Competition as a Discovery Procedure: Hayek in a Model of Innovation

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Abstract

I model a mechanism through which competition can encourage innovation and growth. Although the oft-cited 'Schumpeterian effect' of competition is to decrease the expected rents from an innovation, the competitive process also causes a 'thousand flowers to bloom', and provides a mechanism for discovering the best among them. When firms are uncertain about the best direction in which to innovate, more competition implies more innovations, resulting in a higher expected value for the 'best' innovation. By endogenizing the level of competition and introducing this uncertainty into a general equilibrium model of vertical innovation, I show how this Hayekian effect of competition can work against the Schumpeterian effect, resulting simultaneously in a positive relationship between competition and growth at the industry level, and an inverted-U relationship between competition and firm-level innovation. Finally, I show how this framework can be used to examine the effects on innovation of antitrust policy and regulation.

1 Introduction

Schumpeter (1942) argued that some expectation of monopoly power is necessary to induce innovation. Although his views were certainly more nuanced, a simplistic implication is that a higher level of competitive rivalry should translate into lower levels of innovation, and thus lower productivity growth. Early theoretical models of both vertical and horizontal innovation reinforced this view, sharing the conclusion that competition diminishes the expected rents to innovators, thereby dampening innovation and growth.[1] At the same time, however, evidence is accumulating that suggests competition is *positively* related to productivity growth.[2] Related studies, including Aghion *et al.* (2005)

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[1]For example, see Aghion and Howitt (1992) and Romer (1990).

[2]For example, see Nickell (1996), Blundell *et al.* (1999), and Dutz and Hayri (2000).

and Aghion *et al.* (2008), have provided evidence of an inverted-U relationship between firm-level innovation and competition, even when the relationship is strictly positive at the industry level.

The purpose of this paper is to model a mechanism through which competition can encourage growth, in a general equilibrium setting. I do this by building a simple model of vertical innovation, similar in spirit to earlier Schumpeterian growth models. I assume that entrepreneurs are uncertain about the relative value of each potential direction of innovation, until after an innovation has actually been introduced. I also endogenize the level of competition by introducing Bertrand competition. Once firms realize the value of their innovations, the best firm can capture the relevant market by pricing at or below its closest competitor's marginal cost. Contrary to the negative relationship between competition and growth implied by conventional endogenous growth models, such as Grossman and Helpman (1991), and Aghion and Howitt (1992, 2005), the model developed here generates a positive relationship between competition and growth, as well as an inverted-U relationship between competition and firm-level innovation.

Certainty of the relative demand for new goods, and of the value of innovations in general, is the source of the consistent negative relationship between competition and growth in endogenous growth theory. Assuming that demand is known assumes away much of the value that competition brings to a society, leaving only the benefit of a higher allocative efficiency at some fixed point in time. A survey of the empirical evidence of the magnitude of the deadweight loss due to market power suggests that any benefit from greater static allocative efficiency is most likely overwhelmed by the supposed ill effects of greater competition on the returns to research.[3]

What else of value does competition bring to the table? In Hayek's words, competition is (2002, p.9):

> a procedure for discovering facts which, if the procedure did not exist, would remain unknown or at least would not be used.

In a static setting, the process of competition tends to result in higher profits for more efficient producers, and losses for inefficient producers. Equilibrium prices start to emerge, or are *discovered*, as a result of the competitive process. In a dynamic setting, the value of technical efficiency becomes more tenuous. Schumpeter's "perennial gale of creative destruction" (1942, p.85) destroys not just the profits of incumbents, but the very relevance of the information that

[3]Harberger (1954) pioneered the empirical measurement of deadweight loss due to various distortions. Jenny (1983) provides a partial survey of the subsequent literature. Estimates of the deadweight loss due to monopoly power in the U.S. economy range from about 0.1% to 8% of G.N.P. To justify my characterization of these magnitudes as inconsequential, imagine that the U.S. stopped enforcing intellectual property rights, effective immediately. Further, imagine this reform would result in an increase in the level of output of 8%, due to efficiency gains, as well as a decrease in subsequent growth from 3% to 2% − a decrease that seems conservative, given most economists' priors about the efficacy of intellectual property rights in spurring innovation. In such a senario, the reformed economy would perform better for just 15 years before permanently falling below its previous potential.

firms previously relied on to determine their 'optimal' size, product space, and organizational structure. As technology advances and consumer behavior adapts to new conditions, yesterday's best practices become today's mistakes. Over time, static efficiency becomes less important, and profits tend toward those producers who best anticipate changes in the economic system.

Why are entrepreneurs so uncertain about the value of innovations? When firms set out to improve a product, there is no one-dimensional measure of quality that they can progress along. Lancaster (1966) and others have explicitly modeled the demand for a good as being derived from the demand for the various characteristics of that good. A consumer's willingness to pay for a car depends on how comfortable it is, its appearance, level of safety, durability, and a host of other characteristics − not to mention the characteristics of other goods that are available. For example, a consumer may choose safety over style in his car, and then purchase more style through other goods, like clothing and furniture.

Likewise, an entrepreneur cannot be certain of the value of a process innovation. When McDonald's restaurants began outsourcing order-taking at certain drive-thrus to out-of-state call centers in 2005, they could not know in advance whether the value of faster service would outweigh the costs of any unexpected problems that might arise. Even if the benefits did outweigh the costs, they still could not be certain that a different innovation (perhaps undertaken by a competitor) would not have turned out better.

Entrepreneurs, then, are constantly making informed guesses about how to change their production processes or product lines. They may decide to invest in R&D to improve their product or production process in what they perceive to be the best direction. If this research succeeds, however, firms must still incur further costs before realizing the value of their innovations. These costs are made necessary by both the uncertainty inherent in a market and any regulations that govern the activities of firms.

The market portion of these costs may flow from the necessity of holding an inventory for a length of time, the need to disseminate information about the new product to consumers, or the desire to provide sales training − all before demand is realized. Similarly, new machinery must be purchased and employees trained before a new production process can be implemented. Production can then take place, but time must still be spent using the new process before any unexpected problems or benefits make themselves known.

An obvious example of regulatory costs that must be incurred before introducing a new good is the extensive safety and efficacy testing required by the FDA before a new drug is allowed to be marketed.[4] A new restaurant must obtain licenses and a new car requires safety testing. In addition to these costs, new products must often include a plethora of mandatory features, such as airbags, seatbelts and catalytic converters in automobiles, various ingredient and warning labels on food and appliances, and Vitamin D in milk.

Regulatory compliance can be just as burdensome for process innovations.

[4] Klein and Tabarrok (ongoing) provide a detailed history of U.S. Food and Drug Administration regulations.

Labor and wage restrictions can increase the transitional cost of changing production processes, and a process innovation that involves a change in the size of a firm may run afoul of competition policy. Again, the value of a new product or process only becomes fully known after all of these costs are incurred.

In the next section, I present a general equilibrium model of quality-improving innovations that features this uncertainty. When firms decide to improve a product, they must first choose the direction in which to innovate. One firm might try to improve the speed of a car more than its safety, while another firm may eschew both in favor of better handling or a larger carrying capacity. The mechanics and implications of the model are equivalent to those of a model with cost-reducing innovations. In such a model, one firm might try to routinize its production process, while another may try to incorporate a new computer network.

By combining this uncertainty with an endogenously determined level of competition, I show how a model of vertical innovation can generate a positive relationship between competition and growth. By allowing each firm to invest in research in an effort to improve the value of its innovation, the model also generates an inverted-U relationship between competition and firm-level innovation.

Two types of models have been developed that generate the possibility of a positive correlation between competition and growth. The first is by Aghion *et al.* (2005).[5] They model an economy with a continuum of intermediate sectors, each structured as a duopoly. Each pair of firms is characterized by the leader's level of technology and the gap between its own technology and its competitor's. Competition is measured as the inverse of the ability to collude when the two firms are 'neck-and-neck', with respect to their level of technology. The model implies that less ability to collude results in more research being done by neck-and-neck firms, in an attempt to escape competition. When the duopolists are far from each other, however, less ability to collude results in less research being done by the lagging firm, since there is less reward for catching up. A very low level of collusion results in an industry exhibiting a greater tendency to be in a leader-follower situation, where even less collusion reduces innovation. A relatively high level of collusion, on the other hand, results in a greater tendency for an industry to be in a neck-and-neck situation, where less ability to collude would *increase* the average level of innovation. This is the idea behind the inverted-U result of the model.

The level of collusion in an industry, however, is a troublesome way to define competition. While the effect of differing levels of collusion on innovation is interesting in itself, the question remains as to whether or not the model can explain the connection between firms' innovative activity and the Lerner Index. One might presume that the model is representative of an economy with more firms, where the level of collusion could be redefined more generally as an innovator's markup over marginal cost. But then the assumption that more

[5] The model presented in Aghion *et al.* (2005) is actually a variant of the model developed in Aghion *et al.* (2001), and has subsequently been used in other papers and by other authors.

competition leads to a lower markup for innovating followers, but not for innovating leaders, would no longer seem plausible. Dropping this assumption would remove any positive effect of competition on innovation, as competition would no longer be increasing the relative value of innovating for neck-and-neck firms.[6] The present paper instead relies on a more plausible source of variation in measured competition across industries - namely, variation in the costs of introducing an innovation. By incorporating free entry into the market, I also retain a connection between the conventional definition of 'competition' - more firms in the market - and the *measure* of competition used in recent empirical studies. Finally, the structure of the Aghion *et al.* (2005) model does not allow for the differing relationships between competition and growth at the firm and industry levels suggested by studies like Aghion *et al.* (2008).

The second class of model in which competition can be positively correlated with growth is analyzed by Vives (2008), who incorporates differentiated products with cost-reducing innovations. As in the present paper, he analyzes the effects of changes in primitives on both competition and innovation, finding that changes in some parameters lead to increases in both growth and competition. By using strictly differentiated firms, however, Vives can only analyze competition *between* markets, and not *for* a market. Although the model can generate correlations between competition and growth that differ at the firm and economy level, it does not explain an inverted-U at the firm level.

This paper is somewhat related to papers by Boldrin and Levine (for example, 2008), which study the conditions under which innovation can occur under perfect competition. They do not, however, consider the consequences to innovation of varying levels of competition, as I do here.

In Section 3, I review some empirical studies and discuss how well this paper's results hold up to the available evidence. Section 4 uses the Hayekian model to evaluate the effects of antitrust policies on growth and innovation. To preview the results, I find that efforts to limit markups *ex post* reduce growth, as do efforts to prevent collusion or 'anti-competitive' mergers. The last section concludes.

2 The Model

2.1 Setup

Consider a closed economy where time is discrete, indexed by t=0,1,2... There are two sectors; a final-good (y) sector, and an intermediate-good (x) sector, made up of a measure 1 of intermediate-good industries. The final-good market is perfectly competitive, with a representative final-good firm producing

[6] It should be noted that in earlier versions of their paper, as well as in Aghion *et al.* (2001), the variation in measured competition is caused by changes in the elasticity of substitution between varieties. This places them in the second class of models, discussed below.

according to

$$y = \int_0^1 (A_{[1]}(j)X(j))^\alpha L_y^{1-\alpha} dj,$$

where j indexes intermediate-good industries, $X(j)$ denotes the amount of intermediate-good j used, and L_y the labor input in final good production. $A_{[1]}(j)$ is a measure of the quality of $X(j)$, and y will double as the numeraire.

