Review of Possibilities and Paradox: An Introduction to Modal and Many-Valued Logic, J.C. Beall and Bas C. Van Fraassen

By Roy T. Cook << DRAFT. FOR PHIL. QUARTERLY>>

Possibilities and Paradox: An Introduction to Modal and Many-Valued Logic is, the authors write in the preface, intended "to give philosophy students a basic grounding in philosophical logic, in a way that connects with the motivations they derive elsewhere from philosophy" (p. ix). In providing a uniform framework within which one can study the behaviour of, and interactions between, modal, many-valued, intuitionistic, and paraconsistent logic, the text provides a useful contribution along these lines.

The authors restrict their attention to propositional logics. This alone is commendable, as too many textbooks intended to introduce non-specialists to the wonders of formalization seem to be written under the misapprehension that nothing philosophically or mathematically interesting occurs in formal logic until one has added quantification to the language. As a result, the book is presented at a level that is appropriate for advanced undergraduates with little exposure to mathematical logic or to advanced mathematics in general (although much of the material in the book will be of interest to non-specialists of any level of sophistication).

They begin the book with a philosophical introduction to the two main issues to be addressed in the text: modality and non-standard truth values. The chapter begins with an examination of the role of formalization within philosophy, adopting a position by which we can "see different logics as exemplifying a single logical point of view applying itself to... different circumstances." The reader who is familiar with Beall's publications on logical pluralism (or who read the last sentence of the chapter, recommending Beall and Restall [2000] and [2003] as further reading) is likely to emerge somewhat confused over a textbook written from a perspective that is in conflict with the published views of one of the authors. While some philosophical compromise is inevitable within a co-authored book (and perhaps especially so in a co-authored textbook), one cannot help but think that perhaps the discussion of the role of formalization in general, and logical pluralism in particular, should have been avoided (it only merits direct discussion for two pages anyway, although they happen to be the first two pages of the main body of the text!).

The remainder of the first chapter, which introduces issues motivating modal and many-valued logics through short discussions of Aristotle's sea battle, 'ways of being true', and the notion of a 'possible world', is well-informed and well intentioned. At a mere 9 pages (including the discussion of logical pluralism), however, it is altogether too short.

After a nice (but, again, rather short) chapter on the fundamentals of naive set theory (including a useful warning that "careless combination of [naive set abstraction] with other principles leads to contradiction" (p. 12)), the book moves on to its main task: providing a

uniform framework within which all of the logics mentioned above can be studied. Chapter 3 provides general definitions of notions such as language, valuation, deductive system, theoremhood, validity, soundness, and completeness. Chapter 4 provides the crucial materials for the rest of the book – a deductive system for classical propositional logic based on tableaux (or trees). The tableaux method of proof is then extended or modified so as to apply to normal modal logics (Chapter 5), intuitionistic and non-normal modal logics (Chapter 6), the so-called 'four corners of truth, i.e. true/false/both/neither (Chapter 7), various 3-valued logics (Chapter 8), and supervaluational and degree-theoretic semantics (Chapter 8). Finally, chapters 10 through 12 provide more sophisticated mathematical machinery and methods which allow for the presentation of metatheoretical results only discussed in the previous chapters.

Throughout the book, short sections are devoted to philosophical issues that (might) motivate the logics in question, including discussion of the paradoxes of material implication (Chapter 6), the Liar paradox (Chapter 8), and the Sorites paradox (Chapter 9). Like the discussion of modality and non-standard truth values found in the first chapter, this material is both illuminating and philosophically up-to-date. These discussions, however, like that of the first chapter, strike one as too concise to provide the student with any deep understanding of the problems at hand. The superficial treatment is especially unfortunate given the author's expertise in the philosophical areas surrounding truth and vagueness.

The book does provide, as advertised, a systematic treatment of a wide variety of logics, but there are worries regarding the particular methods employed. The authors provide tableaux as a proof procedure (p. 45), pointing out that the tableaux method meets the definition of a syntactically defined consequence relation (contrasting it with admissible valuations). Be that as it may, tableaux are originally motivated in the text (as they usually are) in terms of a test for (semantic) validity (p. 36-37). This ambiguity – tableau as both a test for semantic validity and as a derivation – threatens to cause confusion in less logically sophisticated readers regarding the crucial difference between deduction and semantic entailment (a difference which becomes more important when we move on to logics substantially stronger than those considered here). Even if confusion is avoided, adopting a 'deductive system' that is motivated by semantic concerns, instead of one that is modeled on the actual moves one makes when reasoning deductively (such as in a natural deduction or sequent calculus system) eliminates any sense of wonder or surprise when one reaches completeness theorems and other significant metatheoretical results. Instead, one gets the feeling that the deductive system in question was purposefully 'designed' to match up to the semantics, instead of being designed to match up to actual reasoning in natural language (it is worth noting that, in the later chapters of the book, where the metatheorems are rigorously proved, the authors switch from a tableaux system to natural deduction).

