More machine learning for fMRI

Francisco Pereira

[many slides adapted from slides by Ken Norman]

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“Wanderer, your footsteps are the road, and nothing more; wanderer, there is no road, the road is made by walking. By walking one makes the road, and upon glancing behind one sees the path that never will be trod again. Wanderer, there is no road-- only wakes upon the sea.”

Antonio Machado
what questions can be tackled?

- is there information? (pattern discrimination)
- where/when is information present? (pattern localization)
- how is information encoded? (pattern characterization)
where are we?

stimulus → mind → brain → fMRI data

(task)
where are we?

stimulus → mind → brain → fMRI data

(task)  

no dualism implied!
where are we?

stimulus → mind → brain → fMRI data

(task)
where are we?

stimulus (task) \(\rightarrow\) mind \(\rightarrow\) brain \(\rightarrow\) fMRI data

conclusions from feature choice

classifier

- voxel location
- voxel behaviour
- time within trial

dependent on experiment
where are we?

stimulus (task) → mind → brain → fMRI data

- conclusions from structure of the learnt model
- conclusions from feature choice

classifier

- weights on features
- hidden layer activations
- voxel location
- voxel behaviour
- time within trial

dependent on prediction model

dependent on experiment
what do we want?

stimulus → mind → brain → fMRI data

(task)

what is present in the mind as the task is performed?
where can it come from?

stimulus \rightarrow mind \rightarrow brain \rightarrow fMRI data

(task)

what is present in the mind as the task is performed?

- known or constrained from behavioural experiments
- modelled mathematically or computationally
- hypothesized
- learnt elsewhere (text corpora)
how do we know it’s there?

1. stimulus (task) → mind → brain → fMRI data
   - model
   - predicted fMRI data

- Kay et al. 2008
- Mitchell et al. 2008
how do we know it’s there?

2

stimulus
(task)

→ mind

→ brain

→ fMRI data

predicted
stimulus
or task

model

- Thirion et al. 2006
- Miyawaki et al. 2008
- Naselaris et al. 2009
- van Gerven et al. 2010
- Pereira et al. 2011
how do we know it’s there?

3

stimulus (task) → mind → brain → fMRI data

known relationships between conditions

- Kriegeskorte et al. 2008/2009
- Walther et al. 2009
- ...

pattern similarities or confusion matrices
how do we know it’s there?

1

stimulus (task) → mind → brain → fMRI data

model

predicted fMRI data

- Kay et al. 2008
- Mitchell et al. 2008
Identifying natural images from human brain activity

Kendrick N. Kay¹, Thomas Naselaris², Ryan J. Prenger³ & Jack L. Gallant¹,²
- decode which natural image a subject is seeing (out of 1000s)
- approach:
  - assume that the image perceived is processed into **features** by V1/V2/V3
  - build a model of how voxels respond to the features
  - predict the fMRI response to a new image and match
apply filters with various scales/orientations
the output of each filter is a feature
deterministic function of stimulus image
feature computation + response model

- feature values are fed into a voxel response model
- voxel response model: linear combination of features
feature computation + response model

- one set of weights per voxel
- intuition: find which features a voxel responds to
- learned over training images + fMRI data
model validation

- over test stimuli not used to fit the model
- obtain fMRI data in response to stimuli
- test:
  - given natural image
  - compute features
  - predict voxel responses
  - find most similar fMRI image in test data

Stage 2: image identification
1. Measure brain activity for an image

![Diagram showing image processing stages](image)

2. Predict brain activity for a set of images using receptive-field models

   - Set of images
   - Receptive-field models for multiple voxels
   - Predicted voxel activity patterns

3. Select the image (★) whose predicted brain activity is most similar to the measured brain activity
how do we know it’s there?