In any particular industry j, there are a large number of potential intermediate firms. Any firm-i that invests in innovation in period $t-1$ can build on that period's highest-quality good and then produce in period t according to

$$x_{it}(j) = L_{it}(j),$$

where $x_{it}(j)$ is the amount of intermediate good-j produced by firm-i of quality $A_{it}(j)$, and $L_{it}(j)$ is its labor input. The investment required to innovate is made up of a fixed cost, $zy_{t-1}\frac{A^\alpha_{t-1[1]}(j)}{\overline{A}^\alpha_{t-1[1]}}$, and the cost of research, $my_{t-1}\frac{A^\alpha_{t-1[1]}(j)}{\overline{A}^\alpha_{t-1[1]}}n_{i,t-1}(j)$, where $n_{i,t-1}(j)$ is the level of research chosen by firm-i, $A_{t-1[1]}(j)$ is the quality of the first-best good of industry-j in period $t-1$, and $\overline{A}_{t-1[1]}$ is an average over the best quality of each industry in $t-1$. The presence of the term $\frac{A^\alpha_{t-1[1]}(j)}{\overline{A}^\alpha_{t-1[1]}}$ reflects the phenomenon that each incremental quality improvement is more costly than the last.[7] The fixed cost can be interpreted as the cost of marketing a new product, carrying an inventory, etc..., before demand is realized, while the research investment is the cost of actually improving the quality of the product. Firms finance these investments by issuing equity to households.

Once firms have decided whether to invest in innovation, and all the associated costs are sunk, only then is each innovating firm's quality revealed. For each firm-i, the introduction of a new product results in a quality index of

$$A_{it}(j) = A_{t-1[1]}(j)h_{it}(j),$$

where $A_{t-1[1]}(j)$ is the highest quality in industry-j in period $t-1$, and $h_{it}(j)$ is the realized value of a random draw. I assume $h_{it}(j) \sim U(0, n^\theta_{i,t-1}(j))$, $\theta \in (0,1)$, where $n_{i,t-1}(j)$ is the level of research invested in by firm-i. Intuitively, the randomness of the draw represents each firm's uncertainty of the value of the dimension in which it has chosen to innovate, relative to other dimensions, while the level of research, n, determines the magnitude of the improvement along that dimension. Note that the lower bound of zero implies that innovators may misread their their potential customers so badly that the new product may be less valuable than the previous period's best. I call this the *New Coke Phenomenon.*

There is a mass of households of measure 1, each of which supplies L_y units of labor inelastically to the final-good sector, and L_x units of labor to each intermediate-good industry. Households only value consumption, and have a constant discount rate $\beta \in (0,1)$. Income can be consumed or saved, and the

[7]This is similar to the 'fishing-out' effect in Aghion and Howitt (2005).

only vehicle for savings is the purchase of equity in innovating intermediate firms, offering a gross rate of return of R.[8] The household's problem, then, is to choose consumption, $\{c_t\}_{t=0}^{t=\infty}$, and savings, $\{s_t\}_{t=0}^{t=\infty}$, to maximize

$$\sum_{t=0}^{\infty} \beta^t E_0 \left[\ln(c_t)\right], \quad \text{s.t. } c_t + s_t \leq L_y w_{yt} + L_x \int_0^1 w_t(j)dj + s_{t-1}R_t,$$

where $L_y w_{yt}$ and $L_x \int_0^1 w_t(j)dj$ are the wage incomes from labor supplied to the final and intermediate sectors, respectively. Finally, I assume no population growth.

2.2 Market Structure

The final-good sector is perfectly competitive, and so the representative final-good producer will demand $X_t^d(j)$, such that

$$\alpha A_{t[1]}^{\alpha}(j) \left(\frac{L_{yt}^d}{X_t^d(j)}\right)^{1-\alpha} = P_t(j) \tag{1}$$

in equilibrium, where $P_t(j)$ is the price of $X_t^d(j)$ in terms of the numeraire y_t, $A_{t[1]}(j)$ is the highest realized quality in industry-j, and $X_t^d(j)$ is an aggregate measure of $A_{t[1]}(j)$-equivalent inputs demanded from industry-j. That is,

$$X_t^d(j) = x_{t[1]}^d(j) + x_{t[2]}^d(j)\frac{A_{t[1]}(j)}{A_{t[2]}(j)} + ...,$$

where $x_{t[k]}^d$ is the demand for the input with the k-th best quality.

Taking the wage rate $w_t(j)$ as given, the j-industry firm with the highest realized quality, $A_{t[1]}(j)$, would prefer to charge the monopoly price,

$$P_t(j) = \frac{w_t(j)}{\alpha}, \tag{2}$$

and supply the entire market for intermediate-good j. It may be constrained, however, by the second-best firm. Given that the second-best firm can produce one unit of $A_{t[1]}(j)$-equivalent intermediate input at a cost of $w_t(j)\frac{A_{t[1]}(j)}{A_{t[2]}(j)}$, it will have an incentive to start producing if the price rises above a certain threshold. In particular, the best firm can only maintain its monopoly price when

$$\frac{A_{t[2]}(j)}{A_{t[1]}(j)} \leq \alpha. \tag{3}$$

When condition (3) is violated, however, the best firm can still capture the entire market for good-j by setting a price equal to the second-best firm's marginal cost. That is,

$$P_t(j) = w_t(j)\frac{A_{t[1]}}{A_{t[2]}}. \tag{4}$$

[8] With *ex ante* identical firms, households will diversify their investments over all intermediate firms.

Labor-market clearing will combine with the final-good firm's demand for $X_t^d(j)$ (equation (1)) and the two pricing strategies ((2) and (4)) to determine the wage rate in industry-j;

$$w_t(j) = \begin{cases} \alpha^2 A_{t[1]}^{\alpha}(j)\left(\frac{L_y}{L_x}\right)^{1-\alpha}, & \text{if } \frac{A_{t[2]}(j)}{A_{t[1]}(j)} \leq \alpha \\ \alpha \frac{A_{t[2]}(j)}{A_{t[1]}^{1-\alpha}(j)}\left(\frac{L_y}{L_x}\right)^{1-\alpha}, & \text{if } \frac{A_{t[2]}(j)}{A_{t[1]}(j)} > \alpha \end{cases}$$

Any innovating firm-i in period t, therefore, will face the following expected discounted profits;[9]

$$\begin{aligned} E_t\left(\frac{\pi_{i,t+1}}{R_{t+1}}\right) = & \\ & \text{Prob}\left[i \text{ wins}, \frac{A_{t+1[2]}}{A_{t+1[1]}} \leq \alpha\right] \cdot E_t\left[\frac{P_{t+1} - w_{t+1}}{R_{t+1}} \,\middle|\, i \text{ wins}, \frac{A_{t+1[2]}}{A_{t+1[1]}} \leq \alpha\right] \cdot L_x \\ + & \text{Prob}\left[i \text{ wins}, \frac{A_{t+1[2]}}{A_{t+1[1]}} > \alpha\right] \cdot E_t\left[\frac{P_{t+1} - w_{t+1}}{R_{t+1}} \,\middle|\, i \text{ wins}, \frac{A_{t+1[2]}}{A_{t+1[1]}} > \alpha\right] \cdot L_x \\ - & y_t \frac{A_{t[1]}^{\alpha}}{\overline{A}_{t[1]}^{\alpha}}(z + mn_{it}), \end{aligned}$$

where $zy_t \frac{A_{t[1]}^{\alpha}}{\overline{A}_{t[1]}^{\alpha}}$ is the cost of introducing an innovation, and $my_t \frac{A_{t[1]}^{\alpha}}{\overline{A}_{t[1]}^{\alpha}} n_{it}$ is the cost of firm-i's research n_{it}.

Substituting the final-good production function, equilibrium prices and wage rates, and $\overline{A}_{t[1]}^{\alpha} \equiv \int_0^1 A_{t[1]}^{\alpha}(j)dj$, expected discounted profits are

$$\begin{aligned} E_t\left(\frac{\pi_{i,t+1}}{R_{t+1}}\right) = & \\ & \alpha(1-\alpha)L_x^{\alpha}L_y^{1-\alpha}\text{Prob}\left[i \text{ wins}, \frac{A_{t+1[2]}}{A_{t+1[1]}} \leq \alpha\right] \cdot E_t\left[\frac{A_{t+1[1]}^{\alpha}}{R_{t+1}} \,\middle|\, i \text{ wins}, \frac{A_{t+1[2]}}{A_{t+1[1]}} \leq \alpha\right] \\ + & \alpha L_x^{\alpha}L_y^{1-\alpha}\text{Prob}\left[i \text{ wins}, \frac{A_{t+1[2]}}{A_{t+1[1]}} > \alpha\right] \cdot E_t\left[\frac{A_{t+1[1]} - A_{t+1[2]}}{A_{t+1[1]}^{1-\alpha}R_{t+1}} \,\middle|\, i \text{ wins}, \frac{A_{t+1[2]}}{A_{t+1[1]}} > \alpha\right] \\ - & A_{t[1]}^{\alpha}L_x^{\alpha}L_y^{1-\alpha}(z + mn_{it}). \end{aligned}$$

Further substituting $A_{t[1]}h_{t+1[k]}$ for $A_{t[k]}$ and $\Delta_t \equiv A_{t[1]}^{\alpha}L_x^{\alpha}L_y^{1-\alpha}$, we get

$$\begin{aligned} E_t\left(\frac{\pi_{i,t+1}}{R_{t+1}}\right) = & \qquad (5) \\ & \alpha(1-\alpha)\Delta_t\text{Prob}\left[i \text{ wins}, \frac{h_{t+1[2]}}{h_{t+1[1]}} \leq \alpha\right] \cdot E_t\left[\frac{h_{t+1[1]}^{\alpha}}{R_{t+1}} \,\middle|\, i \text{ wins}, \frac{h_{t+1[2]}}{h_{t+1[1]}} \leq \alpha\right] \\ + & \alpha\Delta_t\text{Prob}\left[i \text{ wins}, \frac{h_{t+1[2]}}{h_{t+1[1]}} > \alpha\right] \cdot E_t\left[\frac{h_{t+1[1]} - h_{t+1[2]}}{h_{t+1[1]}^{1-\alpha}R_{t+1}} \,\middle|\, i \text{ wins}, \frac{h_{t+1[2]}}{h_{t+1[1]}} > \alpha\right] \\ - & \Delta_t(z + mn_{it}), \end{aligned}$$

[9] The industry subscript j will henceforth be dropped unless required for the sake of clarity.

where $h_{t+1[k]}$ denotes the k-th highest realized draw in period $t+1$.

I assume free entry into the intermediate sector, and so the number of entrants in each industry, $e_t(j)$, will adjust to satisfy $E_t\left(\frac{\pi_{i,t+1}}{R_{t+1}}\right) = 0$, given each firm's optimal choice of n_{it}, its level of research.

