta)F(\sigma^v) + \delta} \right)^2 \tag{9}$$

The following lemma characterizes equilibria. Part (i) describes limit equilibrium outcomes in terms of the critical states $s^l(\sigma^v)$ and $s^m(\sigma^v)$. Part (ii) characterizes the equilibrium voter cutoff σ^v .

Characterization Lemma: *If $\sigma \in \mathcal{E}$ and $\phi \in \Phi(\sigma)$ then (i) $\sigma^b = s^l(\sigma^v)$, $\sigma^v \in [\underline{\sigma}^v, 1]$ and*

$$\begin{aligned} \phi^l(s) &= 1 && \text{if } s < s^l(\sigma^v) \\ \phi^m(s) &= 1 && \text{if } s^l(\sigma^v) < s < s^m(\sigma^v) \\ \phi^r(s) &= 1 && \text{if } s^m(\sigma^v) < s \end{aligned}$$

(ii) *Either*

$$\sigma^v = \min\left\{\frac{\epsilon}{1 - \mu} \cdot \ell(\sigma^v), 1\right\} \geq \underline{\sigma}^v$$

or

$$\sigma^v = \underline{\sigma}^v \geq \frac{\epsilon}{1 - \mu} \cdot \ell(\underline{\sigma}^v)$$

Proof: see Appendix

For $s < s^l(\sigma^v)$, part (i) above establishes that candidate b chooses the partisan policy l and wins the election. The probability that a randomly selected voter will vote for candidate b if b chooses l is greater than $1/2$ at any state $s < s^l(\sigma^v)$. Hence, in a large electorate, b wins with probability 1 when he chooses l , and since he strictly prefers l to m , he chooses l for sure. Therefore, $\phi^l(s) = 1$ at states $s < s^l(\sigma^v)$.

Part (i) also asserts that candidate b chooses the moderate policy at state $s > s^l(\sigma^v)$. At such states, b loses the election with probability close to 1 if he chooses l . In that case, b is better off if he chooses m and thereby secures the vote of informed agents who prefer his personality. At $s \in (s^l(\sigma^v), s^m(\sigma^v))$, informed agents are decisive and candidate b wins the election. At $s > s^m(\sigma^v)$, candidate a chooses m and wins the election. The following figure summarizes part (i) of the Characterization Lemma.

–Insert figure 1 here–

Part (ii) describes the voters' equilibrium strategy. In all cases, the limiting value of voters' cutoff satisfies $\sigma^v \in [\underline{\sigma}^v, 1]$. For the case of an “interior” equilibrium (i.e., $\underline{\sigma}^v < \sigma^v < 1$) the lemma yields

$$\sigma^v = \epsilon \cdot \frac{\ell(\sigma^v)}{1 - \mu}$$

At an interior equilibrium the voter type (σ^v, b) must be indifferent between a and b . Therefore,

$$\sigma^v = \epsilon \cdot \frac{\theta}{1 - \theta}$$

where θ is the probability that b chooses m conditional on the vote by type (σ^v, b) being pivotal. Hence, characterizing the voters' limit strategy amounts to characterizing the limit beliefs of voters conditional on a vote being pivotal. The lemma shows that in the interior case we have

$$\frac{\theta}{1 - \theta} = \frac{\ell(\sigma^v)}{1 - \mu}$$

Next, we provide intuition for this characterization of beliefs. Note that as the number of voters becomes large, the probability of being pivotal is concentrated around states in which the election is expected to be tied. There are two such states, $s^l(\sigma^v)$ and $s^m(\sigma^v)$. Hence, conditional on being pivotal, a voter knows that the state is in one of two small “critical intervals” around the critical states. Therefore, the inference problem for the uninformed voter is reduced to determining the relative likelihoods of the events

$$E_l := [s \text{ is close to } s^l(\sigma^v) \text{ and } b \text{ chooses } l]$$

and

$$E_m := [s \text{ is close to } s^m(\sigma^v) \text{ and } b \text{ chooses } m]$$

conditional on a vote being pivotal.

Consider candidate b 's incentives when s is close to $s^l(\sigma^v)$. If the probability of winning with l is less than μ -times the probability of winning with m then b strictly prefers the moderate policy m . Therefore, E_l is truncated at the state where the probability of winning drops below μ . (The probability of winning the election with m in a neighborhood of $s^l(\sigma^v)$ is close to 1). Hence, the closer the parameter μ is to 1 the smaller is E_l . The key step in the proof is to show that the relative likelihood of E_l and E_m conditional on a vote being pivotal is related to μ by the simple formula given in the Characterization Lemma.

Proposition 3, our main result, examines the case where b is an office seeker (i.e., μ is close to 1), and shows that when uninformed voters face an office seeker they choose $\sigma^v = 1$. Hence, uninformed voters cast their vote for the candidate with the preferred personality. As a result, candidate b wins the election whenever $s < 1/2$, i.e., whenever the majority prefers his personality.

Proposition 3: *Fix F, G, ϵ . Then, there is $\bar{\mu}$ (independent of δ) such that for $\mu > \bar{\mu}$ the limit equilibrium $\sigma \in \mathcal{E}(\mu)$ satisfies $\sigma^v = 1, \sigma^b = (1 - 2\delta)/(2 - 2\delta)$ and the limit outcome $\phi \in \Phi(\sigma)$ satisfies*

$$\phi^o(s) = \begin{cases} 1 & \text{if } o = l, \quad s \in \left[0, \frac{1-2\delta}{2-2\delta}\right) \\ 1 & \text{if } o = m_b, s \in \left(\frac{1-2\delta}{2-2\delta}, \frac{1}{2}\right) \\ 1 & \text{if } o = m_a, s \in \left(\frac{1}{2}, 1\right] \end{cases}$$

Proof: It is easy to see that for a fixed F, G satisfying the assumptions, $\ell(\sigma)$ is uniformly bounded below by some $\eta > 0$. This follows since g, f are bounded above and below, $\underline{\sigma}^v$ is uniformly bounded below, $s^m(\sigma^v) \leq 1/2$ and $\delta < 1/2$. Choose $1 - \mu$ so that $\frac{\eta\epsilon}{1-\mu} > 1$. Then, the Characterization Lemma implies that $\sigma^v = 1$. Since $s^l(1) = (1 - 2\delta)/(2 - 2\delta)$ the Proposition follows from part (ii) of the Characterization Lemma. \square

Although we have assumed that voters are strategic, equilibrium behavior – as characterized in Proposition 3 – seems very naive: uninformed voters simply vote according to their personality preference. This behavior constitutes an equilibrium because voters expect candidate b to choose the moderate policy whenever the election is close. After all, candidate b cares mostly about winning the election and therefore will be reluctant to choose the partisan policy when the state indicates a close election. Therefore, conditional on a vote being pivotal, it is very likely that the office seeker chooses the moderate policy and hence it make sense for uninformed voters to vote according to their personality preference.

