Draft Only

J. C. Beall and Bas C. van Fraassen, *Possibilities and Paradox; an Introduction to Modal and Many-Valued Logic*, Oxford: Oxford University Press, 2003, ISBN 0 19 925987 9, US$29.95 (paperback).

It is impossible nowadays to pursue many areas of philosophy (philosophy of logic, philosophy of language, philosophy of mind, metaphysics) without understanding something about possible worlds, counterfactual conditionals, truth value gaps and gluts, and other standard fare in non-classical logics. These are not normally covered in introductory text books on logic; nor in more advanced books, which tend to concentrate on metatheory of various kinds. This book aims to fill the gap, presenting various systems of non-classical propositional logic. It is written by two notable philosophical logicians, one (van Fraassen), a well-known member of the profession, and the other (Beall) an up-and-coming one. And as one would expect from philosophers of this calibre, it is an excellent book: clear, elegant, independent minded. It is also explicitly linked to the websites of the two, for further information, updates, etc. This is an excellent idea.

The first part of the book covers various preliminary matters, including some philosophical motivations and the set-theoretic tools necessary to engage with the material. The second part covers normal modal logics, with a brief foray into other logics with possible-world semantics, such as conditional and intuitionistic logic. The third part covers many-valued logics, such as first-degree entailment and (briefly) continuum-valued logic, with an especial eye on the way that these are often applied to the paradoxes of self-reference and vagueness. The final part is entitled 'Metatheory' and covers some of the metatheory of the logical systems already introduced, including various completeness results, but also introduces further "first order" features concerning them. At the end of each chapter, there are exercises and problems of various degrees of difficulty.

The book was put together from notes used by the authors to (independently) teach the material. A result of this, I thought, is that it lacks a certain unity. For example, tableaux are used to provide the proof theory some times, natural deduction others. And soundness and completeness are proved for some of these systems but not others. (Of course, it is good for students to know about different systems of proof, but it is harder for a student when things jump around.) I had to work quite hard to keep track of what had been established by the end of the book. Here is a table I compiled which may be useful for readers.

Logical System	**Proof Theory Used**	**S&C Proved**
Classical	Tableaux	Yes
	Nat. Ded.	Yes
Normal Modal	Tableaux	No
	Nat. Ded.	Yes
Non-Normal Modal	None	
Basic Conditional Logic (CK)	Nat. Ded.	Yes
Other Conditional Logics	None	
Intuitionist Logic	Tableaux	No
	Nat. Ded.	Yes
Many-Valued: K_3, LP, FDE	Tableaux	FDE only
Many-Valued: B_3, RM_3	None	
Continuum-Valued: Ł$_{\aleph}$	None	
Finite-Valued Functionally Complete	Nat. Ded.	Yes

A certain lack of unity is also revealed in the fact that material from the first three parts of the book is repeated in the last part. This is not necessarily a bad thing in a text book. But when the material is repeated, it is sometimes done in a slightly different way (e.g., the semantics for intuitionist logic in 12.4). It might have been better to employ a uniform approach throughout the book.

Whilst still on pedagogical matters, I thought that the book could have been improved by more worked examples. For example, tableaux for the modal logic K are explained carefully, and worked examples are given. But this is not the case for the extensions of K, where the interplay of the tableaux rules for the accessibility relations often cause students to stumble. Nor are there any examples for the more intricate tableaux for intuitionist logic or the basic conditional logic, CK. The use of diagrams would also have made some of the discussion more perspicuous, especially, for example, in the specification of (counter-)models for logics with world-semantics. It also seemed to me that important information was too often relegated to footnotes; and also that some of the definitions and proofs left as exercises in the text were really quite hard for students meeting the material for the first time. In short, then, though the book is clearly written, I thought that it could have been improved from a pedagogical point of view.

Turning to content, this is reliable and instructive. I noted only one significant error. The completeness proof for CK is incorrect. The proof-theory given for CK on p. 200 cannot establish, e.g., that $\vdash \mathcal{A} \Rightarrow \mathcal{A}$, which is valid on the semantics given. It is complete with respect to the truth conditions for $\Rightarrow$ as given on p. 198, with '$\nu(w', \mathcal{A}) = 1$ and' deleted. (The canonical model construction given verifies only this.)

On a smaller point, the construction employed in the "priming lemma", 12.7 (p. 206), is unnecessary. If one's proof theory is axiomatic, a special construction to ensure priming is necessary. But with natural deduction employing the rule $\vee$ Elim (p. 203):

$$\frac{\chi, \mathcal{A} \vdash \mathcal{C} \qquad \chi, \mathcal{B} \vdash \mathcal{C}}{\chi, \mathcal{A} \vee \mathcal{B} \vdash \mathcal{C}}$$

the set obtained by the usual Henkin-style construction is already prime. (If neither $\mathcal{A}$ nor $\mathcal{B}$ is in the set constructed, then $\mathcal{A}$ and $\mathcal{B}$ both give some forbidden $\mathcal{C}$. But then so does $\mathcal{A} \vee \mathcal{B}$, which is, therefore, not in the set.)