When forming expectations about the future, I assume (without loss of generality) that each firm-i believes that every other firm will choose the same level of research $-$ i.e., $n_{kt}(j) = n_{lt}(j), \forall k, l \neq i \in \{1, .., e_t(j)\}$. It follows that the joint density function of $h_{t+1[1]}$, $h_{t+1[2]}$, and $h_{i,t+1} = h_{t+1[1]}$, conditional on $e_t, n_{it},$ and n_{kt}, is

$$f(h_{t+1[1]} = v, h_{t+1[2]} = u, h_{i,t+1} = h_{t+1[1]} \mid e_t, n_{it}, n_{kt}) = \frac{(e_t - 1)u^{e_t - 2}}{n_{it}^{\theta} n_{kt}^{\theta(e_t - 1)}}, \quad (6)$$

defined over the relevant intervals of u and v.[10]

2.3 Competitive Equilibrium

A competitive equilibrium is defined as a sequence of allocations $\{c_t, s_t, y_t, x_t(j), X_t^d(j), L_y, L_{yt}^d, L_x, L_t^d(j), e_t(j), n_{it}(j)\}$ and prices $\{w_t(j), w_{yt}, P_t(j), R_t\}$, $\forall t \geq 0,\ \forall i \in \{1, ..., e_t(j)\},\ \forall j \in [0,1]$, such that;

(i) Households take $e_t(j)$, $n_t(j)$, $w_t(j)$, w_{yt}, and R_{t+1} as given, $\forall t \geq 0, \forall i \in \{1, .., e_t(j)\}$, $\forall j \in [0,1]$, and choose $\{c_t\}_{t=0}^{t=\infty}$ and $\{s_t\}_{t=0}^{t=\infty}$ to solve

$$\max_{\{c_t, s_t\}_{t=0}^{t=\infty}} \left\{ \sum_{t=0}^{\infty} \beta^t E_0 \left[\ln(c_t)\right] \right\},$$

$$\text{s.t. } c_t + s_t \leq L_y w_{yt} + L_x \int_0^1 w_t(j) dj + s_{t-1} R_t$$

(ii) In each period t, each innovating intermediate firm-i in each industry-j takes $e_t(j)$, $n_{k \neq it}(j)$, and R_{t+1} as given, and chooses $n_{it}(j)$ to solve

$$\max_{n_{it}(j)} \left\{ E_t \left(\frac{\pi_{i,t+1}(j)}{R_{t+1}} \right) \right\},$$

where $E_t\left(\frac{\pi_{i,t+1}(j)}{R_{t+1}}\right)$ is given by equation (5)

(iii) Free entry is satisfied:

- $E_t\left(\frac{\pi_{i,t+1}(j)}{R_{t+1}}\right) = 0,\ \ \forall t \geq 0, \forall i \in \{1, .., e_t(j)\}, \forall j \in [0,1]$

[10] This is derived in Appendix A.1.

(iv) In each period t, the final-good firm takes $P_t(j)$, and w_{yt} as given, and chooses L_{yt}^d and $X_t^d(j)$, $\forall j \in [0,1]$, to solve

$$\max_{\left\{L_{yt}^d, X_t^d(j)\right\}} y_t - w_{yt}L_{yt}^d - \int_0^1 P_t(j)X_t^d(j)dj,$$

$$\text{s.t. } y_t = \int_0^1 (A_{t[1]}(j)X_t^d(j))^\alpha L_{yt}^d dj$$

(v) Markets clear:

- $y_t = c_t + s_t, \ \ \forall t \geq 0$
- $L_t(j) \equiv x_t(j) = X_t^d(j), \ \ \forall t \geq 0, \ \ \forall j \in [0,1]$
- $L_y = L_{yt}^d, \ \ \forall t \geq 0$
- $s_t = \int_0^1 A_{t[1]}^\alpha(j)L_x^\alpha L_y^{1-\alpha}e_t(j)(z + mn_t(j))dj, \ \ \forall t \geq 0, \ \ \forall j \in [0,1]$

(vi) Prices satisfy;

$$P_t(j) = \begin{cases} w_t(j)/\alpha, & \text{if } \frac{A_{t[2]}(j)}{A_{t[1]}(j)} \leq \alpha \\ w_t(j)\frac{A_{t[1]}(j)}{A_{t[2]}(j)}, & \text{if } \frac{A_{t[2]}(j)}{A_{t[1]}(j)} > \alpha \end{cases}$$

I focus on the balanced growth equilibrium, in which $e_t(j) = e(j)$ and $n_{it}(j) = n(j)$, $\forall t$, for each industry-j For this equilibrium, $e(j)$ (the number of firms), $n(j)$ (the level of research per firm), and R (the gross rate of return on equity) can be solved for by using the following three conditions;

The *free-entry condition*;

$$E_t\left(\frac{\pi_{i,t+1}(j)}{R_{t+1}}\right) = 0$$

The *research condition*;

$$\frac{\partial}{\partial n_{it}(j)}E_t\left(\frac{\pi_{i,t+1}(j)}{R_{t+1}}\right) = 0$$

The *Euler condition*;

$$\frac{1}{c_t} = \beta E_t\left[\frac{R_{t+1}}{c_{t+1}}\right],$$

where the first two conditions use equation (5), and the *Euler condition* can be derived from the household's problem.

The economy's growth rate, g_t, is equal to

$$g_t = \frac{y_t}{y_{t-1}} - 1 = \frac{\int_0^1 A^{\alpha}_{t-1[1]}(j) h^{\alpha}_{t[1]}(j) dj}{\int_0^1 A^{\alpha}_{t-1[1]}(j) dj} - 1.$$

From the law of large numbers, we know that the right-hand-side of the equation above will be approximately equal to $\int_0^1 h^{\alpha}_{t[1]}(j)dj - 1$. With a large number of industries, then, the balanced growth rate in a symmetric economy is approximately equal to the expected growth rate in each industry, or

$$g \approx E(h^{\alpha}_{[1]}) - 1 = \frac{en^{\alpha\theta}}{e+\alpha} - 1.^{11} \tag{7}$$

Given the economy's growth rate g, the gross interest rate is

$$R_{t+1} = R = \frac{1+g}{\beta}. \tag{8}$$

Using the *free-entry* and *research conditions* above, we can now characterize a symmetric equilibrium with the following two equations;[12]

$$n = \frac{z\theta(e+\alpha-1)}{m[1-\theta(e+\alpha-1)]} \tag{9}$$

$$z = \frac{\alpha(1-\alpha^e)\beta[1-\theta(e+\alpha-1)]}{e^2} \tag{10}$$

All other variables (or their expected values) are functions of e and n.

The next section compares equilibria in which the value of one parameter is varied. One important result will come from a comparison of equilibria in which a parameter in just one industry is varied, while keeping the gross interest rate constant. This industry equilibrium is characterized by the following two equations;

$$n = \frac{z\theta(e+\alpha-1)}{m[1-\theta(e+\alpha-1)]} \tag{11}$$

$$z = \frac{\alpha(1-\alpha^e)n^{\alpha\theta}}{Re(e+\alpha)} - mn, \tag{12}$$

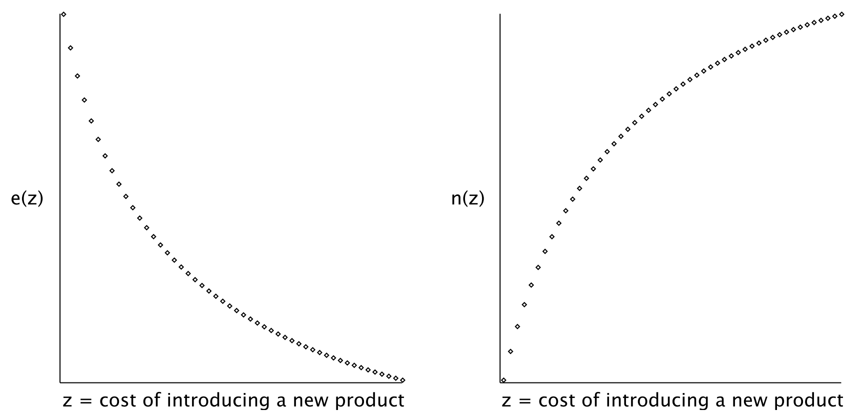
where the number of firms e, the level of research per firm n, and all parameter values except R, are specific to the industry being examined.

[11] This is derived in Appendix A.2.

[12] Equations (9) through (12) are derived in Appendix A.3.

2.4 Comparative Statics

Figures 1a and 1b show the equilibrium number of innovations (equivalently, firms) and level of research per innovation (respectively), as functions of the economy-wide cost of introducing a new product.[13] That is, each point represents the equilibrium value of e or n for the associated value of parameter z.[14] Not surprisingly, the number of innovations, e, is a decreasing function of the cost of introducing an innovation. Free entry ensures that e adjusts downwards as the fixed cost of innovating increases. As the overall level of innovation decreases along this *extensive* margin, however, remaining innovators partially compensate with a higher optimal level of research n, as the relative cost of research has decreased. The overall level of innovation thus increases at the *intensive* margin.



(a) $e(z)$ = Number of Innovations (b) $n(z)$ = Firm-Level Research

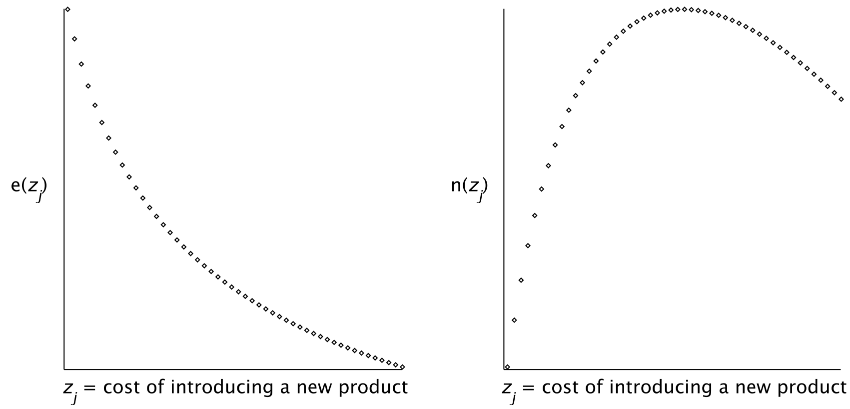
Figure 1: $e(z)$ and $n(z)$ with economy-wide variation in z

Figures 2a and 2b show a similar experiment at the industry level. Here, equilibria for one industry are compared for different values of z_j (the cost of introducing a new product) while z remains fixed for the rest of the economy.[15] While the qualitative relationship between $e(j)$ and z_j remains the same, a new relationship emerges between $n(j)$ and z_j. As z_j increases, $e(j)$ must decrease, and so the probability of industry-j ending up with a monopoly price increases (since $\frac{A_{[2]}}{A_{[1]}}$ will tend to be lower), while the probability of a limit price decreases. A limit price provides more of an incentive for research at the margin, as a larger

[13] All graphs that follow were produced using Maple, copyright (c) Maplesoft, a division of Waterloo Maple Inc. 2010.

[14] Unless otherwise specified, the direction of the relationship shown in each graph is independent of the values of all other parameters (as long as only feasible values are used). Accordingly, I omit the values used to produce each graph.

[15] For each industry experiment, I calculate the gross interest rate R using a value for the economy-wide z equal to the midpoint of the range used for z_j.



(a) $e(z_j)$ = Number of Innovations (b) $n(z_j)$ = Firm-Level Research

Figure 2: $e(z_j)$ and $n(z_j)$ with variation in z_j for industry-j

gap between $A_{[1]}$ and $A_{[2]}$ means higher profits for the best firm.[16] All else equal, this effect will depress $n(j)$ as z_j increases. After a certain threshold, this effect dominates, resulting in a decrease in $n(j)$ when z_j increases.[17] Why does this effect never dominate in Figure 1b, where z is increased for the entire economy? As z increases, a decrease in the equilibrium interest rate R offsets this effect by lowering the entire cost of innovating, resulting in an unambiguously positive relationship between n and z.

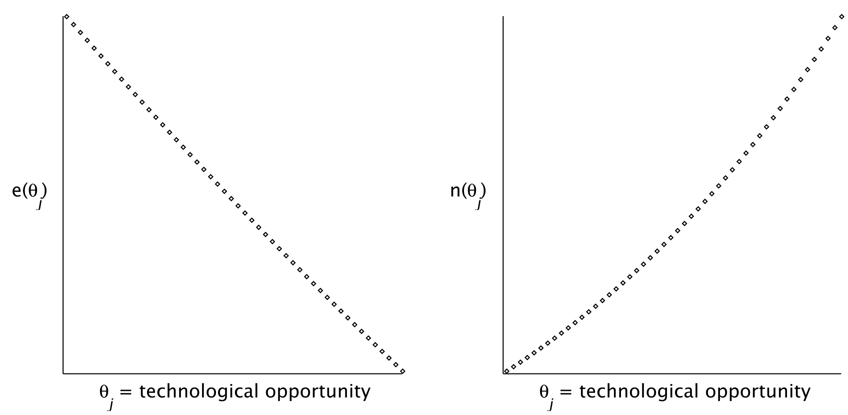
Equations (9) and (10) make it clear that the number of innovations is independent of the economy-wide value of m (the unit cost of research), as is $n \cdot m$, the level of research expenditure per innovation. While a lower m will be shown to increase the growth rate of the economy, it does so entirely through a proportionate increase in the level of research n, and not through a change in e. Given the relationship illustrated by Figure 1a, equations (9) and (10) also show that both the number of innovations and the level of research per innovation are increasing in β. This is unsurprising, as households with more patience will invest more into innovating firms.