- Kay et al. 2008: specified computation model + learnt response model
- Mitchell et al. 2008
Predicting Human Brain Activity Associated with the Meanings of Nouns

Tom M. Mitchell,¹* Svetlana V. Shinkareva,² Andrew Carlson,¹ Kai-Min Chang,³,⁴ Vicente L. Malave,⁵ Robert A. Mason,³ Marcel Adam Just³
goal:

- understand how semantic information is represented in the pattern of activation

approach

- stimulus: word + drawing
- subject visualizes object, thinks of properties, using it, etc
- assume that each concept is represented in terms of features
- learn a mapping between each feature and the voxels it affects
## Stimuli
60 Exemplars

<table>
<thead>
<tr>
<th>Categories</th>
<th>Exemplars</th>
</tr>
</thead>
<tbody>
<tr>
<td>BODY PARTS</td>
<td>leg, arm, eye, foot, hand</td>
</tr>
<tr>
<td>FURNITURE</td>
<td>chair, table, bed, desk, dresser</td>
</tr>
<tr>
<td>VEHICLES</td>
<td>car, airplane, train, truck, bicycle</td>
</tr>
<tr>
<td>ANIMALS</td>
<td>horse, dog, bear, cow, cat</td>
</tr>
<tr>
<td>KITCHEN UTENSILS</td>
<td>glass, knife, bottle, cup, spoon</td>
</tr>
<tr>
<td>TOOLS</td>
<td>chisel, hammer, screwdriver, pliers, saw</td>
</tr>
<tr>
<td>BUILDINGS</td>
<td>apartment, barn, house, church, igloo</td>
</tr>
<tr>
<td>PART OF A BUILDING</td>
<td>window, door, chimney, closet, arch</td>
</tr>
<tr>
<td>CLOTHING</td>
<td>coat, dress, shirt, skirt, pants</td>
</tr>
<tr>
<td>INSECTS</td>
<td>fly, ant, bee, butterfly, beetle</td>
</tr>
<tr>
<td>VEGETABLES</td>
<td>lettuce, tomato, carrot, corn, celery</td>
</tr>
<tr>
<td>MAN MADE OBJECTS</td>
<td>refrigerator, key, telephone, watch, bell</td>
</tr>
</tbody>
</table>
feature extraction

- we have models of the computation being carried out by the visual cortex
- but how do we find semantic features?
feature extraction

- we have models of the computation being carried out by the visual cortex
- but how do we find semantic features?

- consider the use of the nouns naming stimuli in a very large text corpus!
- in particular, consider their co-occurrence with certain verbs...
feature extraction

- 25 verbs reflecting sensory/motor/function aspects
  - sensory: see, hear, listen, taste, touch, smell, fear, ...
  - motor: rub, lift, run, push, move, say, eat, ...
  - abstract: fill, open, ride, approach, drive, enter, ...

- 25 features
  - co-occurrence of a stimulus noun with each of 25 verbs
  - co-occurrence is normalized to be length 1
feature extraction

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- 25 features
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- example: “airplane”
  - 0.87, ride
  - 0.29, see
  - 0.17, near
  - 0.08, run
  - ...

28
predicting activation

stimulus word (+ picture)
e.g. celery

average activation during 3 seconds
predicting activation

stimulus word
(+ picture)
e.g. celery

hypothesis:
verb semantic features
e.g. taste/eat/fill
word co-occurrence

learnt model:
mapping between semantic features and their influence in activation

average activation during 3 seconds
predicting activation

semantic feature values

“eat”

Predicted “celery” = 0.84

+ 0.35

“taste”

+ 0.32

“fill”

+...

feature basis images capture influence
basis images

semantic features

"eat"  "push"  "run"

the basis images put weight in locations involved in tasks related to the verbs

Participant P1

Mean over participants

Pars opercularis (z=24 mm)  Postcentral gyrus (z=30 mm)  Superior temporal sulcus (posterior) (z=12 mm)
- learn basis images from 58 of 60 nouns
- use semantic features for 2 test nouns as weights for **combining** basis images
- result: predicted fMRI for the 2 nouns

**classification:** match predicted with observed
how do we know it’s there?

1. stimulus (task) → mind → brain → fMRI data → model → predicted fMRI data

- Kay et al. 2008: specified computation model + learnt response model
- Mitchell et al. 2008: feature values from text data + learnt response model
how do we know it’s there?

