Ec 501
Game Theory

Rationalizability

The book's description of rationalizable strategies is a little vague. So, here we will present a formal definition:

For any collection of pure strategies A_i, let $\bar{A}_i$ denote the set of all mixed strategies σ_i such that $\sigma_i(s_i) > 0$ implies $s_i \in A_i$.

Let $S_i(0) = S_i$ for all $i \in I$. Define inductively, $S_i(t)$ for all $t \geq 1$ as follows: $S_i(t+1) = S_i \cap B_i(\times_{j \neq i} \bar{S}_j(t))$.

Fact 1: $S_i(t+1) \subset S_i(t)$ for all t.

Fact 2: There exits $T < \infty$ such that $S_i(T) = S_i(t)$ for all $t \geq T$.

The sets of pure strategies $S_i^* = S_i(T)$ are called rationalizable pure strategies. The strategies $\Sigma_i^* = B_i(\times_{j \neq i} \bar{S}_j(t))$ are called the rationalizable strategies.

Fact 3: $\Sigma_i^* \cap S_i = S_i^*$ for all $i \in I$.

Exercise 1: Is it true that $\Sigma_i^* = \bar{S}_i^*$?

For any collection of pure strategies $A_j \subset S_j$ for $j = 1, \ldots, n$, $j \neq i$, let $UD_i(\times_{j \neq i} \bar{A}_j)$ denote the set of $s_i \in S_i$ such that there exists no $\sigma_i \in \Sigma_i$ such that

$$U(s_i, \sigma_{-i}) < U(\sigma_i, \sigma_{-i})$$

for all $\sigma_{-i} \in \times_{j \neq i} \bar{A}_j$. Let $S_i'(0) = S_i$ for all $i \in I$. Define inductively, $S_i'(t)$ for all $t \geq 1$ as follows: $S_i'(t+1) = S_i(t) \cap UD_i(\times_{j \neq i} \bar{S}_j'(t))$.

Exercise 2: Show that there exits $T < \infty$ such that $S_i(T) = S_i(t)$ for all $t \geq T$

The sets of pure strategies $S_i^{**} = S_i(T)$ are the iterative strict dominance solution of the game G.

Exercise 3: Show that $S_i^* \subset S_i^{**}$.