Because voters choose according to their personality preference, candidate b wins the election whenever he has a personality advantage, i.e., whenever $s < 1/2$. Note that this is also true of the Downsian benchmark analyzed in section 3. Hence, the uncertainty about the state does not affect the probability that an office seeker is elected. However, in contrast to the Downsian benchmark, Proposition 3 shows that there is a strictly positive probability that the outcome is l , the partisan policy. Note that *all* voters strictly prefer m to l irrespective of the candidate who implements m . Therefore, the outcome l represents an *aggregation failure*, i.e., a failure of the election to choose the median preferred alternative.

Proposition 3 shows a close relationship between the ex ante probability that candidate b chooses the moderate (partisan) policy and the proportion of informed voters. For $o \in \{l, m_a, m_b\}$ let $\bar{\phi}^o = E[\phi^o]$ denote the ex ante probability of outcome o . Proposition 3 shows that when b is an office seeker the ex ante probability of outcome l is

$$\bar{\phi}^l = G\left(\frac{1-2\delta}{2-2\delta}\right) \tag{10}$$

and the probability of outcome m_b is

$$\bar{\phi}^{m_b} = G(1/2) - \bar{\phi}^l \quad (11)$$

The ex ante probability that b wins the election is the sum of $\bar{\phi}^l$ and $\bar{\phi}^{m_b}$ and therefore is equal to $G(1/2)$. Note that this probability is independent of the fraction of informed voters. The fraction of informed voters determines the range of states over which the partisan outcome is implemented. Equations (10) and (11) imply that $\bar{\phi}^l$ is decreasing and $\bar{\phi}^{m_b}$ is increasing in δ when b is an office seeker. Moreover,

$$\lim_{\delta \rightarrow 0} \bar{\phi}^l \rightarrow G(1/2), \lim_{\delta \rightarrow 0} \bar{\phi}^{m_b} \rightarrow 0$$

Hence, when the fraction of informed voters is small then an office seeker implements the partisan policy in almost all states in which he has a personality advantage. Conversely, we have

$$\lim_{\delta \rightarrow 1/2} \bar{\phi}^l \rightarrow 0, \lim_{\delta \rightarrow 1/2} \bar{\phi}^{m_b} \rightarrow G(1/2)$$

Hence, if the proportion of informed voters approaches $1/2$ then the election outcome is close to the Downsian benchmark. In that case, the informed voters are almost always pivotal and candidate b must choose m to get elected.

When the proportion of informed voters is small uninformed voters vote for the opportunistic office seeker despite the fact that he is unlikely to implement their favored policy. This is true even though (for small ϵ) personality is a significant consideration only for a small fraction of the electorate. The disconnect between the ex ante choices of the candidate and voter behavior comes about because voters condition on being pivotal. Even though the candidate chooses the partisan policy in most states, he chooses the moderate policy conditional on a vote being pivotal. Knowing this, uninformed voters behave as if candidate b *always* chooses m .

Next, we analyze the polar opposite case where b has a strong partisan preference. To be able to state a result analogous to Proposition 3 we assume that the personality preference of voters is small. Let λ^* denote the median type, i.e., $F(\lambda^*) = 1/2$ and let

$$\bar{\epsilon} = \lambda^* \cdot \frac{\max g(s)}{\min g(s)}$$

Proposition 4 shows that if μ is sufficiently small then candidate b always chooses the moderate policy and wins only if he has a significant personality advantage.

Proposition 4: *Fix $\epsilon < \bar{\epsilon}$ and F, G satisfying the assumptions. There is $\underline{\mu}$ (independent of δ) such that for $\mu < \underline{\mu}$ the limit equilibrium $\sigma \in \mathcal{E}(\mu)$ and the limit outcome $\phi \in \Phi(\sigma)$ satisfy $\sigma^b = 0, \sigma^v = \underline{\sigma}^v$ and*

$$\phi^o(s) = \begin{cases} 0 & \text{if } o = l, \quad \forall s \in [0, 1] \\ 1 & \text{if } o = m_b, s \in \left(0, \frac{2\delta}{1+2\delta}\right) \\ 1 & \text{if } o = m_a, s \in \left(\frac{2\delta}{1+2\delta}, 1\right] \end{cases}$$

Proof: Note that for all $\delta \in (0, 1/2)$ we have $\underline{\sigma}^v > \lambda^*$ and $\ell(\sigma) \leq \frac{\max g(s)}{\min g(s)}$. Therefore, the Characterization Lemma implies that for

$$\frac{\epsilon}{1 - \mu} < \bar{\epsilon} \tag{10}$$

the unique limit equilibrium is $\sigma^v = \underline{\sigma}^v$. Choose $\underline{\mu}$ so that (10) holds for all $\mu < \underline{\mu}$. Note that $s^l(\underline{\sigma}^v) = 0$. Then, Proposition 4 follows from part (ii) of the Characterization Lemma. \square

When b is a partisan, voters expect him to choose l with high probability even if the election is close. As a result, uninformed voters are reluctant to vote for a partisan. This in turn, forces b to choose m since otherwise he is almost sure to lose the election. Proposition 4 seems paradoxical because candidate b never chooses l yet voters assume that conditional on a vote being pivotal there is a significant probability that b chooses l . Note that what is described in Proposition 4 is the limit of a sequence of equilibria with $\sigma^b \rightarrow 0$. Along the sequence b chooses l if the state s is close to 0. Hence, for any finite electorate the probability that b chooses l is strictly positive and conditional on a vote being pivotal the probability that b chooses l stays bounded away from 0 for all n .

The probability that a partisan wins the election is equal to $\bar{\phi}^{m_b}$, the probability of outcome m_b . Proposition 4 shows that

$$\bar{\phi}^{m_b} = G(2\delta/1+2\delta)$$

Hence, the ex ante probability that a partisan wins the election is smaller than in the benchmark Downsian case of section 3. Moreover, that probability is increasing in the fraction of informed voters. We have

$$\lim_{\delta \rightarrow 1/2} \bar{\phi}^{m_b} \rightarrow G(1/2), \lim_{\delta \rightarrow 0} \bar{\phi}^{m_b} \rightarrow 0$$

Hence, when the proportion of informed voters is small, a partisan almost never wins the election. When the proportion of informed voters approaches $1/2$ we converge to the Downsian benchmark.

The case analyzed in Proposition 4 demonstrates a second type of aggregation failure. In the states $2\delta/(1+2\delta) < s < 1/2$ the partisan candidate b does not win the election even though he has chosen the moderate policy and he has a personality advantage. Hence, in those states b is the median preferred alternative but fails to get a majority of votes.

Propositions 3 and 4 analyze the extreme cases of very weak or very strong partisan preferences. In both cases, the probability of outcome m_b is close to zero when δ is close to zero and close to $G(1/2)$ when δ is close to $1/2$. The next Corollary points out that this result is true for all values of μ .