Although not shown, both $e(j)$ and $n(j)$ increase with α_j when α_j is low and there is little possibility of a monopoly price emerging in the industry. As α increases and a monopoly price becomes more likely, both $e(j)$ and $n(j)$ begin to fall with α_j. A similar result holds for a change in the economy-wide α, although the relationships are somewhat flatter.

[16] This is similar to the Aghion *et al.* (2005) 'escape-competition' effect in their two-firm model of innovation.

[17] An important caveat to this result is that this effect is only strong enough to turn the relationship between $n(j)$ and z_j negative when α (the parameter governing the elasticity of substitution between industries) is close to 1. This is likely due to the use of a uniform distribution, which makes it very unlikely that $A_{[2]} < \alpha A_{[1]}$ for any number of identical draws. Unfortunately, distributions of order statistics derived from any other distribution are too intractable to solve for. I address this problem in Appendix A.6.

Figures 3a and 3b compare equilibria when θ_j is changed for one industry. θ governs the upper bound of the distribution from which firms draw the value of their innovations, as $h_i \sim U(0, n_i^{\theta})$ for each firm-i. An increase in θ_j increases the marginal effect of changes in the level of research per firm n, and so increases overall innovation at the *intensive* margin (through an increase in research per innovation), while decreasing overall innovation at the *extensive* margin (a decrease in the number of innovations). The same qualitative relationships hold when θ is changed for all industries. For several decades, now, researchers have taken steps to control for 'technological opportunity' when estimating the correlation between competition and innovation, reasoning that R&D may be more or less important for different industries.[18] The parameter θ can be interpreted as a measure of this 'technological opportunity.'



(a) $e(\theta_j)$ = Number of Innovations (b) $n(\theta_j)$ = Firm-Level Research

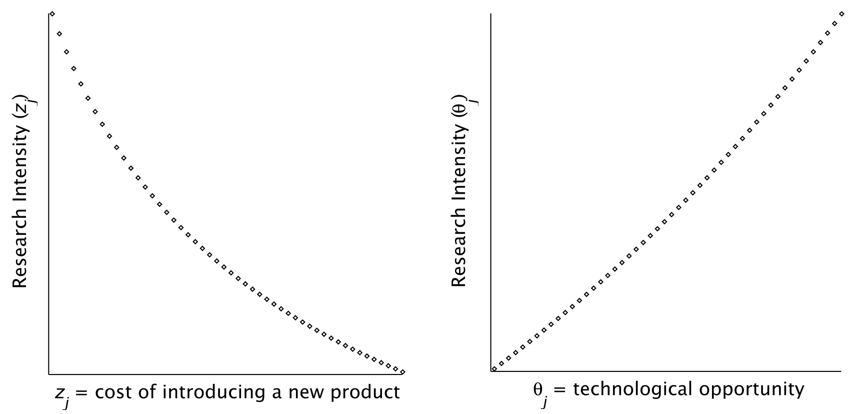
Figure 3: $e(\theta_j)$ and $n(\theta_j)$ with variation in θ_j for industry-j

A number of studies have examined the empirical relationship between growth and the 'research intensity' of an industry, defined as total research expenditure as a fraction of output, or *emn*. Since $e'(m) = 0$ and $nm'(m) = 0$ for economy-wide changes in m, research intensity for the entire economy is independent of the cost of research.

Figure 4a plots the research intensity of an industry as its cost of introducing an innovation z_j changes, while Figure 4b plots the same for changes in θ_j. When z_j increases for industry-j, we have seen that the equilibrium value of $e(j)$ will decrease, while that of $n(j)$ will first increase, and then decrease (if α is high enough). Figure 4a shows that although $n(j)$ may be increasing, it does not offset the drop in $e(j)$ as z_j increases. As a result, research intensity is always a decreasing function of the cost of introducing an innovation. The same holds for economy-wide changes in z. Since e and n exhibit the same qualitative relationship with respect to β and α, research intensity will also exhibit that

[18]Cohen and Levin (1989) discuss this literature, beginning with Scherer (1965).

same relationship - enm is an increasing function of β, and has an inverted-U relationship with α. While the effect on e from a change in z dominates the effect on n, however, the reverse is true for changes in θ. Figure 4b shows that as $\theta(j)$ increases, the level of research per innovation $n(j)$ increases enough to raise industry-j's research intensity, even while the number of innovations $e(j)$ is falling. Changes in θ for all industries produces the same result.



(a) $enm(z_j)$ = Research Intensity (b) $enm(\theta_j)$ = Research Intensity

Figure 4: $enm(j)$ with variation in z_j and θ_j for industry-j

Following Aghion *et al.* (2005), I use one minus the average Lerner Index per firm as the measure of competition when analyzing the relationship between competition, growth, and the average value of innovation per firm. Also following Aghion *et al.* (2005), I attribute a Lerner Index value of zero to firms that do not produce in equilibrium.[19] The expected value of this measure of competition for industry-j is thus

$$E\left[1-\frac{Lerner(j)}{e(j)}\right]=E\left[1-\left(\frac{P(j)-MC(j)}{P(j)e(j)}\right)\right]=\frac{e(j)-1}{e(j)}+E\left[\frac{MC(j)}{P(j)e(j)}\right].$$

In this model, this is equal to

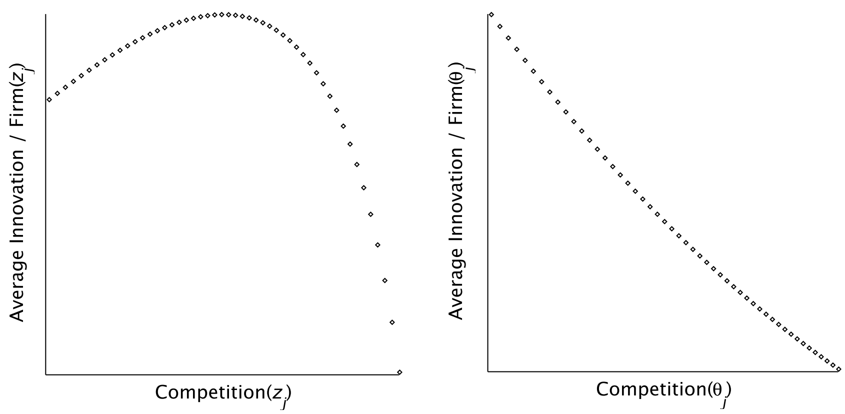
$$Competition=\frac{e^2+\alpha^e-1}{e^2}.^{20} \tag{13}$$

In equilibrium, then, the expected level of competition will be positively correlated with the number of innovations e.

[19] The essentially means the measure of competition is equal to $1-E(Lerner)/e$, where $Lerner$ is the Lerner Index for the winning firm and e is the number of firms. A better industry measure might weight each firm's Index by its market share, so that competition would be measured here as $1-E(Lerner)$. The choice does not affect any qualitative results, and so I use the same measure as Aghion *et al.* (2005) for the sake of easier comparison.

[20] This is derived in Appendix A.4.

The average value of innovation per firm (the variable of interest in recent empirical studies like Aghion *et al.* (2005)) is closely tied to firm-level research. In equilibrium, each firm is drawing from the same distribution, $h \sim U(0, n^{\theta})$, so the average value of innovation per firm can be measured simply as $n^{\theta}/2$, the expected value of each firm's draw. Figures 5a and 5b plot the expected average level of innovation per firm against the expected value of measured competition, for different values of the cost of introducing an innovation z_j (Figure 5a) and θ_j - the parameter governing the effect of research on the upper bound of the distribution (Figure 5b).



(a) Varying z_j

(b) Varying θ_j

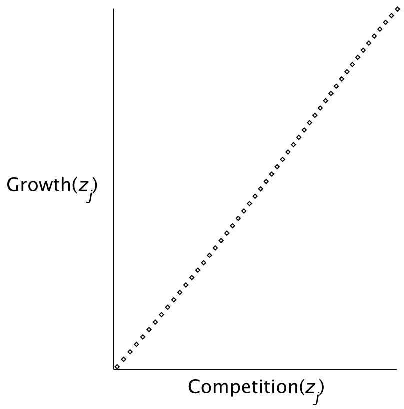
Figure 5: Average Value Of Innovation Per Firm

When an increase in the level of competition is driven by an industry-specific decrease in the cost of introducing an innovation, competition and the average value of innovation exhibit an inverted-U relationship. When changes in competition are driven by different θ_j, however, $n^{\theta}(j)/2$ is unambiguously negatively correlated with competition. As with the inverted-U relationship between z_j and $n(j)$, the relationship illustrated by Figure 5a only holds for high values of α, and does not hold at all when z is changed for the whole economy, for the same reasons explained above. When the economy-wide cost of introducing an innovation increases, the average value of innovation per firm will always tend to increase.

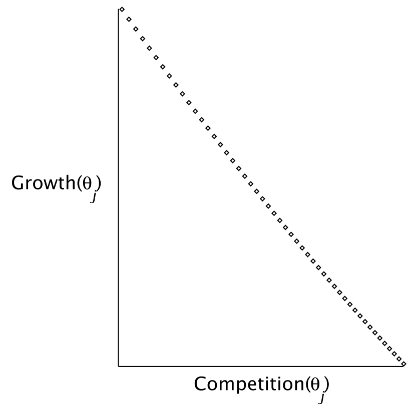
Finally, the relationship between growth and competition is illustrated in Figures 6a and 6b, where z_j is varied in 6a, and θ_j is varied in 6b, for a particular industry-j. Again, the expected growth rate of an industry (equation (7)) is

$$\frac{en^{\alpha\theta}}{e+\alpha} - 1.$$

Regardless of whether the average level of innovation per firm follows an inverted-U or unambiguously decreases with the cost of introducing an innovation z_j, the expected growth rate of the industry is decreasing with z_j, and



(a) Varying z_j



(b) Varying θ_j

Figure 6: Competition and Growth

thus increasing with competition when z_j is the underlying parameter being allowed to vary. As in the case of research intensity, however, this relationship is reversed when θ_j is varied. As θ_j increases, the level of competition decreases while the expected growth rate of the industry increases. These relationships hold for economy-wide changes in either parameter.

In Figure 6a, the decrease in the expected markup of the winning firm from more competition will tend to discourage research, all else equal, since a drop in the fraction of value captured by the winner means firms will be less interested in winning the market − this is the Schumpeterian effect. Besides the fraction of value captured by the winner, however, competition also tends to *increase* the expected value of the best innovation − this is the Hayekian effect. At low levels of competition, an increase in competition is associated with *more* firm-level research, more innovations, and higher growth. At relatively high levels of competition, where the Schumpeterian effect causes firm-level research and *average* innovation to decline with competition, the Hayekian effect more than compensates, increasing the expected value of the *best* innovation, and thus increasing the expected rate of growth.