2

stimulus  mind  brain  fMRI data
(task)

predicted
stimulus
or task

- Thirion et al. 2006
  - forward model with retinotopy task + inversion
- Miyawaki et al. 2008
- Naselaris et al. 2009
- van Gerven et al. 2010
Visual Image Reconstruction from Human Brain Activity using a Combination of Multiscale Local Image Decoders

Yoichi Miyawaki,1,2,6 Hajime Uchida,2,3,6 Okito Yamashita,2 Masa-aki Sato,2 Yusuke Morito,4,5 Hiroki C. Tanabe,4,5 Norihiro Sadato,4,5 and Yukiyasu Kamitani2,3,*
1National Institute of Information and Communications Technology, Kyoto, Japan
2ATR Computational Neuroscience Laboratories, Kyoto, Japan
3Nara Institute of Science and Technology, Nara, Japan
4The Graduate University for Advanced Studies, Kanagawa, Japan
5National Institute for Physiological Sciences, Aichi, Japan
6These authors contributed equally to this work
*Correspondence: kmtn@atr.jp
DOI 10.1016/j.neuron.2008.11.004
goal

- reconstruct 10 x 10 pixel binary images
- from fMRI data of subjects seeing them
goal

- reconstruct 10 x 10 pixel binary images
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- approach:
  - assume stimulus images can be represented as linear combinations of local image elements (multiple scales)
goal

- reconstruct 10 x 10 pixel binary images
- from fMRI data of subjects seeing them

- approach:
  - assume stimulus images can be represented as linear combinations of local image elements (multiple scales)
  - for each image element, learn a predictor of its state for any stimulus from corresponding fMRI data
goal

- reconstruct 10 x 10 pixel binary images
- from fMRI data of subjects seeing them

approach:
- assume stimulus images can be represented as linear combinations of local image elements (multiple scales)
- for each image element, learn a predictor of its state for any stimulus from corresponding fMRI data
- reconstruct image from linear combination of predictors
any 8x8 binary image can be represented by a linear combination of the basis elements
the basis is fixed
the coefficients depend on the image represented
A

Presented image (10 x 10 patches)

Local image bases (elements)

Reconstructed contrast pattern

fMRI signals

Multi-voxel pattern decoders

Multi-scale image representation

\[ C_i \times w_i \]

\[ C_j \times w_j \]

\[ C_k \times w_k \]

\[ C_l \times w_l \]
- pattern decoders work over all voxels (regularized)
- basis elements are 1x1, 1x2, 2x1 and 2x2 patches
- at every location
- linear combination of decoder outputs with non-negative coefficients (sparse)
evaluation

- trained with fMRI responses to random images
- test reconstruction of figures, letters, other random
- generalizes to novel stimuli
- can be used for classification
how do we know it’s there?

2

stimulus (task) ➔ mind ➔ brain ➔ fMRI data

predicted stimulus or task

- Miyawaki et al. 2008
  - local decoders of basis elements + linear combination
- Naselaris et al. 2009
Bayesian Reconstruction of Natural Images from Human Brain Activity

Thomas Naselaris,¹ Ryan J. Prenger,² Kendrick N. Kay,³ Michael Oliver,⁴ and Jack L. Gallant¹,³,⁴,*
¹Helen Wills Neuroscience Institute
²Department of Physics
³Department of Psychology
⁴Vision Science Program
University of California, Berkeley, Berkeley, CA 94720, USA
*Correspondence: gallant@berkeley.edu
DOI 10.1016/j.neuron.2009.09.006
goal

- find a natural scene that matches the stimulus from fMRI data of a subject
- match both visual properties and semantic content

approach:
- learn a model from stimulus to fMRI (same as [Kay 2008])
- label training images with one of 23 categories
- learn a model from category label to fMRI (AOC)
goal

- find a natural scene that matches the stimulus from fMRI data of a subject
- match both visual properties and semantic content

approach:
- learn a model from stimulus to fMRI (same as [Kay 2008])
- label training images with one of 23 categories
- learn a model from category label to fMRI (AOC)