There is also one point at which the lucidity of the book leaves it. In chapters 11 and 12, the symbol '$\vdash$' is used both as the relation of derivability and for what is, in effect, the main connective of a sequent calculus (so that things of the form '$\chi \vdash \mathcal{A}$ ' may themselves be proved). A trained eye can tell when it is functioning in which role, but for a student unfamiliar with the material, this is likely, I think, to lead to confusion. It caused me some confusion too. On p. 161 '$\chi \vdash \mathcal{A}$ ' is defined in the usual way for an axiom system. In particular, χ is an arbitrary set, finite or infinite. But immediately after this, things of this form appear as sequents in a natural deduction system, the preferred proof-theory of this part of the book. No restriction to finite χ is mentioned. So one might naturally assume that the χ can be infinite—or, if not, we are not told how to understand '$\chi \vdash \mathcal{A}$ ' in this context when χ is infinite. In all the completeness proofs that come up thereafter it is essential to use the fact that $\vdash$ is compact (that is, if $\chi \vdash \mathcal{A}$ then there is some finite $\chi' \subseteq \chi$ such that $\chi' \vdash \mathcal{A}$). But this is never proved, and is always left as an exercise (e.g., p. 182). In virtue of the unclarity over $\vdash$, it was not clear to me how this was supposed to be proved. And if the χ in the natural deduction systems may be infinite, the proof is hardly of the trivial kind that can safely be left too the student reader.

The book also has, I thought, a rather large number of typos and minor infelicities. Here are some of the ones that I noted:

- p. 31, last sentence of 3.5: tableaux procedures provide a decision procedure only when the tableaux must be finite.

- p. 40, fn. 4: '$\circ$' should be '0'.

- p. 62, l. 7: the notion of a characteristic sentence for a modal logic is introduced without any real explanation.

- p. 75, l. -7 of text: this should read 'Ignore all boxed-up sentences in *b and those where the formula starts with a "$\Box$"*'. And the recipe being provided here does not say what to do with a propositional parameter when neither it nor its negation occurs on a line. (Moreover, the counter-model constructed on p. 79 does not follow the recipe given.)

- p. 79: the method of "climbing" provides a counter-model only when all possible rules have been applied. When the branch is infinite, some algorithm it needed to ensure that this is done.

- p. 93, first sentence of 6.3: the notions of indicative and subjunctive conditionals are referred to with no explanation.

- p. 95, intuitionistic proof conditions: a proof condition for $\perp$ needs to be given.

- p. 98, first part para: the connection envisaged between intuionist disjunction and *reductio ad absurdum* is not clear.

- p. 99: closure for intuitionistic tableaux is not defined.

- p. 105, first para: it would be nice to have an explanation of what, exactly, a relevance logic is here.

- p. 110, first para. of 7.3: in FDE everything implies all tautologies, since there aren't any! And it would be nice to be given an idea of what the "paradoxes of material and strict implication" are.

- p. 142, l. -5 of text: 'just $\nu(B)$' should be 'just $1 - \nu(B)$'.

- p. 150, bottom: some examples to illustrate the definitions here would be helpful.

- p. 151, l. -1: any maths student is going to rebel at the expression '$1+2+4+8+...$'.
- p. 156, PROOF OF LEM 10.2: The point of withdrawing from `Synt` to `Synt`* is not explained. (What is it?)
- p. 163: conjunction has turned into a multigrade connective without warning.
- p. 164, l. 20: 'Lemma 4' should be 'Lemma 11.1'.
- p. 178, l. 8: 'compact' might be more usual than 'finitary'; and l. 16: in this context, 'trivial' would be better than 'inconsistent'.
- p. 190, fn. 1: the Henle matrices for $S5$ have not been given or referenced.
- p. 197, l. 10: the second occurrence of '$\chi_k \vdash \mathcal{A}_k$' should be '$\chi_k \Vdash \mathcal{A}_k$'.
- p. 198, l. 17: the first occurence of '$\nu(w', \mathcal{A}) = 1$' should be '$\nu(w', \mathcal{B}) = 1$'.
- p. 200, ll. 4-5: a bit more might be said to help the reader see why the Union and Order conditions might be plausible.
- p. 205, l. -8: 'prime theories' should be 'non-trivial prime theories'.
- p. 215: it might help to point out that the truth values here are natural numbers (they are not always, in the book); and l. 19: 'the expression "(k, j)"' should be 'the expression "$u(k, j)$"'.
- p. 218, fn. 19: it would be helpful to have a hint as to what sorts of corollaries the authors have in mind.
- p. 219: 'PROOF THM 12.4' should be 'PROOF THM 12.3'.

Of course, any text is going to have features of this kind, but I did think that this one had perhaps too many.

In short, this is an excellent book, well conceived in principle and clearly written. But I think that it would have benefitted from a bit more tender loving care in execution and production.

Graham Priest
Universities of Melbourne and St Andrews
ggp@unimelb.edu.au