It is worthwhile to step back here and explain how the introduction of uncertainty and Bertrand competition has resulted in a change in the direction of the relationship between competition and growth suggested by previous models (except in the case illustrated by Figure 6b). Here I assume away monopoly pricing for simplicity, and force winning firms to charge a limit price equal to the second-best firm's marginal cost. The operating profits of the *ex post* winning firm are

$$(P - MC)x.$$

Aghion and Howitt (2005, p.4) proceed from this point by assuming an exogenous markup, denoted $\chi \equiv P/MC$. Given that $MC = w$ in this economy, price can be expressed as $P = \chi w$. Combining this equality with the final-good firm's demand for x (equation (1)), $P = \alpha A_{[1]}^{\alpha} x^{\alpha-1} L_{yt}^{1-\alpha}$, we get

$$w = \frac{\alpha A_{[1]}^{\alpha} x^{\alpha-1} L_{y}^{1-\alpha}}{\chi}.$$

Substituting this expression for w, and L_x for x, the operating profits of the winning firm become

$$\alpha L_x^{\alpha} L_y^{1-\alpha} A_{[1]}^{\alpha} \left(1 - \frac{1}{\chi}\right) = \alpha L_x^{\alpha} L_y^{1-\alpha} A_{-1[1]}^{\alpha} h_{[1]}^{\alpha} \left(1 - \frac{1}{\chi}\right).$$

To simplify this comparison, I take away each firm's ability to choose its level of research (thereby removing the only source of the Schumpeterian effect in the Hayekian model), imposing an upper bound of λ on the distribution of each draw. It is now possible to write the *free-entry condition* as

$$\alpha \left(1 - \frac{1}{\chi}\right) \text{Prob[i wins]} \cdot E[h_{t[1]}^{\alpha} \mid i \text{ wins}] - zR = 0,$$

or

$$\left(1 - \frac{1}{\chi}\right) \cdot \frac{\lambda^{\alpha}}{e + \alpha} = \frac{zR}{\alpha},$$

since $f(h_{[1]} = v, h_{[2]} = u, h_i = h_{[1]}) = (e-1)u^{e-2}\lambda^{-e}$.

In equilibrium, then, $e = \left(1 - \frac{1}{\chi}\right) \cdot \frac{\alpha \lambda^{\alpha}}{zR} - \alpha$. Substituting this expression for e into the expected growth rate,

$$E[g] = E\left[h_{[1]}^{\alpha}\right] - 1 = \frac{e\lambda^{\alpha}}{e+\alpha} - 1 = \lambda^{\alpha} - 1 - zR\left(\frac{\chi}{\chi - 1}\right),$$

which is increasing in χ (markup), and thus decreasing with competition (as is e, actually).

Intuitively, when the markup $\chi \equiv P/MC$ is exogenous to the model, a higher markup encourages more firms to enter, which encourages higher growth. When this markup is instead made a decreasing function of the number of firms or innovations, an exogenous change that induces more firms to enter the market (such as a drop in regulatory or marketing costs) may both decrease the expected markup *and* increase the expected rate of growth.

3 Empirical Support

The primary goal of this Hayekian model of innovation is to show that when the best direction of innovation is uncertain, whether due to uncertain shadow prices for the characteristics of a good or inexperience with a new production process, it is often necessary to just try a new product or process, and hope for the best. Depending on the definition of product failure, anywhere from 50-90% of products fail within one or two years. $9.5 billion was spent on market research in North America in 2007, presumably by firms attempting to mitigate this uncertainty.[21] These casual observations suggest that the assumption of uncertainty is well-grounded.

Introducing uncertainty and endogenous competition into an endogenous growth model can reverse the negative relationship between competition and growth generated by previous models, such as Romer (1990), and Aghion and Howitt (1992). The main result of the model, that competition and growth are positively related, is well supported in empirical studies of both across-industry and across-country differences in productivity growth.[22] To my knowledge, no studies have provided evidence to the contrary. The stipulation for this result in the model, however, is that differences in the level of competition are being driven by changes in the cost of introducing a new product or process. As evidence that this condition is satisfied, consider first that each of the empirical studies cited in this paper make efforts to control for differences across industries in 'technological opportunity'. By controlling for the importance of R&D across industries, it seems likely that uncontrolled differences such as varying marketing costs and regulatory environments are driving the variation in levels of competition. In addition, a number of studies have suggested that both higher marketing costs and more burdensome regulations are associated with lower growth, which is consistent with the model.[23] Although changes in other parameters in the model produce different (even inverted-U) relationships between competition and growth, those relationships are in every case identical for both firm-level and industry-level observations. Only variation in z, the cost of introducing an innovation, can produce different changes in equilibrium growth at different levels of aggregation. A final piece of evidence comes from Aghion *et al.* (2005). In an attempt to control for endogeneity in their test of the effect of competition on firm-level innovation, they employ a number of instruments that track closely the parameter z in the model presented here. In particular, they use a series of stuttered waves of deregulation in different industries as their source of exogenous changes to competition. In the end, their IV results are almost identical to their OLS results (with fixed effects), with a reduced-form R^2 of 0.8, suggesting that much of the correlation between competition and innovation at the firm level can be explained using the present model with variation in z over time.

There have been a number of studies that find a positive relationship between research intensity and growth, both across industries and across countries.[24] The model's results are fully consistent with these findings.

Aghion *et al.* (2005) test the relationship between competition and the average level of innovation per firm in an industry. For a measure of the level of innovation by each firm, the authors total the number of patents in the industry, each weighted by the number of citations from other patent applications. They

[21]See ESOMAR (2008).

[22]Nickell (1996), Blundell *et al.* (1999), and Aghion *et al.* (2008), for example, all present evidence of this relationship.

[23]See Graham et al. (1983) and Nicoletti and Scarpetta (2003) for examples.

[24]OECD (2003) is one such study that also surveys part of the literature.

report an inverted-U relationship between the average level of innovation per firm and the level of competition. The present model results in the same relationship between competition and average innovation (Figure 5a), although this result must again be conditioned on the variation in competition being driven by changes in the cost of innovating.

In recent publications, economists have interpreted evidence of an inverted-U relationship between firm-level innovation and competition as evidence of a similar relationship between industry-level productivity growth and competition.[25] Aghion *et al.* (2008) present evidence that these relationships may differ, however, even in the same sample. Their results are consistent with those of the model presented here, where variation in the cost of innovating induces positively correlated differences in competition and productivity growth at the industry level, but an inverted-U relationship between competition and firm-level innovation.

Many empirical tests of the relationship between competition and innovation seem to be looking for some causal effect of competition on innovation, without specifying any particular theory of how competition could affect innovation and where differences in the level of competition are coming from.[26] When the authors discuss their results, however, they seem to have in the back of their minds some kind of principal-agent model similar to Hart (1983), where competition increases the chance of bankruptcy and encourages slacking managers to work harder. The model presented here, on the other hand, makes no use of exogenous links between competition and firm behavior. There are two points I would like to make in support of the Hayekian model as the proper framework with which to explain the link between competition, innovation, and productivity growth. The first is that linking Hayekian uncertainty to both innovation and measured competition provides an explanation for the different relationships between competition and innovation at the firm and industry levels, whereas other theories have not. The second point is somewhat broader. Principal-agent problems have been with us since before Adam Smith (1776) argued that the separation of ownership from control creates perverse incentives for firm behavior. It seems implausible that after more than two centuries, the market has been unable to mitigate these problems to such an extent that the negative effect of lower rents on the incentive to innovate is dominated by the extra effort put forth by inefficient firms when competition threatens their solvency.

[25]See Bianco (2007), for example. Indeed, this view seems to have become the conventional wisdom.

[26]See, for example, Dutz and Hayri (2000), Geroski (1990), and Nickell (1996). Nickell (1996) is explicit about this, noting that his analysis cannot determine whether competition forces firms to be efficient, or rather lets "many flowers bloom and [ensures] that only the best survive." (p.741)

4 Applications

In the conclusion to the last section, I discussed the lack, in many empirical studies, of a specified theory that could explain how competition could affect innovation and where differences in the level of competition are coming from. This becomes especially problematic when the authors go on to offer policy proposals based on their results. For example, evidence of a positive link between competition and innovation or productivity growth does not provide a basis for a proposal to more strictly enforce antitrust policy unless the evidence supports a theory of how that policy will affect innovation.[27]

The model developed in this paper implies that the regulation of products or production processes will act to slow innovation and growth, in addition to any positive or negative effects on the allocation of resources that fall outside the scope of this paper. In the following two subsections, I use this Hayekian framework to evaluate the effects of antitrust policies. I first consider a cap on markups of price over marginal cost, and then look at legal restrictions on collusion or (equivalently) 'anti-competitive' mergers.

4.1 Ceilings on Markups

I start by considering the effect of a legislated cap on a firm's markup of price over marginal cost. I assume this cap is known before hand, so there is no uncertainty about its level or about whether it will be enforced.

Whereas in the competitive model the winning firm will charge a monopoly price when $\frac{h_{[2]}}{h[1]} < \alpha$, it will now be constrained to charging a markup less than or equal to $q < \frac{1}{\alpha}$. The only change in the model, then, is that the price of firm-i in industry-j will be

$$P_t(j) = \begin{cases} qw_t(j), & \text{if } \frac{h_{[2]}(j)}{h_{[1]}(j)} \leq \frac{1}{q} \\ w_t(j)\frac{h_{[1]}(j)}{h_{[2]}(j)}, & \text{if } \frac{h_{[2]}(j)}{h_{[1]}(j)} > \frac{1}{q}. \end{cases} \tag{14}$$

Equation (1) gave us the final-good firm's inverse demand function for $X_t^d(j)$ as

$$P_t(j) = \alpha A_{t[1]}^{\alpha} \left(\frac{L_y}{X_t^d(j)} \right)^{1-\alpha}.$$

Together with equation (14), this gives us the following equilibrium wage rate for industry-j;

$$w_t(j) = \begin{cases} \alpha q^{-1} h_{t[1]}^{\alpha}(j) A_{t-1[1]}^{\alpha} \left(\frac{L_y}{L_x}\right)^{1-\alpha}, & \text{if } \frac{h_{t[2]}(j)}{h_{t[1]}(j)} \leq \frac{1}{q} \\ \alpha \frac{h_{t[2]}(j)}{h_{t[1]}^{1-\alpha}(j)} A_{t-1[1]}^{\alpha} \left(\frac{L_y}{L_x}\right)^{1-\alpha}, & \text{if } \frac{h_{t[2]}(j)}{h_{t[1]}(j)} > \frac{1}{q}. \end{cases}$$

[27] Examples of this particular policy proposal can be found in the concluding sections of Geroski (1990) and Dutz and Hayri (2000). I should note that I use them as examples only because the rigorous empirical analysis in these papers is well known and often referenced in the competition and innovation literature.

.

The expected discounted profits of any innovating firm-i in any period are thus

$$
\begin{aligned}
E\left(\frac{\pi_i}{R}\right) = & \\
& \alpha\left(\frac{q-1}{q}\right)\frac{\Delta}{R}\text{Prob}\left[i \text{ wins}, \frac{h_{[2]}}{h_{[1]}} \leq \frac{1}{q}\right] \cdot E\left[h_{[1]}^{\alpha} \,\middle|\, i \text{ wins}, \frac{h_{[2]}}{h_{[1]}} \leq \frac{1}{q}\right] \\
+ & \frac{\alpha\Delta}{R}\text{Prob}\left[i \text{ wins}, \frac{h_{[2]}}{h_{[1]}} > \frac{1}{q}\right] \cdot E\left[\frac{h_{[1]} - h_{[2]}}{h_{[1]}^{1-\alpha}} \,\middle|\, i \text{ wins}, \frac{h_{[2]}}{h_{[1]}} > \frac{1}{q}\right] \\
- & \Delta(z + mn_i),
\end{aligned}
$$

where $\Delta \equiv A_{-1[1]}^{\alpha} L_x^{\alpha} L_y^{1-\alpha}$.