Corollary: Fix F, G, ϵ, μ . Then, $\bar{\phi}^{m_b} \rightarrow 0$ as $\delta \rightarrow 0$ and $\bar{\phi}^{m_b} \rightarrow G(1/2)$ as $\delta \rightarrow 1/2$.

Proof: The first part follows from the definition of s^l, s^m and the Characterization Lemma. For the second part note that $\underline{\sigma}^v \rightarrow 1$ as $\delta \rightarrow 1/2$. Substituting into s^l, s^m yields $s^l \rightarrow 0, s^m \rightarrow 1/2$. Then, the Characterization Lemma yields the result. \square

In general, the probability of the outcome m_b may not be monotone in the fraction of informed voters δ . Nevertheless, as Corollary 1 demonstrates, information plays a key role in determining the extent to which outcomes depart from the standard Downsian prediction. In our model, a well informed electorate is one where at least one half of the voters observe the candidate's policy choices. When the fraction of informed voters is smaller than this critical number, we observe deviations from the Downsian prediction. If candidate b is an office seeker this deviation will be in the form of more partisan outcomes. If candidate b has strong policy preferences then this deviation will be in the form of candidate b not getting elected even though he chooses the moderate policy. In the extreme

case where δ is close to 0 we get extreme departures from the Downsian prediction: the outcome m_b occurs with negligible probability.

5. Control of Information

In this section, we allow candidate b to choose the fraction of voters that will be informed. Since the probability of being pivotal is very small, voters have little incentive to acquire information. Hence, there is a tendency for voters to remain ignorant. Our objective is to investigate if candidates have incentives to combat this tendency.

As in the previous section, we assume that b chooses a policy $o \in \{l, m\}$. Candidate b also chooses the fraction of voters $\delta^* \in \{\delta, \Delta\}$ ($0 < \delta < \Delta < 1/2$) that will be informed of his policy choice. We assume that voters cannot observe δ^* .

One interpretation of this model is the following. Suppose b must decide how many informative campaign commercials to run. The more informative commercials are run the more likely a voter observes the policy choice.

Since all swing voters strictly prefer m to l and voters never use weakly dominated strategies, b will choose $\delta^* = \Delta$ whenever he chooses the moderate policy m . Moreover, choosing $\delta^* = \Delta$ and l cannot be optimal unless all uninformed swing voters vote for a . But, if all uninformed swing voters vote for a then a wins regardless of b 's actions. Therefore, without loss of generality, we assume that b either chooses (Δ, m) or (δ, l) .

The following proposition establishes that when μ is close to 1 the equilibrium outcome is as if δ^* is fixed at δ . In other words, b 's ability to disclose additional information (choose $\delta^* = \Delta$) has no effect on the equilibrium outcome if he is an office seeker. Let \mathcal{E}^i denote the set of limit equilibria for this game and let $\Phi^i(\sigma)$ denote the corresponding limit outcome. Limit equilibria and limit outcomes are defined as in the previous section.

Proposition 5: *Fix F, G, ϵ, Δ . Then, there is $\bar{\mu}$ (independent of δ) such that for $\mu > \bar{\mu}$ the limit equilibrium $\sigma \in \mathcal{E}^i$ satisfies $\sigma^v = 1, \sigma^b = (1 - 2\delta)/(2 - 2\delta)$ and the limit outcome $\phi \in \Phi^i(\sigma)$ satisfies*

$$\phi^o(s) = \begin{cases} 1 & \text{if } o = l, \quad s \in \left[0, \frac{1-2\delta}{2-2\delta}\right) \\ 1 & \text{if } o = m_b, \quad s \in \left(\frac{1-2\delta}{2-2\delta}, \frac{1}{2}\right) \\ 1 & \text{if } o = m_a, \quad s \in \left(\frac{1}{2}, 1\right] \end{cases}$$

Proof: It is straightforward to adapt the analysis of the previous section to this new game. The definitions of the critical states are modified as follows:

$$\hat{s}^l(\sigma^v) = 1 - \frac{1}{2(1 - \delta)F(\sigma^v)}$$

$$\hat{s}^m(\sigma^v) = 1 - \frac{1}{2((1 - \Delta)F(\sigma^v) + \Delta)}$$

Let $\hat{\underline{\sigma}}^v$ be such that $F(\hat{\underline{\sigma}}^v) = 1/2$. Define

$$\hat{\ell}(\sigma^v) = \ell(\sigma^v) \frac{1 - \Delta}{1 - \delta}$$

With these modified definitions the characterization lemma holds for the modified game. Now, we can repeat the argument for Proposition 3 to prove Proposition 5. \square

Uninformed voters must take into account the fact that b can choose the informativeness of the campaign. However, for μ close to 1 this has no effect on σ^v and hence has no effect on the probability that b is elected.

Proposition 5 shows that if δ is close to 0 – that is, if voters remain ignorant unless b voluntarily discloses information – equilibrium is *as if* voters are uninformed. Candidate b chooses l and wins the election whenever he has a personality advantage. All uninformed voters who prefer b 's personality vote for b . Hence, when candidate b has control over information, the effect described in section 4 is exacerbated. In a limit equilibrium, partisan politics and aggregation failure will occur whenever b has a personality advantage.

6. Uninformed Candidates

In the analysis of the previous section, we assumed that candidates observe the distribution of voter preferences but voters do not. To illustrate the importance of this assumption we examine a version of the model with symmetric information, i.e., voters and candidate b are uncertain about s . We modify the election game in section 4 so that candidate b cannot observe the parameter s . For simplicity, we assume that F and G are uniform on $[0, 1]$.

In this modified game, b 's policy choice cannot depend on s . As before, we consider symmetric equilibria in weakly undominated strategies. Voters who prefer b 's personality or are informed have the same dominant strategy as in the section 4. An optimal strategy for uninformed voters who prefer b 's personality can again be described by a cutoff σ^v . Candidate b 's strategy σ^b is simply his probability of choosing m .

We define the set of limit equilibria \mathcal{E}^c the same way we defined limit equilibria in section 4. Proposition 6 below shows that if μ is above $\frac{1-2\delta}{1-\delta}$ then $\sigma^b = 1, \sigma^v = 1$ is the unique equilibrium. The unique equilibrium outcome is exactly as in the benchmark case examined in Proposition 1.

Proposition 5: *For F, G uniform and $\mu > \frac{1-2\delta}{1-\delta}$ the unique limit equilibrium is $(\sigma^v, \sigma^b) = (1, 1)$. In the unique limit outcome, m_b is implemented with probability 1 at $s < 1/2$, and m_a is implemented with probability 1 at $s > 1/2$.*

Proof: First assume $\sigma^v < \underline{\sigma}^v$. In that case, the limit vote share of candidate b is less than $1/2$ in every state if he chooses l . It is easy to see that in this case candidate b must choose m with probability 1. Therefore, $\sigma^v = 1$ as well.