- invert both models to find most probable stimulus
- prior probability has a large influence
semantic model

mostly animate
    human
        many  (crowd/gathering)
        few   (body parts/full bodies/portrait)
    animal
        mammal  (land/water)
        non-mammal (bird/fish/other)
mostly inanimate
    man-made
        non-building  (vehicle/artifacts)
        building    (indoor/outdoor)
    natural
        plant  (edible/non-edible)
        non-plant  (land/water/sky)
texture
effects of model and prior

Target image

Reconstructions with structural encoding model

Flat prior

Sparse Gabor prior

Natural image prior

Target image

Reconstructions with structural encoding model

Flat prior

Sparse Gabor prior

Natural image prior

Target image

Reconstructions with structural encoding model

Flat prior

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Natural image prior

Target image

Reconstructions with structural encoding model

Flat prior

Sparse Gabor prior

Natural image prior
effects of model and prior

<table>
<thead>
<tr>
<th>Target image</th>
<th>Reconstructions with natural image prior</th>
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<tbody>
<tr>
<td></td>
<td>Structural model only</td>
</tr>
<tr>
<td></td>
<td>Structural and semantic models (hybrid method)</td>
</tr>
</tbody>
</table>

- **Target image**: Images of various scenes and objects.
- **Reconstructions with natural image prior**: Comparisons of reconstructions using different models.

The images show different scenes and objects, with annotations indicating the model used for each reconstruction.
evaluation

- reconstruction accuracy
  - complex wavelet-domain correlation with target image

- classification accuracy
  - probability that the predicted image has the same class label as the target image
  - at multiple levels of granularity
how do we know it’s there?

stimulus (task) → mind → brain → fMRI data

predicted stimulus or task

- Miyawaki et al. 2008
  - local decoders of basis elements + linear combination
- Naselaris et al. 2009
  - model of visual processing + semantic information
how do we know it’s there?

stimulus → mind → brain → fMRI data

known relationships between conditions

- Kriegerkorte et al. 2008/2009
- Walther et al. 2009
- ...

pattern similarities or confusion matrices
representational similarity analysis

goal:

- compare fMRI patterns elicited by stimuli without assuming a priori structure
- relate similarity/distance in fMRI space to behavioural or hypothesized similarity/distance

- a range of ideas more than a technique
stimuli and similarity

<table>
<thead>
<tr>
<th>animate</th>
<th>human</th>
<th>body</th>
<th>face</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>nonhuman</td>
<td>body</td>
<td></td>
<td></td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>inanimate</td>
<td>natural</td>
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</table>
stimuli and similarity

stimulus 1 pattern

stimulus 2 pattern
<table>
<thead>
<tr>
<th>animate</th>
<th>human</th>
<th>body</th>
<th>face</th>
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<td></td>
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<tr>
<td>inanimate</td>
<td>natural</td>
<td></td>
<td></td>
</tr>
<tr>
<td>artificial</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **stimulus 1 pattern**
- **stimulus 2 pattern**

Correlation
stimuli and similarity

correlation

stimulus 1 pattern

stimulus 2 pattern
stimuli and similarity

- summarizes a lot of information
- a bit hard to make sense of
multidimensional scaling

- represent 96 items in a 2D space
- euclidean distance in 2D space corresponds (if possible) to dissimilarity in the matrix
multidimensional scaling

human IT
compare multiple areas

A: human IT

human early visual cortex
(1057 voxels)

- animate vs. inanimate
- natural vs. artificial
- human body, face
- not human body, face

Dissimilarity (percentile of 1-ρ)

36 %-ile points***

5 %-ile points*

within animates
within inanimates
between ani. & inani.
other aspects

- can compute matrices in different ROIs

- can look for matrices compatible with a certain hypothesis (or use it to compare competing ones)

- matrices matching behaviour, ratings, etc

- more later...
how do we know it’s there?