Given the joint density function of $h_{[1]}$ and $h_{[2]}$,

$$
f(h_{[1]} = v, h_{[2]} = u, h_i = h_{[1]} | e, n_i, n_{k \neq i}) = \frac{(e-1)u^{e-2}}{n_i^{\theta} n_k^{\theta(e-1)}},
$$

expected discounted profits will be equal to

$$
\begin{aligned}
E\left(\frac{\pi_i}{R}\right) = & \\
& \alpha\left(\frac{q-1}{q}\right)\frac{\Delta}{R}\int_0^{n_i^{\theta}}\int_0^{vq^{-1}} \frac{(e-1)v^{\alpha}u^{e-2}}{n_i^{\theta} n_k^{\theta(e-1)}} du dv \\
+ & \frac{\alpha\Delta}{R}\int_0^{n_i^{\theta}}\int_{vq^{-1}}^{v} \frac{(e-1)(v-u)u^{e-2}}{v^{1-\alpha} n_i^{\theta} n_k^{\theta(e-1)}} du dv \\
- & \Delta(z + mn_i),
\end{aligned}
$$

or

$$
E\left(\frac{\pi_i}{R}\right) = \frac{\alpha(1 - q^{-e})\Delta n_i^{\theta(e+\alpha-1)}}{Re(e+\alpha) n_k^{\theta(e-1)}} - \Delta(z + mn_i).
$$

Using the *free-entry* and *research* conditions (equations (7) and (8)), equilibrium in an industry with a cap on markups can be characterized by the following two equations;

$$
n = \frac{z\theta(e+\alpha-1)}{m[1 - \theta(e+\alpha-1)]} \tag{15}
$$

$$
z = \frac{\alpha(1 - q^{-e})n^{\alpha\theta}}{Re(e+\alpha)} - mn. \tag{16}
$$

For all feasible parameter values, both the number of innovations e, and the level of research per innovation n, are lower when markups are capped, relative to the competitive equilibrium. Figure 7 compares the growth rates for two otherwise identical industries for different values of z_j, the cost of introducing a new product. Growth in the competitive industry is always higher, for any $q < \frac{1}{\alpha}$. Although not shown, the results are the same when comparing a competitive economy to one in which a cap is enforced in every industry.

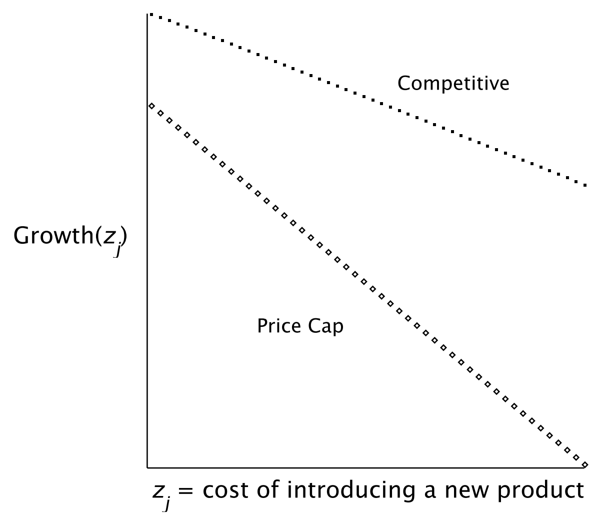


Figure 7: Expected Growth Rates - Varying z_j

The results of this experiment should not be surprising. If innovators face a positive probability of making less profit than they would absent this policy, we should expect less innovation, notwithstanding the positive relationship between competition and productivity growth in the data.

4.2 Price-Fixing and Anti-Competitive Mergers

While ceilings on markups have perhaps been commonly understood to have potentially negative consequences for innovation and investment, the consequences for growth and innovation of restrictions on collusion and 'anti-competitive' mergers have received little attention. Williamson (1968), Mathewson and Winter (1987), and others have considered situations in which price-fixing and mergers thought to be anti-competitive might actually improve allocative efficiency. The possible benefits of joint research ventures with respect to technology diffusion have also been analyzed in studies like Katz (1986).

What I instead want to consider in this section is the effect on innovation of restrictions on price-fixing and mergers for which no efficiency defence exists - that is, behavior undertaken by firms that increases profits at the expense of consumers, and provides no benefit to allocative efficiency.

Building on the benchmark model, I now allow the winning firm in any particular period to either purchase, or collude with, any other firm that might constrain the winning firm's pricing decision. I model the cost of purchasing or colluding with all firms that pose a threat as an exogenous fraction τ^C of the additional profit firm-[1] stands to gain by charging a monopoly price, rather than a limit price. In addition, I assume firm-[1] must incur a cost to either monitor its co-conspirators, or manage its larger size after merging, which I model in a similar fashion as a fraction τ^M.

The total cost of achieving a monopoly price when $\frac{h_{[2]}}{h_{[1]}} > \alpha$, is thus

$$\alpha(\tau^C + \tau^M)\Delta\left(\frac{h_{[2]} - \alpha h_{[1]}}{h_{[1]}^{1-\alpha}}\right), \tag{17}$$

where $\Delta \equiv A_{-1[1]}^{\alpha} L_x^{\alpha} L_y^{1-\alpha}$. The payoff for each firm-$[r]$ for which $\frac{h_{[r]}}{h_{[1]}} > \alpha$ is equal to the fraction τ^C of the additional profit the winning firm receives by colluding with that particular firm. The payoff for an eligible firm-$[r]$ is thus

$$\alpha\tau^C\Delta\left(\frac{h_{[r]} - h_{[r+1]}}{h_{[1]}^{1-\alpha}}\right), \text{ if } \frac{h_{[r+1]}}{h_{[1]}} > \alpha,$$

and

$$\alpha\tau^C\Delta\left(\frac{h_{[r]} - \alpha h_{[1]}}{h_{[1]}^{1-\alpha}}\right), \text{ if } \frac{h_{[r+1]}}{h_{[1]}} \le \alpha.$$

When $\frac{h_{[2]}}{h_{[1]}} > \alpha$, the operating profits of a winning firm that chooses a limit price are

$$\alpha\Delta\left(\frac{h_{[1]} - h_{[2]}}{h_{[1]}^{1-\alpha}}\right),$$

while those of a winning firm that chooses to collude are

$$\alpha(1-\alpha)\Delta h_{[1]}^{\alpha} - \alpha(\tau^C + \tau^M)\Delta\left(\frac{h_{[2]} - \alpha h_{[1]}}{h_{[1]}^{1-\alpha}}\right).$$

Firm-[1] will therefore choose to collude if $\frac{h_{[2]}}{h_{[1]}} > \alpha$ and

$$\alpha(1-\alpha)h_{[1]}^{\alpha} - \alpha\left(\frac{h_{[1]} - h_{[2]}}{h_{[1]}^{1-\alpha}}\right) \ge \alpha(\tau^C + \tau^M)\left(\frac{h_{[2]} - \alpha h_{[1]}}{h_{[1]}^{1-\alpha}}\right),$$

or if $\tau^C + \tau^M \le 1$.

Since the decision to collude or not depends only on τ^C and τ^M, and not on the realization of any random variables, I assume $\tau^C + \tau^M \le 1$. Expected discounted profits for any firm-i are therefore

$$
\begin{aligned}
E\left(\frac{\pi_i}{R}\right) = & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (18)\\
& \frac{\alpha(1-\alpha)\Delta}{R}\mathrm{Prob}[i \text{ wins}] \cdot E\left[h_{[1]}^{\alpha} \;\middle|\; i \text{ wins}\right] \\
- \quad & \frac{\alpha\Delta(\tau^C+\tau^M)}{R}\mathrm{Prob}\left[i \text{ wins}, \frac{h_{[2]}}{h_{[1]}} > \alpha\right] \\
& \qquad \cdot E\left[\frac{h_{[2]}-\alpha h_{[1]}}{h_{[1]}^{1-\alpha}} \;\middle|\; i \text{ wins}, \frac{h_{[2]}}{h_{[1]}} > \alpha\right] \\
+ \quad & \frac{\alpha\Delta\tau^C}{R}\sum_{r=2}^{e-1}\left(\mathrm{Prob}\left[h_i = h_{[r]}, \frac{h_{[r+1]}}{h_{[1]}} > \alpha\right]\right. \\
& \qquad \cdot E\left[\frac{h_{[r]}-h_{[r+1]}}{h_{[1]}^{1-\alpha}} \;\middle|\; h_i = h_{[r]}, \frac{h_{[r+1]}}{h_{[1]}} > \alpha\right] \\
+ \quad & \mathrm{Prob}\left[h_i = h_{[r]}, \frac{h_{[r]}}{h_{[1]}} > \alpha, \frac{h_{[r+1]}}{h_{[1]}} \leq \alpha\right] \\
& \qquad \left.\cdot E\left[\frac{h_{[r]}-\alpha h_{[1]}}{h_{[1]}^{1-\alpha}} \;\middle|\; h_i = h_{[r]}, \frac{h_{[r]}}{h_{[1]}} > \alpha, \frac{h_{[r+1]}}{h_{[1]}} \leq \alpha\right]\right) \\
+ \quad & \frac{\alpha\Delta\tau^C}{R}\mathrm{Prob}\left[h_i = h_{[e]}, \frac{h_{[e]}}{h_{[1]}} > \alpha\right] \cdot E\left[\frac{h_{[e]}-\alpha h_{[1]}}{h_{[1]}^{1-\alpha}} \;\middle|\; h_i = h_{[e]}, \frac{h_{[e]}}{h_{[1]}} > \alpha\right] \\
- \quad & \Delta(z+mn_i),
\end{aligned}
$$

where the first term is the expected monopoly profits from production, the second term is the total cost of ensuring the monopoly price, the third term is the payoff for firm-[r] when $h_{[r+1]} > \alpha h_{[1]}$, the fourth term is the payoff for firm-[r] when $h_{[r]} > \alpha h_{[1]}$, but $h_{[r+1]} \leq \alpha h_{[1]}$, and the fifth term is the payoff for the worst firm when $h_{[e]} > \alpha h_{[1]}$.