Next, assume that $\sigma^v \geq \underline{\sigma}^v$. We note that in any limit equilibrium, candidate b wins at any state $s < s^l(\sigma^v)$ if he adopts l ; wins with policy m at states $s < s^m(\sigma^v)$; and he loses at any state $s > s^m(\sigma^v)$ no matter what policy he chooses. This follows from the law of large numbers. Hence, (since F, G are uniform), the probability that b wins if he chooses m is $s^m(\sigma^v)$, while the probability that b wins if he chooses l is $s^l(\sigma^v)$. Candidate b chooses $\sigma^b = 1$ if

$$\mu > \frac{s^l(\sigma^v)}{s^m(\sigma^v)}$$

Substituting for s^l, s^m we can rewrite this equation as

$$1 - \mu < \frac{\delta}{(1 - \delta)\sigma^v(2\sigma^v(1 - \delta) + 2\delta - 1)}$$

Since $\sigma^v \leq 1$ this will hold if

$$1 - \mu < \frac{\delta}{1 - \delta}$$

Then, $\mu > \frac{1-2\delta}{1-\delta}$ yields $\sigma^b = 1$ which in turn implies $\sigma^v = 1$ as desired. \square

In the game where candidates are uninformed an office seeker will choose the moderate policy. Voters will vote for b if and only if they prefer the personality of b . Hence, partisan politics and aggregation failure cannot occur. In this case, the model re-produces the standard Downsian prediction of median preferred policy outcomes. This shows that asymmetric information between candidates and voters is essential for the results of the previous section.

7. Conclusion

We have analyzed how candidate competition is altered when only a fraction of voters is informed of the candidate's policy choice. We show that when a candidate is an office seeker with a weak partisan preference, voter ignorance will enable him to implement partisan policies without suffering a reduced probability of winning the election. Uninformed voters behave as if the office seeker always chooses the median preferred policy.

One consequence of this effect is that candidates have little incentive to spend resources on informing voters. Providing an office seeker with the opportunity to inform voters costlessly has no effect on the equilibrium outcome. As long as voters are convinced that a candidate will “do what it takes” to get elected, his chance of getting elected is not harmed by voter ignorance. At the same time, a less well informed electorate allows the candidate to choose policies that closer match his policy preference.

To simplify the exposition we have considered a one-sided model where only candidate b has a non-trivial policy choice and assumed that candidate a is committed to the median preferred policy. Hence, a provides the stiffest possible competition for the opportunistic candidate b . As our main result shows, even in this case, the median preferred moderate policy may not be implemented. In a symmetric model where both candidates can choose between the moderate and a partisan policy, we would expect this failure of median preferred outcomes to be even more pronounced.

8. Appendix

Lemma 1: (i) $B_n(x) = \frac{\int_0^x \theta^n (1-\theta)^n d\theta}{\int_0^1 \theta^n (1-\theta)^n d\theta}$; (ii) B_n is strictly log-concave.

Proof: (i) The binomial theorem implies that

$$\begin{aligned} \int_0^x \theta^n (1-\theta)^n d\theta &= \int_0^x (\theta - \theta^2)^n d\theta = \int_0^x \sum_{k=0}^n \binom{n}{k} (-1)^k \theta^{n+k} \\ &= \sum_{k=0}^n \binom{n}{k} \frac{x^{n+k+1}}{n+k+1} (-1)^k \end{aligned} \quad (A1)$$

Next, we show that

$$B_n(x) = \frac{(2n+1)!}{n!n!} \sum_{k=0}^n \binom{n}{k} \frac{x^{n+k+1}}{n+k+1} (-1)^k \quad (A2)$$

The binomial theorem yields

$$\begin{aligned} B_n(x) &= \sum_{k=n+1}^{2n+1} \binom{2n+1}{k} x^k (1-x)^{2n+1-k} \\ &= \sum_{k=n+1}^{2n+1} \sum_{m=0}^{2n+1-k} x^{k+m} (-1)^m \binom{2n+1-k}{m} \binom{2n+1}{k} \end{aligned}$$

Hence, letting $t = m + k$ and rearranging terms yields

$$\begin{aligned} B_n(x) &= \sum_{k=n+1}^{2n+1} \sum_{t=k}^{2n+1} x^t (-1)^{t-k} \binom{2n+1-k}{t-k} \binom{2n+1}{k} \\ &= \sum_{t=n+1}^{2n+1} x^t \sum_{k=n+1}^t \binom{2n+1-k}{t-k} (-1)^{t-k} \binom{2n+1}{k} \\ &= \sum_{t=n+1}^{2n+1} \frac{(2n+1)! x^t}{(2n+1-t)!} \sum_{m=0}^{t-(n+1)} \frac{(-1)^m}{m!(t-m)!} \end{aligned}$$

Feller (1967) pg 65 provides the following identity:

$$\binom{a}{k} - \binom{a}{k-1} + \dots - \binom{a}{0} = \binom{a-1}{k}$$

Hence, the last equation implies

$$\begin{aligned}
B_n(x) &= \sum_{t=n+1}^{2n+1} \frac{(2n+1)! x^t}{(2n+1-t)!} \frac{(-1)^{t-(n+1)}}{t!} \frac{(t-1)!}{(t-(n+1))! n!} \\
&= \frac{(2n+1)!}{n! n!} \sum_{t=n+1}^{2n+1} \frac{(-1)^{t-(n+1)} n! x^t}{(2n+1-t)! (t-(n+1))! t} \\
&= \frac{(2n+1)}{n! n!} \sum_{k=0}^n \binom{n}{k} \frac{x^{n+k+1}}{n+k+1} (-1)^k
\end{aligned}$$

We conclude from (A1) and (A2) that

$$A \cdot \frac{\int_0^x \theta^n (1-\theta)^n d\theta}{\int_0^1 \theta^n (1-\theta)^n d\theta} = B_n(x)$$

for some constant $A > 0$. Clearly, $A = 1$ since

$$1 = B_n(1) = A \cdot \frac{\int_0^1 \theta^n (1-\theta)^n d\theta}{\int_0^1 \theta^n (1-\theta)^n d\theta} = A$$

which proves part (i).

(ii) We must show that

$$\frac{d}{dx} \left(\frac{B'_n(x)}{B_n(x)} \right) < 0 \tag{A3}$$

Substituting the left hand side expression from part (i) and computing the derivative, straightforward computation shows that inequality (A3) is equivalent to

$$n(1-2x) \left(\int_0^x \theta^n (1-\theta)^n d\theta \right) - x^{n+1} (1-x)^{n+1} < 0 \tag{A4}$$

For $x \geq 1/2$ the inequality is obviously correct. To see that it holds for $x < 1/2$ note that for $x < 1/2$ we have

$$(1-2x) \left(\int_0^x \theta^n (1-\theta)^n \right) \leq \left(\int_0^x \theta^n (1-\theta)^n (1-2\theta) d\theta \right) = \frac{x^{n+1} (1-x)^{n+1}}{n+1} \tag{A5}$$

Substituting (A5) into (A4) proves part (ii). □

Lemma 2: Let $\bar{x} > x, s \in [0, 1)$. Then, $\frac{B_n((1-s)x)}{B_n((1-s)\bar{x})}$ is decreasing in s .