This complaint is, in all fairness, one that should be aimed at the present vogue for tableaux systems in general, and the authors should, perhaps, not be blamed overly much for hopping on the logical bandwagon. As Beall and van Fraassen demonstrate here, tableaux are extremely elegant and useful tools for studying formal logics. Nevertheless, they should not be used in a textbook intended to introduce the methods of formal logic to the uninitiated – the danger of students failing to attend to the crucial distinction between syntax and semantics is too critical to ignore.

The text fails, as most do, to be immune from mistakes. Fortunately such confusions are rare and usually rather subtle. For example, in a footnote Beall and Restall claim that:

> It is essential that a proof is finite. To see why, imagine that we allow infinitely long proofs and have the rule that A follows from A Δ A. Then for any sentence A, such as 'God exists', one could offer the "proof":
>
> ..., A Δ A Δ A Δ A, A Δ A Δ A, A Δ A, A
>
> which has no beginning (it is infinitely long), uses no axioms, but has each member follow from the preceding by means of that rule. (p. 160)

Beall and van Fraassen are half right – given the possibility of such arguments, we do not want to allow proofs where there are infinitely long chains of inference-rule applications. However, we might (even in the propositional case) adopt rules of inference that allow us to move from infinitely many premises to a single conclusion (i.e. proofs cannot be infinitely long, but might be infinitely wide, see, e.g. Dummett [***], p. **). This mistake is surprising in a book based on the method of tableaux, since the relevant distinction is between a tree with an infinitely long path versus a tree with an infinitely branching node. Nevertheless, this sort of error is perhaps excusable in a textbook intended for undergraduate students that are first encountering formal logic, since in such a context some simplification is unavoidable.

One of the main disappointments of the book, however, is an opportunity that the authors had but did not take. The tableaux approach allows the authors to place a multitude of standard formal results about modal and many-valued logics in a single uniform setting, but there is little if anything truly new in the book (other than, perhaps, the treatment of supervaluational semantics as a many-valued system). Since the focus is on modal logics and many-valued logics, the reader is left wondering why a chapter was not devoted to man-valued modal logics, i.e. modal logics where sentences can take values other than the traditional two at the possible worlds. Presumably the mathematics required to explore such logics at the propositional level would be little more advanced than that required to prove the more advanced metatheoretic results contained in the final three chapters. Had such an examination been included, then Beall and van Fraassen's contribution could be viewed as a philosophical (or at least logical) achievement as well as a pedagogical one. Given the

intrinsic interest of (extensions of) such logics as a possible means for dealing with puzzles arising from varying the domains of possible worlds, an introductory presentation of logics of this sort would have been valuable.

It is, perhaps, unfair to judge a textbook in terms of what novel material it fails to cover or explore, however, instead of judging it in terms of the manner in which it presents difficult, but standard, material to the student who is encountering these ideas for the first time. And to be fair, Beall and van Fraassen's book is a clear, concise, and thorough presentation of the standard metatheoretical results for (propositional) modal and many-valued logics. Given the lack of decent logic textbooks aimed at advanced undergraduate students, their book should be viewed as a genuine contribution to logic instruction.

As a final note, I must express some misgivings at the inclusion of the following quotation from Lord Dunsany at the conclusion of Chapter 11:

> Logic, like whiskey, loses its beneficial effect when taken in too large quantities. (p. 185)

My doubts about the appropriateness of this quote stem more from its implications regarding the mental state of professional logicians, however, than from worries regarding the encouragement of social drinking.

References:
Beall, J. and G. Restall [2000], "Logical Pluralism", Australasian Journal of Philosophy 78: 475-493.
Beall, J. and G. Restall [2003], Logical Pluralism, Oxford, Oxford University Press.
Dummett, M. [1991], The Logical Basis of Metaphysics, Cambridge, Harvard University Press.