This extension requires new density functions, as all innovating firms may receive a payoff after innovating. In Appendix A.5, I derive the joint density function for $h_{[1]}$, $h_{[r]}$, and $h_{[r+1]}$, conditional on e, n_i, and $n_{k\neq i}$;

$$
\begin{aligned}
& f(h_{[1]} = \omega, h_{[r]} = v, h_{[r+1]} = u, h_i = h_{[r]} \mid e, n_i, n_{k\neq i}) = \\
& \qquad\qquad \frac{(e-1)!(\omega - v)^{r-2}u^{e-r-1}}{(r-2)!(e-r-1)!n_i^{\theta}n_k^{\theta(e-1)}}, \text{ for } e > r > 1
\end{aligned}
$$

In that same Appendix, I derive the joint density function for $h_{[1]}$ and $h_{[e]}$;

$$
f(h_{[1]} = \omega, h_{[e]} = v, h_i = h_{[e]} \mid e, n_i, n_{k\neq i}) = \frac{(e-1)(\omega - v)^{e-2}}{n_i^{\theta}n_k^{\theta(e-1)}}
$$

Finally, the joint density function for $h_{[1]}$, $h_{[r]}$, and $h_{[r+1]}$ must be summed over all values of r between 2 and $e-1$;

$$\sum_{r=2}^{e-1} f(h_{[1]}=\omega, h_{[r]}=v, h_{[r+1]}=u, h_i=h_{[r]} \mid e, n_i, n_{k\neq i}) = \frac{(e-1)(e-2)(\omega-v+u)^{e-3}}{n_i^{\theta} n_k^{\theta(e-1)}}$$

The expected discounted profits of firm-i can now be expressed as

$$\begin{aligned}
E\left(\frac{\pi_i}{R}\right) &= \\
& \frac{\alpha(1-\alpha)\Delta}{R}\int_0^{n_i^\theta}\int_0^v v^\alpha f(h_{[1]}=v, h_{[2]}=u, h_i=h_{[1]})dudv \\
- & \frac{\alpha\Delta(\tau^C+\tau^M)}{R}\int_0^{n_i^\theta}\int_{\alpha v}^v \frac{(u-\alpha v)}{v^{1-\alpha}} f(h_{[1]}=v, h_{[2]}=u, h_i=h_{[1]})dudv \\
+ & \frac{\alpha\Delta\tau^C}{R}\int_0^{n_k^\theta}\int_{\alpha\omega}^{\omega}\int_{\alpha\omega}^{v} \frac{(v-u)}{\omega^{1-\alpha}} \sum_{r=2}^{e-1} f(h_{[1]}=\omega, h_{[r]}=v, h_{[r+1]}=u, h_i=h_{[r]})dudvd\omega \\
+ & \frac{\alpha\Delta\tau^C}{R}\int_0^{n_k^\theta}\int_{\alpha\omega}^{\omega}\int_0^{\alpha\omega} \frac{(v-\alpha\omega)}{\omega^{1-\alpha}} \sum_{r=2}^{e-1} f(h_{[1]}=\omega, h_{[r]}=v, h_{[r+1]}=u, h_i=h_{[r]})dudvd\omega \\
+ & \frac{\alpha\Delta\tau^C}{R}\int_0^{n_k^\theta}\int_{\alpha\omega}^{\omega} \frac{(v-\alpha\omega)}{\omega^{1-\alpha}} f(h_{[1]}=\omega, h_{[e]}=v, h_i=h_{[e]})dvd\omega \\
- & \Delta(z+mn_i),
\end{aligned}$$

or

$$\begin{aligned}
E\left(\frac{\pi_i}{R}\right) &= \\
& \frac{\alpha(1-\alpha)e\Delta n_i^{\theta(e+\alpha-1)}}{Re(e+\alpha)n_k^{\theta(e-1)}} - \frac{\alpha(\tau^C+\tau^M)\Delta n_i^{\theta(e+\alpha-1)}}{Re(e+\alpha)n_k^{\theta(e-1)}}[e(1-\alpha)+\alpha^e-1] \\
+ & \frac{\alpha\tau^C\Delta n_k^{\theta(1+\alpha)}}{Re(e+\alpha)n_i^{\theta}}[e(1-\alpha)+\alpha^e-1] \qquad -\Delta(z+mn_i).
\end{aligned} \tag{19}$$

Using the *free-entry* and *research* conditions, the collusive equilibrium can now be characterized by the following two equations;

$$\begin{aligned}
nm = & \frac{\alpha\theta n^{\alpha\theta}}{Re(e+\alpha)}(e+\alpha-1)(e(1-\alpha)-\tau^M[e(1-\alpha)+\alpha^e-1]) \\
& - \frac{\alpha\theta n^{\alpha\theta}}{Re(e+\alpha)}(e+\alpha)\tau^C[e(1-\alpha)+\alpha^e-1]
\end{aligned} \tag{20}$$

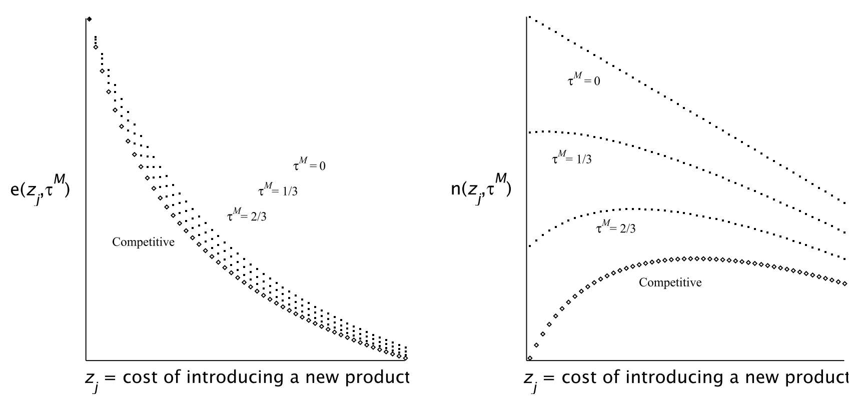
$$z + mn = \frac{\alpha n^{\alpha\theta}}{Re(e+\alpha)}(e(1-\alpha) - \tau^M[e(1-\alpha) + \alpha^e - 1]) \tag{21}$$

Note that if total payoffs are zero ($\tau^C = 0$) and monitoring costs eat up the entire benefit of collusion ($\tau^M = 1$), then these conditions reduce to those of a competitive equilibrium (equations (11) and (12));

$$n = \frac{z\theta(e+\alpha-1)}{m[1-\theta(e+\alpha-1)]}$$

$$z = \frac{\alpha(1-\alpha^e)n^{\alpha\theta}}{Re(e+\alpha)} - mn$$

Figures 8a through 9b plot the number of innovations and the level of research per firm, for various values of z_j (the cost of introducing a product), and for both a collusive and a competitive industry. The interest rate is held constant throughout.

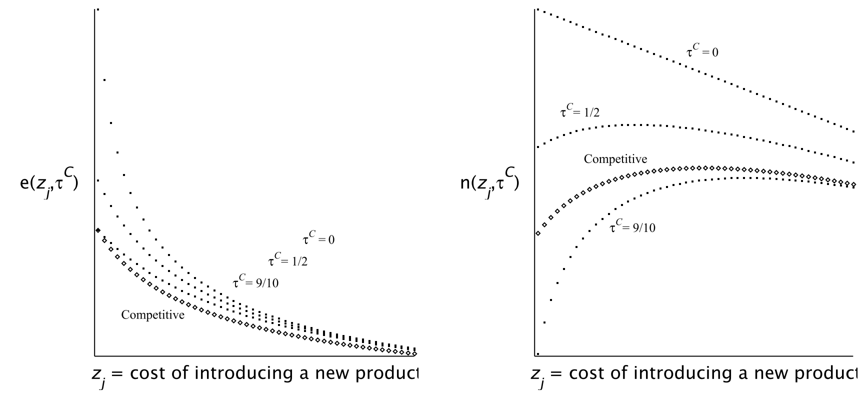


(a) $e(z_j, \tau^M)$ = Number of Innovations (b) $n(z_j, \tau^M)$ = Firm-Level Research

Figure 8: $e(z_j, \tau^M)$ and $n(z_j, \tau^M)$ with variation in z_j and τ^M for industry-j

Figures 8a and 8b show how outcomes change as monitoring / management costs (τ^M) increase, while keeping τ^C equal to zero. Both e and n are decreasing in τ^M, but stay above the associated competitive equilibrium values until $\tau^M = 1$.

Figures 9a and 9b illustrate the same experiment with respect to τ^C, keeping monitoring costs at zero. As payouts to competing firms increase, it becomes less profitable to be the best firm, but *more* profitable to be a lower-ranked firm. This results in lower research per firm, but encourages more firms to enter in an effort to capture some of the payouts. While the number of innovations



(a) $e(z_j, \tau^C)$ = Number of Innovations (b) $n(z_j, \tau^C)$ = Firm-Level Research

Figure 9: $e(z_j, \tau^C)$ and $n(z_j, \tau^C)$ with variation in z_j and τ^C for industry-j

always remains above the competitive level, research per firm drops below this level once total payouts increase past a certain threshold. This pattern holds when monitoring costs are positive.

If payoffs to colluding firms were zero, figures 8a and 8b imply that growth must be higher when collusion is possible, relative to the competitive outcome, for any level of τ^M lower than one. Since collusion increases the profits of the winning firm without otherwise distorting incentives, collusion in this case increases growth.

On the other hand, when the cost of buying-out or buying-off competitors is positive, a higher number of innovations comes at the cost of less research per innovation. Ultimately, however, growth will still increase with collusion in this case. Figure 10 shows how the expected growth rate varies with τ^C, keeping τ^M equal to zero. Growth remains higher with collusion for any value of τ^C lower than 0.95. As long as the best firm retains enough profit to make winning more profitable than losing, the negative effect of payoffs on research is more than offset by the increase in the number of innovations.

Testing these results against real outcomes is difficult, due to the ubiquitousness of antitrust laws throughout the last century. The story of Standard Oil in the late nineteenth and early twentieth centuries does, however, provide a case study that seems consistent with the implications of the present model.[28] Over a forty-year period, the company used a combination of process and product innovations, acquisitions, and price-fixing agreements to dominate the market, maintaining a market share of close to 90% until both politics and more able rivals began to drag them back down to earth. Just as the model predicts, a

[28] McGee (1958) provides a detailed analysis of the competitive behavior of Standard Oil and its competitors. Boudreaux and Folsom (1999) provide a short summary of the innovations and price reductions that accompanied J.D. Rockefeller's entertaining quest to dominate the market.

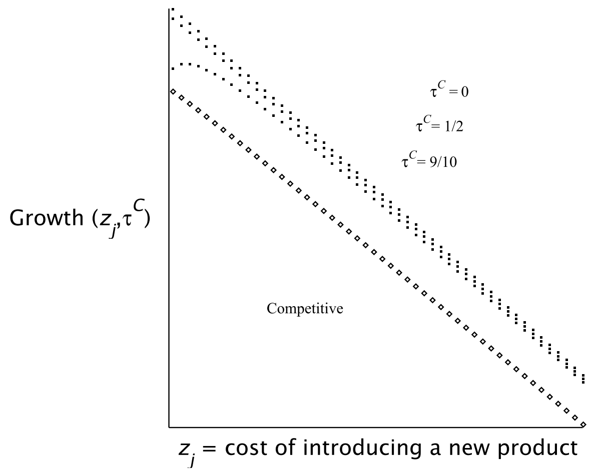


Figure 10: Expected Growth Rates - Varying z_j, τ^C

large number of new entrants appeared each year, many seemingly with the intention of becoming just competitive enough to get bought-out by Standard Oil. While some refineries were used to add to capacity, many were just bought and then shut down in an effort to reduce competition. Nevertheless, the period saw an enormous number of new process innovations, as well as new ways to turn the 'waste' from the production of kerosene into products like gasoline, paving tar, and petroleum jelly. Over the course of thirty years, just as Standard Oil acquired and continued to maintain its monopoly, the deflation-adjusted price of kerosene dropped by over 65%, even while petroleum output increased by more than a factor of three.

It is important to point out that the model is not appropriate for evaluating the welfare implications of collusion, as it does not allow for any static inefficiency resulting from a monopoly price. That collusion may be welfare-reducing even while increasing growth is most obvious in the limiting case where monitoring costs erode the entire benefit of charging a monopoly price. When τ^M is equal to one, growth will be no higher than in a competitive industry, but resources will nonetheless be wasted enforcing the collusive agreement. The policy implication of this model is that courts should either enforce price-fixing agreements like most other contracts, thereby lowering monitoring costs, or else raise monitoring costs to a prohibitive level. The appropriate choice depends on the trade-off between dynamic and static efficiency (as well as on enforcement costs and ideology).

5 Conclusion

Hayek (2002) argued that the competitive process could be thought of as a procedure for discovering and making use of knowledge that would otherwise not emerge. When firms are uncertain of which direction to innovate in, the best innovation to emerge will tend to be of higher value when more innovations are tried. Although competition can lower the expected rents to innovators, the Hayekian effect can, and in this model does, dominate the Schumpeterian effect. When both the number of innovations and the level of research are endogenized, along with the level of competition, the model presented here mimics the relationships found in the data more closely than current endogenous growth models, where competition is kept exogenous.