Proof: Clearly it suffices to show that $\ln B_n((1-s)x) - \ln B_n((1-s)\bar{x})$ is decreasing in s . Let $B'(z) = \partial B(z)/\partial z$. We must show that

$$(-x) \frac{B'_n(x(1-s))}{B_n(x(1-s))} - (-\bar{x}) \frac{B'_n(\bar{x}(1-s))}{B_n(\bar{x}(1-s))} < 0 \quad (A6)$$

By the characterization in Lemma 1, (A6) is equivalent to

$$\frac{(1-s)^n x^{n+1} (1 - (1-s)x)^n}{\int_0^{(1-s)x} z^n (1-z)^n dz} - \frac{(1-s)^n \bar{x}^{n+1} (1 - (1-s)\bar{x})^n}{\int_0^{(1-s)\bar{x}} z^n (1-z)^n dz} > 0 \quad (A7)$$

To prove (A7), we will show that

$$h(y) := \frac{\partial}{\partial x} \frac{(1-s)^n x^{n+1} (1 - (1-s)x)^n}{\int_0^{(1-s)x} z^n (1-z)^n dz} < 0$$

for $y \in (0, 1)$. Setting $y = (1-s)x$ we can simplify

$$\begin{aligned} h &= \frac{((n+1)(1-y) - ny) \int_0^y z^n (1-z)^n dz - xy^n (1-y)^{n+1}}{(\int_0^y z^n (1-z)^n dz)^2} \\ &< \frac{((n+1)(1-y) - ny) \int_0^y z^n (1-z)^n dz - y^{n+1} (1-y)^{n+1}}{(\int_0^y z^n (1-z)^n dz)^2} \end{aligned} \quad (A8)$$

Integration by parts yields $\int_0^y z^n (1-z)^n (1-2z) dz = \frac{y^{n+1} (1-y)^{n+1}}{n+1}$. Hence, showing that the last term in (A8) is negative is equivalent to proving

$$\hat{h}(y) := \int_0^y z^n (1-z)^n \left(2z - \frac{2n+1}{n+1} y \right) dz < 0$$

Since $\hat{h}(0) = 0$, to conclude the proof, it suffices to show that $\frac{\partial}{\partial y} \hat{h}(y) < 0$ for $y \in (0, 1)$.

$$\begin{aligned} (n+1) \frac{\partial}{\partial y} \hat{h}(y) &= y^{n+1} (1-y)^n - (2n+1) \int_0^y z^n (1-z)^n dz \\ &= y^{n+1} (1-y)^n - \frac{2n+1}{n+1} y^{n+1} (1-y)^n - \frac{n(2n+1)}{n+1} \int_0^y z^{n+1} (1-z)^{n-1} dz \\ &= -\frac{n}{n+1} y^{n+1} (1-y)^n - \frac{n(2n+1)}{n+1} \int_0^y z^{n+1} (1-z)^{n-1} dz \end{aligned}$$

Hence, $\frac{\partial}{\partial y} \hat{h}(y) < 0$ as desired. \square

Lemma 3: Assume (i) $\lim a_n = 1/2$, $\lim \alpha_n = \alpha \in (1/2, 1]$, $\lim b_n = b \in [0, 1/2)$, and $\lim \beta_n = \beta \in (1/2, 1]$. (ii) $\{f_1, h_1, f_2, h_2, \dots\}$ are equicontinuous functions on $[0, 1]$ such that for some $c, C \in \mathbb{R}_+$ $c \leq f_n \leq C$, $c \leq h_n \leq C$ for all n , and (iii) $\lim f_n(1/2)$, $\lim h_n(1/2)$, $\gamma := \lim \frac{\int_{a_n}^1 x^n (1-x)^n dx}{\int_0^1 x^n (1-x)^n dx}$ exist. Then,

$$\lim \frac{\int_{a_n}^{\alpha_n} x^n (1-x)^n f_n(x) dx}{\int_{b_n}^{\beta_n} x^n (1-x)^n h_n(x) dx} = \gamma \lim \frac{f_n(1/2)}{h_n(1/2)}$$

Proof: Define

$$\begin{aligned} q_n(x) &= x^n (1-x)^n \\ z_n &= \frac{(\frac{1}{2} - \eta)^n (\frac{1}{2} + \eta)^n}{(\frac{1-\eta}{2})^n (\frac{1+\eta}{2})^n} \\ X_n(r, t) &= \int_r^t q_n(x) dx \end{aligned}$$

Step 1: $\lim r_n = r < t = \lim t_n$ and $1/2 \notin [r, t]$ implies

$$\lim \frac{X_n(r_n, t_n)}{X_n(0, 1)} = 0$$

Assume that $1/2 < r$ and choose $\eta \in (0, t - 1/2)$. (The proof for the $1/2 > t$ is symmetric and omitted.) Note that q is a strictly quasiconcave function on $[0, 1]$ which attains its unique maximum at $1/2$. Let $y = r - \eta$ and $z = \min\{t + \eta, 1\}$. Hence, $q_n(x) \leq q_n(y)$ for all $x \in [y, z]$ and $q_n(x) \geq q_n((1-\eta)/2)$ for all $x \in [(1-\eta)/2, (1+\eta)/2]$. Therefore, for n sufficiently large

$$\begin{aligned} \frac{X_n(r_n, t_n)}{X_n(0, 1)} &\leq \frac{X_n(y, z)}{X_n((1-\eta)/2, (1+\eta)/2)} \\ &\leq \frac{(A-a)z_n}{\eta} \end{aligned}$$

Since $\lim z_n = 0$ step 1 follows.

Step 2: $\lim r_n = r < t = \lim t_n$ and $1/2 \in (r, t)$ implies

$$\lim \frac{X_n(r_n, t_n)}{X_n(0, 1)} = 1$$

Choose $\eta \in (0, \min\{1/2 - r, t - 1/2\})$. Then, for n large enough

$$1 \geq \lim \frac{X_n(r_n, t_n)}{X_n(0, 1)} \geq \frac{X_n(1/2 - \eta, 1/2 + \eta)}{X_n(0, 1)} = \frac{1}{1 + \frac{X_n(0, 1/2 - \eta)}{X_n(1/2 - \eta, 1/2 + \eta)} + \frac{X_n(1/2 + \eta, 1)}{X_n(1/2 - \eta, 1/2 + \eta)}}$$

By step 1, the second and third terms in the denominator go to 0 as n goes to 0, proving step 2.

Let

$$\begin{aligned} N_n &= \int_{a_n}^{\alpha_n} q_n(x) f_n(x) dx \\ D_n &= \int_{b_n}^{\beta_n} q_n(x) h_n(x) dx \\ T_n &= \frac{N_n}{D_n} \end{aligned}$$

Step 3: $\lim T_n = \gamma \frac{\lim f_n(1/2)}{\lim h_n(1/2)}$.