3

stimulus
(task)

→

mind

→

brain

→

fMRI data

known relationships between conditions

- Kriegerkorte et al. 2008/2009
- Walther et al. 2009
- ...

pattern similarities or confusion matrices
Natural Scene Categories Revealed in Distributed Patterns of Activity in the Human Brain

Dirk B. Walther,¹ Eamon Caddigan,¹,² Li Fei-Fei,³* and Diane M. Beck¹,²*
¹Beckman Institute for Advanced Science and Technology, University of Illinois Urbana-Champaign, Urbana, Illinois, 61801-2325, ²Department of Psychology, University of Illinois at Urbana-Champaign, Champaign, Illinois 61820-6232, and ³Computer Science Department, Stanford University, Stanford, California 94305-9025
rapid scene categorization

- humans are good at extracting information about a scene
- 100ms can be enough
relating classifiers to behaviour

goal:
- identify brain regions
  - V1, FFA, LOC, PPA, RSC (retrosplenial)
- involved in rapid natural scene categorization
  - beach, city, road, …
relating classifiers to behaviour

goal:

- identify brain regions
  - V1, FFA, LOC, PPA, RSC (retrosplenial)
- involved in rapid natural scene categorization
  - beach, city, road, ...

approach:

- in each candidate region
  - train classifier to distinguish the six possibilities
  - compute confusion matrix
- contrast with other matrices...
per-ROI classifier

ROI data

ROI classifier

ROI confusion matrix

true category

predicted category
behavioural information

- collect data on confusability
- flash scenes (11-45 ms)
- have subjects make a 6-way judgement
- behavioural confusion matrix
stimulus information

- compute how similar scenes from various categories are
- low-level visual properties
- stimulus similarity matrix
analysis

- Q1: which ROIs drive scene classification?

ROIs where classifier and behavioural confusion matrices match
Q1: which ROIs drive scene classification?

ROIs where classifier and behavioural confusion matrices match

Q2: which ROIs encode low-level image properties?

ROIs where image and behavioural similarity matrices match
Table 1. Summary of main results

<table>
<thead>
<tr>
<th>ROI</th>
<th>Decoding accuracy</th>
<th>Error correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1</td>
<td>26%*</td>
<td>0.21</td>
</tr>
<tr>
<td>FFA</td>
<td>22%</td>
<td>0.10</td>
</tr>
<tr>
<td>LOC</td>
<td>24%*</td>
<td>0.42*</td>
</tr>
<tr>
<td>RSC</td>
<td>27%*</td>
<td>0.34†</td>
</tr>
<tr>
<td>PPA</td>
<td>31%**</td>
<td>0.57**</td>
</tr>
</tbody>
</table>

Decoding accuracy is measured in percentage of blocks predicted correctly, and significance is assessed relative to chance (17%). Error correlation establishes a correlation between misclassifications (off-diagonal entries in the confusion matrices) (Figs. 2, 3) between decoding from ROIs and human behavior. Image similarity correlation correlates the image similarities matrix with the confusion matrix from fMRI decoding. The inversion effect is defined as the difference in accuracy of a decoder trained and tested with upright versus trained and tested with inverted scene presentations. PPA shows significant effects in all analyses except for the image similarity correlation.

* p < 0.05; ** p < 0.01; † p = 0.069.

- all 5 ROIs have above chance classification
- PPA, LOC, RSC show correlation between classifier and subject errors
Table 1. Summary of main results

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<tr>
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<td>0.21</td>
<td>0.46**</td>
</tr>
<tr>
<td>FFA</td>
<td>22%</td>
<td>0.10</td>
<td>0.03</td>
</tr>
<tr>
<td>LOC</td>
<td>24%*</td>
<td>0.42*</td>
<td>-0.22</td>
</tr>
<tr>
<td>RSC</td>
<td>27%*</td>
<td>0.34 †</td>
<td>-0.24</td>
</tr>
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* p < 0.05; ** p < 0.01; † p = 0.069.

- low-level similarity correlates with behavioural confusion in V1 but not elsewhere
how do we know it’s there?

3

stimulus → mind → brain → fMRI data

(task)

known relationships between conditions

- Krieger'skorte et al. 2008/2009
- Walther et al. 2009
- ...

pattern similarities or confusion matrices
where can it come from?

stimulus → mind → brain → fMRI data

what is present in the mind as the task is performed?