Although empirical studies of competition and growth generally conclude with calls for more antitrust enforcement, the model developed here serves as a reminder that the level of competition in an industry is endogenously determined, as is productivity growth. As such, any policy proposals that attempt to exploit this positive relationship between competition and growth by exogenously manipulating the *measure* of competition (MC/P), say by forcing down prices, should be viewed with skepticism. The results of earlier Schumpeterian growth models notwithstanding, it seems that one of Schumpeter's main messages should be taken to heart - regardless of the static benefits of antitrust enforcement, and regulation more generally, these policies come with the cost of less innovation and growth. Or alternatively, there ain't no such thing as a free lunch.

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A Appendix

A.1 Competitive Density Function

To solve the competitive model, it is necessary to derive the joint density function of $h_{[1]}$, $h_{[2]}$, and $h_i = h_{[1]}$. When deciding its optimal level of research, I assume firm-i believes all other firms will choose levels of research identical to each other. This makes the notation simpler, without sacrificing any generality. The relevant joint density function is thus

$$f(h_{[1]} = u_1, h_{[2]} = u_2, h_i = h_{[1]} \mid e, n_i, n_{k \neq i}) =$$
$$\int_0^{u_2} \cdots \int_0^{u_{e-1}} (e-1)! f(h_i = u_1) \prod_{\ell=2}^{e} f(h_k = u_\ell) du_e \cdots du_3.$$

Given that $h_k \sim U(0, n_k^\theta)$, $\forall k \in \{1, .., e\}$, it follows that $f(h_k = u_\ell) = \frac{1}{n_k^\theta}$. From this, the density function can be derived by integrating over all $u_\ell > 2$;

$$f(h_{[1]} = u_1, h_{[2]} = u_2, h_i = h_{[1]} \mid e, n_i, n_{k \neq i}) = \frac{(e-1)u_2^{e-2}}{n_i^\theta n_k^{\theta(e-1)}},$$

for all $u_1 \geq u_2 \geq 0$.

A.2 Rate of Growth

The growth rate in a symmetric competitive economy is approximately equal to the expected rate of growth in any industry, which is equal to

$$E(h_{[1]}^\alpha) - 1 = \int_0^{n^\theta} u_1^\alpha f(h_{[1]} = u_1 \mid e, n) du \; - 1.$$

The required density function can be derived in the following way;

$$F(h_{[1]} < u_1 \mid e, n) = \text{Prob}[h_k < u_1, \; \forall k \in \{1, ..., e\} \mid e, n].$$

Since $h_k \sim U(0, n^\theta)$, $\forall k$, the probability that all draws are less than u_1 is

$$F(h_{[1]} < u_1) = \left(\frac{u_1}{n^\theta}\right)^e.$$

It follows that

$$f(h_{[1]} = u_1 \mid e, n) = \frac{\partial}{\partial u_1} F(h_{[1]} < u_1 \mid e, n) = \frac{e u_1^{e-1}}{n^{\theta e}}.$$

The expected growth rate of an industry is therefore

$$\frac{e n^{\alpha\theta}}{e + \alpha} - 1,$$

which is approximately the growth rate of the entire economy.

A.3 Competitive Equilibrium

Using equations (5) and (6), firm-i's expected discounted profits can be expressed as

$$E\left(\frac{\pi_{i,t+1}}{R}\right) = \frac{\alpha(1-\alpha)\Delta_t}{R}\int_0^{n_{it}^{\theta}}\int_0^{\alpha v}\frac{(e_t-1)v^{\alpha}u^{e-2}}{n_{it}^{\theta}n_{kt}^{\theta(e-1)}}dudv$$
$$+\quad \frac{\alpha\Delta_t}{R}\int_0^{n_{it}^{\theta}}\int_{\alpha v}^{v}\frac{(e_t-1)(v-u)u^{e-2}}{v^{1-\alpha}n_{it}^{\theta}n_{kt}^{\theta(e-1)}}dudv - \Delta_t(z+mn_i),$$

where $\Delta_t \equiv A_{t[1]}^{\alpha}L_x^{\alpha}L_y^{1-\alpha}$.[29]

Since it is now obvious that decisions are time-independent, expected discounted profits can be expressed as

$$E\left(\frac{\pi_i}{R}\right) = \frac{\alpha^e(1-\alpha)\Delta n_i^{\theta(e+\alpha-1)}}{R(e+\alpha)n_k^{\theta(e-1)}}$$
$$+\quad \frac{\alpha\Delta n_i^{\theta(e+\alpha-1)}}{Re(e+\alpha)n_{kt}^{\theta(e-1)}}[1-\alpha^e-\alpha^{e-1}(1-\alpha)e] - \Delta_t(z+mn_i),$$

or

$$E\left(\frac{\pi_i}{R}\right) = \frac{\alpha(1-\alpha^e)\Delta n_i^{\theta(e+\alpha-1)}}{Re(e+\alpha)n_k^{\theta(e-1)}} - \Delta_t(z+mn_i).$$

Each firm-i will maximize $E\left(\frac{\pi_i}{R}\right)$ with respect to n_i, given $n_{k\neq i}$ and e. Given identical firms, and focusing solely on interior solutions, this results in the following equilibrium condition;

$$\frac{mn}{\theta(e+\alpha-1)} = \frac{\alpha(1-\alpha^e)n^{\alpha\theta}}{Re(e+\alpha)}. \tag{22}$$

Free entry ensures that the number of firms e will adjust until $E\left(\frac{\pi_i}{R}\right) = 0$. Given that $n_i = n_k$, the following condition must also hold in equilibrium;

$$z + mn = \frac{\alpha(1-\alpha^e)n^{\alpha\theta}}{Re(e+\alpha)}. \tag{23}$$

Rearranging conditions (22) and (23) above results in equations (11) and (12);

$$n = \frac{z\theta(e+\alpha-1)}{m[1-\theta(e+\alpha-1)]}$$

$$z = \frac{\alpha(1-\alpha^e)n^{\alpha\theta}}{Re(e+\alpha)} - mn.$$

[29] To be precise, the above is true only if $n_{it} \leq n_{kt}$. But if one were to instead assume $n_{it} \geq n_{kt}$, the same *free-entry* and *research* conditions would result in equilibrium, where $n_{it} = n_{kt}$.

To derive equations (9) and (10), which characterize equilibrium for a symmetric economy, first use equations (7) and (8) to substitute $\frac{en^{\alpha\theta}}{\beta(e+\alpha)}$ for R in equation (23);

$$z + mn = \frac{\alpha(1-\alpha^e)\beta}{e^2}. \tag{24}$$

Equations (24) and (22) can now be combined to get equation (10);

$$z = \frac{\alpha(1-\alpha^e)\beta[1-\theta(e+\alpha-1)]}{e^2}.$$

Equation (9) is simply a restatement of equation (12), derived above.

A.4 Expected Competition

Following Aghion *et al.* (2005), I use a measure of competition equal to

$$1 - \frac{1}{e}\sum_{i=1}^{e} Lerner_i,$$

and attribute a value of zero to each firm that does not produce. Competition is therefore equal to

$$1 - \frac{Lerner_{[1]}}{e} = 1 - \frac{P - MC}{Pe} = \frac{e-1}{e} + \frac{MC}{Pe},$$

which is equal to

$$\frac{e-1}{e} + \begin{cases} \frac{\alpha}{e}, & \text{if } \frac{h_{[2]}}{h_{[1]}} \leq \alpha \\ \frac{h_{[2]}}{h_{[1]}e}, & \text{if } \frac{h_{[2]}}{h_{[1]}} > \alpha. \end{cases}$$

.

The expected value of measured competition is therefore

$$\begin{aligned} Competition &= \frac{e-1}{e} \\ &+ \int_0^{n^\theta}\int_0^{\alpha u_1} \frac{\alpha}{e} f(h_{[1]} = u_1, h_{[2]} = u_2 \mid e, n)du_2 du_1 \\ &+ \int_0^{n^\theta}\int_{\alpha u_1}^{u_1} \frac{u_2}{u_1 e} f(h_{[1]} = u_1, h_{[2]} = u_2 \mid e, n)du_2 du_1. \end{aligned} \tag{25}$$

Given the number of firms e, the joint density function of $h_{[1]}$ and $h_{[2]}$ from any distribution is

$$\int_0^{u_2} \cdots \int_0^{u_{e-1}} e! \prod_{\ell=1}^{e} f(h_k = u_\ell) du_e \cdots du_3.$$

Each draw is distributed according to $h \sim U(0, n^\theta)$, and so

$$f(h_{[1]} = u_1, h_{[2]} = u_2 \mid e, n) = \int_0^{u_2} \cdots \int_0^{u_{e-1}} \frac{e!}{n^{\theta e}} du_e \cdots du_3,$$

or

$$f(h_{[1]} = u_1, h_{[2]} = u_2 \mid e, n) = \frac{e(e-1)u_2^{e-2}}{n^{\theta e}}.$$

Substituting this function into equation (25) results in equation (13);

$$Competition = \frac{e^2 + \alpha^e - 1}{e^2}.$$

A.5 Collusive Density Functions

To calculate expectations in a model with collusion, two additional density functions are required - $f(h_{[1]} = u_1, h_{[r]} = u_r, h_{[r+1]} = u_{r+1}, h_i = h_{[r]} \mid e, n_i, n_{k\neq i})$ for $e > r > 1$, and $f(h_{[1]} = u_1, h_{[e]} = u_e, h_i = h_{[e]} \mid e, n_i, n_{k\neq i})$ for $r = e$. I start with the former;

$$\begin{aligned} f(h_{[1]} = u_1, h_{[r]} = u_r, h_{[r+1]} = u_{r+1}, h_i = h_{[r]} \mid e, n_i, n_{k\neq i}) = \\ \int_0^{u_1} \cdots \int_0^{u_{r-2}} \int_0^{u_{r+1}} \cdots \int_0^{u_{e-1}} (e-1)! f(h_i = u_r) \\ \cdot \prod_{\ell \neq r}^{e} f(h_k = u_\ell) du_e \cdots du_{r+2} du_{r-1} \cdots du_2, \quad \text{if } r > 2, \end{aligned}$$

or

$$\int_0^{u_{r+1}} \cdots \int_0^{u_{e-1}} (e-1)! f(h_i = u_r) \prod_{\ell \neq r}^{e} f(h_k = u_\ell) du_e \cdots du_{r+2}, \text{ if } r = 2.$$

Given that $f(h_k = u_\ell) = \frac{1}{n_k^\theta}$, $\forall k, \ell$, this works out to

$$\begin{aligned} & f(h_{[1]} = u_1, h_{[r]} = u_r, h_{[r+1]} = u_{r+1}, h_i = h_{[r]} \mid e, n_i, n_{k\neq i}) \\ = & \quad \frac{(e-1)!(u_1 - u_r)^{r-2} u_{r+1}^{e-r-1}}{(r-2)!(e-r-1)! n_i^\theta n_k^{\theta(e-1)}}, \quad \text{for } e > r > 1. \end{aligned}$$

The second necessary density function can be derived the same way;

$$\begin{aligned} & f(h_{[1]} = u_1, h_{[e]} = u_e, h_i = h_{[e]} \mid e, n_i, n_{k\neq i}) \\ = & \quad \int_0^{u_1} \cdots \int_0^{u_{e-2}} (e-1)! f(h_i = u_e) \prod_{\ell \neq r}^{e} f(h_k = u_\ell) du_{e-1} \cdots du_2 \\ = & \quad \frac{(e-1)(u_1 - u_e)^{e-2}}{n_i^\theta n_k^{\theta(e-1)}}. \end{aligned}$$

A.6 Alternative Distributions

Not yet complete.