The equicontinuity of f_n, h_n ensures that for any $\eta > 0$ there exists $\eta' > 0$ such that for n large enough

$$\begin{aligned} [f_n(1/2) - \eta] X_n(a_n, 1/2 + \eta') &\leq N_n \leq [f_n(1/2) + \eta] X_n(a_n, 1/2 + \eta') + \\ &\quad C X_n(1/2 + \eta', 1) \\ [f_n(1/2) - \eta] X_n(1/2 - \eta', 1/2 + \eta') &\leq D_n \leq [f_n(1/2) + \eta] X_n(1/2 - \eta', 1/2 + \eta') + \\ &\quad C X_n(0, 1/2 - \eta') + C X_n(1/2 + \eta', 1) \end{aligned}$$

Using the expressions above to bound N_n/D_n , then dividing terms by $X_n(0, 1)$, letting n go to infinity and applying steps 1 and 2 yields

$$\begin{aligned} \frac{\lim f_n(1/2) - \eta}{\lim h_n(1/2) + \eta} \cdot \lim \frac{X_n(a_n, 1/2 + \eta)}{X_n(1/2 - \eta', 1/2 + \eta')} &\leq \\ \lim T_n \leq \frac{\lim f_n(1/2) + \eta}{\lim h_n(1/2) - \eta} \cdot \lim \frac{X_n(a_n, 1/2 + \eta')}{X_n(1/2 - \eta', 1/2 + \eta')} \end{aligned}$$

Applying step 1 and step 2 again yields

$$\frac{\lim f_n(1/2) - \eta}{\lim h_n(1/2) + \eta} \cdot \frac{X_n(a_n, 1)}{X_n(0, 1)} \leq \lim T_n \leq \frac{\lim f_n(1/2) + \eta}{\lim h_n(1/2) - \eta} \cdot \frac{X_n(a_n, 1)}{X_n(0, 1)}$$

Since the equation above holds for any η , we conclude that

$$\lim T_n = \frac{\lim f_n(1/2)}{\lim h_n(1/2)} \cdot \lim \frac{X_n(a_n, 1)}{X_n(0, 1)} = \gamma \cdot \frac{\lim f_n(1/2)}{\lim h_n(1/2)}$$

as desired.

8.1 Proof of Proposition 2

In the text, we have shown that in any equilibrium the voters must use a cutoff strategy. To complete the proof, we will show that b 's best response to any cutoff strategy is also a cutoff strategy.

If σ^b is a best response to voters' cutoff strategy σ^v then $\sigma^b(s) = 1$ whenever

$$\frac{B_n(\pi^m(\sigma^v, s))}{B_n(\pi^l(\sigma^v, s))} > \mu$$

and $\sigma^b(s) = 0$ if this inequality is reversed. To show that this yields a cutoff strategy, it suffices to show that $\frac{B_n(\pi^m(\sigma^v, s))}{B_n(\pi^l(\sigma^v, s))}$ is strictly increasing in s . Recall that $\pi^l(\sigma^v, s) = (1-s)(1-\delta)F(\sigma^v)$ and $\pi^m(\sigma^v, s) = (1-s)[(1-\delta)F(\sigma^v) + \delta]$. Therefore, we can apply Lemma 2 to show that $\frac{B_n(\pi^m(\sigma^v, s))}{B_n(\pi^l(\sigma^v, s))}$ is strictly increasing in s .

To prove that equilibrium exists, define

$$h^*(\sigma^v, \sigma^b) := \frac{\int_{\sigma^b}^1 \pi^m(\sigma^v, s)^n (1 - \pi^m(\sigma^v, s))^n (1-s) g(s) ds}{\int_0^{\sigma^b} \pi^l(\sigma^v, s)^n (1 - \pi^l(\sigma^v, s))^n (1-s) g(s) ds} \quad (A9)$$

Let $h^*(\sigma^v, \sigma^b) = \infty$ if the denominator in (A9) is 0 and define

$$h(\sigma^v, \sigma^b) := \min[\epsilon \cdot h^*(\sigma^v, \sigma^b), 1]$$

Fix the cutoff strategy profile (σ^v, σ^b) and note that voter (λ, ν) will prefer candidate b if and only if

$$\lambda \leq \epsilon \cdot \frac{\theta}{1-\theta} = h(\sigma^v, \sigma^b)$$

where θ is the probability that candidate b chooses m given that (λ, b) is pivotal. It follows that σ^v is a best response to (σ^v, σ^b) if and only if $h(\sigma^v, \sigma^b) = \sigma^v$. Note that $h : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is continuous.

Let $k : [0, 1] \rightarrow S$ be defined as follows:

$$k(\sigma^v) := \begin{cases} 1 & \text{if } \frac{B_n(\pi^l(\sigma^v, 1))}{B_n(\pi^m(\sigma^v, 1))} > \mu \\ 0 & \text{if } \frac{B_n(\pi^l(\sigma^v, 0))}{B_n(\pi^m(\sigma^v, 0))} < \mu \\ \{s \in S \mid \frac{B_n(\pi^l(\sigma^v, s))}{B_n(\pi^m(\sigma^v, s))} = \mu\} & \text{otherwise.} \end{cases}$$

Note that k is in fact a function since $\frac{B_n(\pi^l(\sigma^v, s))}{B_n(\pi^m(\sigma^v, s))}$ is strictly decreasing and continuous in s . Since $\frac{B_n(\pi^l(\sigma^v, s))}{B_n(\pi^m(\sigma^v, s))}$ is jointly continuous in (σ^v, s) , k is also continuous. The cutoff $k(\sigma^v)$ is b 's best response to σ^v . We conclude that a fixed-point of $(\epsilon \cdot h, k) : [0, 1] \times [0, 1] \rightarrow [0, 1] \times [0, 1]$ is an equilibrium in cutoff strategies. Since both h, k are continuous this mapping has a fixed point. \square

8.2 Proof of Characterization Lemma

Lemma 4: *Let (σ_n^v, σ_n^b) be a convergent sequence of equilibria with limit (σ^v, σ^b) such that $\sigma^v \geq \underline{\sigma}^v$. Then, $\sigma^b = s^l(\sigma^v)$.*

Proof: Let $s < s^l(\sigma^v)$. Then there is $\epsilon > 0$ such that $\pi^l(\sigma^v, s) \geq 1/2 + \epsilon$ for n sufficiently large. This implies that b wins the election with probability close to 1 if he chooses policy l . Since $\mu < 1$ this implies that l must be the unique optimal choice at s and hence $\sigma^v > s$. We conclude that $\sigma^b \geq s^l(\sigma^v)$.