- known or constrained from behavioural experiments
- modelled mathematically or computationally
- hypothesized
- learnt elsewhere (text corpora)
take home points

- prediction goes beyond classification
- similarity/distance are useful too
take home points

- prediction goes beyond classification
- similarity/distance are useful too
- data-driven prediction has limits
  - what we can feed the prediction mechanism
  - what kind of bias it has
  - what we can dissect from it
take home points

- prediction goes beyond classification
- similarity/distance are useful too
- data-driven prediction has limits
  - what we can feed the prediction mechanism
  - what kind of bias it has
  - what we can dissect from it
- many other things to use in the mechanism
  - known or constrained from behavioural experiments
  - modelled mathematically or computationally
  - hypothesized
  - learnt elsewhere (text corpora)
Thank you!

questions?
a mathematical perspective for the advanced methods

Francisco Pereira

Botvinick Lab
Princeton Neuroscience Institute
Princeton University
low-dimensional spatial decompositions

example

\( (a, b, c, d) \) in a **basis** of (eigen)images are coordinates
low-dimensional spatial decompositions

\[ \text{example} \quad = \quad a \quad + \quad b \quad + \quad c \quad + \quad d \]

\((a,b,c,d)\) in a basis of (eigen)images

\((a,b,c,d)\) are coordinates

\((a,b,c,d)\) can also be seen as feature values in a low-dimensional feature representation
low-dimensional spatial decompositions

\[ n \text{ examples} \quad = \quad \text{low-dimensional representation} \quad \times \quad l \text{ basis images} \]
low-dimensional spatial decompositions

\[ n \text{ examples} \]

\[ X = Z \times W \]

\[ l \text{ basis images} \]

\[ m \text{ voxels} \]

\[ l \text{-dimensional representation of data} \]
low-dimensional spatial decompositions

Why express an image in terms of a basis of images?
- capture spatial patterns of activity over many voxels
- coordinates give a succinct representation of the data

\[
\begin{align*}
\text{...} & = \begin{array}{c}
\text{...} \\
\text{...} \\
\text{...}
\end{array} \\
\text{...} & = \begin{array}{c}
\text{...} \\
\text{...} \\
\text{...}
\end{array}
\end{align*}
\]

\[\times\]
features

- from data alone: learn Z and basis
  - any matrix factorization
  - SVD, ICA, NMF,…
  - regularization matters

\[ fMRI = ? \]
- from data alone: learn Z and basis
  - any matrix factorization
  - SVD, ICA, NMF,…
  - regularization matters

- hypothesized
  - behavioural experiments
  - mathematical model

- learned from text
two problems

- given Z, learn basis
  - #voxels regression problems
  - predict voxel $i$ from Z using column $i$ of basis matrix as regression coefficients
two problems

- given $Z$, learn basis
  - #voxels regression problems
  - predict voxel $i$ from $Z$ using column $i$ of basis matrix as regression coefficients

- given a new example $X$, find its basis coordinates
  - 1 regression problem
  - predict $x$ vector from basis rows using $z$ as regression coefficients
issues

- if $Z$ columns are orthogonal, finding basis is simple
- if basis images are orthogonal, finding $z$ is simple
issues

- if $Z$ columns are orthogonal, finding basis is simple
- if basis images are orthogonal, finding $z$ is simple

- if they are not, we need regularization
  - make things positive
  - make things have low L2 (small) or L1 (sparse) norms
issues

- if $Z$ columns are orthogonal, finding basis is simple
- if basis images are orthogonal, finding $z$ is simple

- if they are not, we need regularization
  - make things positive
  - make things have low L2 (small) or L1 (sparse) norms

- there might also be constraints
  - e.g. entries of $z$ must be positive and add to 1
a suggestion

- previous problems are likely convex
- CVX
  - a solver for a subset of convex programming
  - uses MATLAB notation to express the problem
  - many functions in the objective and constraints
  - http://cvxr.com/cvx
  - also available in Python
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- if not convex,
  - consider dividing into subproblems that are
  - and solve them in alternation
  - Carl Rasmussen’s minimize.m
Thank you!

questions?