If $s^l(\sigma^v) > s > s^m(\sigma^v)$ then there is $\epsilon > 0$ such that $\pi^m(\sigma_n^v, s') > 1/2 + \epsilon$ and $\pi^l(\sigma_n^v, s') < 1/2 - \epsilon$. This implies that b wins with probability close to 1 if he chooses m but loses with probability close to 1 if he chooses l . Since $0 < \mu$ it follows that the unique optimal choice is m . It follows that $\sigma^b \leq s^l(\sigma^v)$ and hence $\sigma^b = s^l(\sigma^v)$. \square

Lemma 5: *Let (σ_n^v, σ_n^b) be a convergent sequence of equilibria with limit (σ^v, σ^b) . Then, (i) $\sigma^v \geq \underline{\sigma}^v$; (ii) $\sigma^b < 1/2$ and $0 < \sigma_n^b$ for large n ; (iii) $\lim B_n(\pi^l(\sigma_n^v, \sigma_n^b)) = \mu$.*

Proof: (i) Suppose there exists a (sub)sequence of equilibria $(\sigma_n^v, \sigma_n^b) \rightarrow (\sigma^v, \sigma^b)$ with $\sigma^v < \underline{\sigma}^v$. Then, $s^l(0, \sigma_n^v) \leq 1/2 - \eta$ for some $\eta > 0$ and for n sufficiently large. But this implies that $\frac{B_n(\pi^l(\sigma_n^v, 0))}{B_n(\pi^m(\sigma_n^v, 0))} \rightarrow 0$ as $n \rightarrow \infty$. Hence, $\sigma^b = 0$ for sufficiently large n . But then σ^v must be 1 since the probability that b chooses m conditional on a vote being pivotal is 1. Since $\underline{\sigma}^v < 1$ we have a contradiction.

(ii) Note that $s^l(\sigma^v) < 1/2$ for all σ^v and therefore $\sigma^b < 1/2$ follows from Lemma 3. To see that $\sigma_n^b = 0$ cannot be an equilibrium note that $\sigma_n^b = 0$ implies that $\sigma_n^v = 1$. Since $s^l(1) > 0$ Lemma 3 above implies that $\sigma_n^b > 0$ for large n .

(iii) For $0 < \sigma_n^b < 1/2$ we must have

$$\mu B_n(\pi^m(\sigma_n^v, \sigma_n^b)) = B_n(\pi^l(\sigma_n^v, \sigma_n^b))$$

This follows since for $o \in \{l, m\}$, $\pi^o(\sigma_n^v, \cdot)$ is continuous for all n . Further observe that for $s \leq 1/2$ we have that $\pi^m(\sigma^v, s) > \pi^l(\sigma^v, s) + \delta/2$ with $\pi^l(\sigma_n^v, \sigma_n^b) = 1/2$. Therefore, it follows that for large n , $\pi^m(\sigma_n^v, \sigma_n^b) > 1/2 + \delta/2$ and hence $\lim B_n(\pi^m(\sigma_n^v, \sigma_n^b)) = 1$. This yields part (iii) of the Lemma. \square

Let

$$T_n := \frac{\int_0^{\sigma_n^b} \pi^l(\sigma_n^v, s)^n (1 - \pi^l(\sigma_n^v, s))^n (1 - s) g(s) ds}{\int_{\sigma_n^b}^1 \pi^m(\sigma_n^v, s)^n (1 - \pi^m(\sigma_n^v, s))^n (1 - s) g(s) ds}$$

and note that T_n is the likelihood ratio of policy l and policy m conditional on a voter type (λ, ϵ) being pivotal.

Lemma 6: *Let σ_n be a convergent sequence of equilibria with limit σ . (i) If $1 \geq \sigma^v > \underline{\sigma}^v$ then, $T = \lim T_n$ exists and $T = (1 - \mu)/\ell(\sigma^v)$. (ii) If $\sigma^v = \underline{\sigma}^v$ then $\limsup T_n \leq (1 - \mu)/\ell(\underline{\sigma}^v)$.*

Proof: (i) Let $a_n := \pi^l(\sigma_n^v, \sigma_n^b)$, $\alpha_n := \pi_l(\sigma_n^v, 0)$, $b_n := \pi_m(\sigma_n^v, 1)$, $\beta_n := \pi^m(\sigma_n^v, \sigma_n^b)$. Since σ_n is a convergent sequence it follows that $(a_n, \alpha_n, b_n, \beta_n)$ converges to some (a, α, b, β) . Note that $a = 1/2$ and since $\delta > 0$ it follows that $\beta > 1/2$. Since $\sigma^v > \underline{\sigma}^v$ we have $\alpha > 1/2$. Since $\pi_n(\sigma^v, 1) = 0$ we have $b < 1/2$.

Let $q_n(x) := x^n(1 - x)^n$. A change of variables yields

$$T_n = \frac{\int_{a_n}^{\alpha_n} q_n(x) h_n^l(x) dx}{\int_{b_n}^{\beta_n} q_n(x) h_n^m(x) dx}$$

where

$$h_n^l(x) = g(z_n^l(x))(1 - z_n^l(x))/(1 - \delta)F(\sigma^v)$$

$$h_n^m(x) = g(z_n^m(x))(1 - z_n^m(x))/(1 - \delta)(F(\sigma^v) + \delta)$$

and $z_n^o(x)$ is the unique solution to $\pi^o(\sigma_n^v, z_n^o) = x$ for $o \in \{l, m\}$.

Since $\sigma_n^v \rightarrow \sigma$ the collection of functions $\{h_n^l, h_n^m\}$ for $n = 1, \dots$ is equicontinuous.

Also, by Lemma 4, $\sigma^b = a = s^l(\sigma^v)$ and therefore

$$\lim h^l(1/2) = g(s^l(\sigma^v))(1 - s^l(\sigma^v))/((1 - \delta)F(\sigma^v))$$

Similarly,

$$\lim h^m(1/2) = g(s^m(\sigma^v))(1 - s^m(\sigma^v))/((1 - \delta)F(\sigma^v) + \delta)$$

Then, by Lemma 3,

$$\begin{aligned} \lim T_n &= \frac{\lim h^l(1/2)}{\lim h^m(1/2)} \cdot \lim \frac{\int_{a_n}^1 q_n(x) dx}{\int_0^{\beta_n} q_n(x) dx} \\ &= \frac{g(s^l(\sigma^v))}{g(s^m(\sigma^v))} \frac{1 - s^l(\sigma^v)}{1 - s^m(\sigma^v)} \frac{F(\sigma^v) + \delta}{F(\sigma^v)} \cdot \lim \frac{\int_{a_n}^1 q_n(x) dx}{\int_0^{\beta_n} q_n(x) dx} \\ &= \frac{g(s^l(\sigma^v))}{g(s^m(\sigma^v))} \left(\frac{(1 - \delta)F(\sigma^v) + \delta}{(1 - \delta)F(\sigma^v)} \right)^2 \cdot \lim \frac{\int_{a_n}^1 q_n(x) dx}{\int_0^{\beta_n} q_n(x) dx} \end{aligned} \quad (A9)$$

But by Lemma 1,

$$\lim \frac{\int_{a_n}^1 q_n(x) dx}{\int_0^1 q_n(x) dx} = 1 - \lim \frac{\int_0^{a_n} q_n(x) dx}{\int_0^1 q_n(x) dx} = 1 - \lim B_n(a_n)$$

and since $a_n = \pi^l(\sigma^v, \sigma^b)$, Lemma 5(iii) yields $B_n(a_n) = \mu$. Then, (A9) establishes

$\lim T_n = \frac{(1-\mu)}{\ell(\sigma)}$ as desired.

(ii) For $\sigma^v = \underline{\sigma}^v$ note that

$$T_n \leq \frac{\int_{a_n}^1 q_n(x) \hat{h}_n^l(x) dx}{\int_{b_n}^{\beta_n} q_n(x) h_n^m(x) dx}$$

where $\hat{h}_n^l(x) = h_n^l(x)$ for $x \leq \alpha_n$ and $\hat{h}_n^l(x) = h_n^l(\alpha_n)$ for $x > \alpha$. Repeating the argument above then yields the desired bound. \square

Lemma 7: If $\sigma \in \mathcal{E}$ and $\phi \in \Phi(\sigma)$ then

$$\begin{aligned}\phi^l(s) &= 1 && \text{if } s < s^l(\sigma^v) \\ \phi^m(s) &= 1 && \text{if } s^l(\sigma^v) < s < s^m(\sigma^v) \\ \phi^r(s) &= 1 && \text{if } s^m(\sigma^v) < s.\end{aligned}$$

Proof: Suppose $\sigma_n \in \mathcal{E}_n$ for all n . If $s < s^l(\sigma^v)$ then by Lemma 4, there exists N , $s^* \in (s, s^l(\sigma^v))$ such that for all $n \geq N$, $\sigma_n^b \geq s^*$. Hence, in equilibrium (for all $n \geq N$) candidate b chooses l at s . Also, for $\epsilon > 0$, we can choose N sufficiently large so that $B_n(\pi^l(\sigma_n^v, s)) \geq B_n(\pi^l(\sigma_n^v, s^*)) - \epsilon$. Since, $\lim \pi^l(\sigma_n^v, s^*) > 1/2$ we conclude that $\lim B_n(\pi^l(\sigma_n^v, s)) \geq 1 - \epsilon$. Since this statement holds for any ϵ , we have $\lim B_n(\pi^l(\sigma_n^v, s)) = 1$, as desired. The proofs of the other two cases are similar and omitted. \square

Proof of Characterization Lemma: Let $\sigma \in \mathcal{E}$. By Lemma 4, $\sigma^b = s^l(\sigma^v)$. Recall that $\sigma_n^v = \epsilon \cdot \theta_n / (1 - \theta_n)$ where θ_n is the probability that b chooses m given that the voter is pivotal. By Lemma 6(i),

$$\frac{\theta_n}{1 - \theta_n} \rightarrow \frac{\ell(\sigma^v)}{1 - \mu}$$

if $\sigma > \underline{\sigma}^v$. Therefore $\sigma^v > \underline{\sigma}^v$ implies that

$$\sigma^v = \max \left\{ \epsilon \cdot \frac{\ell(\sigma^v)}{1 - \mu}, 1 \right\}$$

If $\sigma^v = \underline{\sigma}^v$ then Lemma 6(ii) implies that

$$\liminf \frac{\theta_n}{1 - \theta_n} \geq \frac{\ell(\sigma^v)}{1 - \mu}$$

and therefore

$$\sigma^v = \underline{\sigma}^v \geq \frac{\ell(\underline{\sigma}^v)}{1 - \mu}$$

in that case. The remainder of the Lemma follows from Lemma 7. \square

References

1. Ansolabehere, S. and J. M. Snyder, "Valence Politics and Equilibrium in Spatial Election Models," *Public Choice*, 103, 327-336.
2. Aragonès, E. and T. R. Palfrey, (2002) "Mixed Equilibrium in a Downsian Model with a Favored Candidate," *Journal of Economic Theory*, 103, 131-161.
3. Aragonès, E. and T. R. Palfrey (2004), "Electoral Competition between Two Candidates of Different Quality," mimeo, Princeton University.
4. Austen-Smith D. and J. Banks, (1995) "Information Aggregation, Rationality and the Condorcet Jury Theorem", *American Political Science Review*, 90, 34-45.
5. Berelson, B. R., P. Lazarsfeld, and W. N. McPhee. 1954. *Voting: A Study of Opinion Formation in a Presidential Campaign*. Chicago: University of Chicago Press.
6. Bernhardt, Duggan and Squintani (2003), "Electoral Competition with Privately Informed Candidates", mimeo, University of Rochester.
7. Calvert, R. (1985), "Robustness of the Multidimensional Voting Model: Candidate Motivations, Uncertainty, and Convergence." *American Journal of Political Science* 29 (February 1985), pp. 69-95.
8. Chan, J. (2001), "Electoral Competition with Private Information", mimeo, Johns Hopkins University.
9. Groseclose, T. (2001) "A Model of Candidate Location When One Candidate has a Valence Advantage," *American Journal of Political Science*, 45, 862-86.
10. Delli Carpini, M. X., and S. Keeter (1993) "Measuring Political Knowledge: Putting First Things First," *American Journal of Political Science* 37:1179-1206.
11. Downs, A., *An Economic Theory of Democracy*, New York, Harper, 1957.
12. Feddersen, T. and W. Pesendorfer, (1996) "The Swing Voter's Curse", *American Economic Review* 86, 381-398.
13. McKelvey, R. D., and P. C. Ordeshook (1985), "Elections with Limited Information: A Fulfilled Expectations Model using Contemporaneous Poll and Endorsement Data as Information Sources. *Journal of Economic Theory*, 36, 55-85.
14. McKelvey, R. D., and P. C. Ordeshook (1986), "Information, Electoral Equilibria, and the Democratic Ideal", *The Journal of Politics*, 48, no 4, 909-937.
15. Redlawsk, D., and R. Lau, (2003), "Do voters want candidates they like or candidates they agree with? Affect vs. cognition in voter decision-making." Working Paper, University of Iowa.

16. Wittman, Donald (1977), "Candidates with Policy Preferences," *Journal of Economic Theory* 14, 180-9.

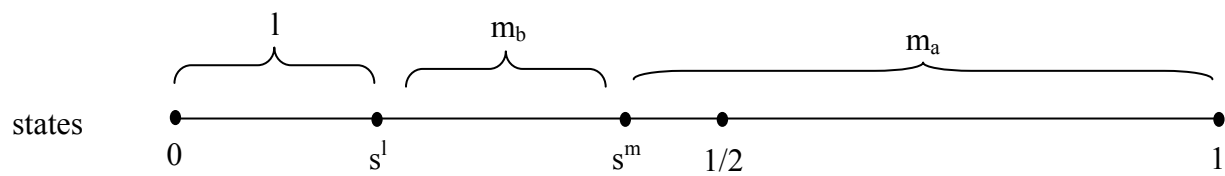


Figure 1