Thermocline Theories and WOCE: A Mutual Challenge

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1 Background

This article was solicited following a lecture on the large-scale circulation and thermocline dynamics given by the author at a recent conference (Vallis 2000). It gives a brief overview of the some recent work on the thermocline problem, and also tries to begin to make a connection between such dynamical theories and observational programs.

WOCE is, as a whole, the most ambitious ocean observing program to date. As the program moves into the AIMS phase it is useful to recall its original objectives, namely: (i) To develop models useful for predicting climate change and to collect the data necessary to test them; (ii) To determine the representativeness of the specific WOCE data sets for the long-term behaviour of the ocean, and to find methods for determining long-term changes in the ocean circulation. Included in these goals is a mandate to 'determine the dynamical balance of the World Ocean Circulation.' This short article asks the questions *can observational data be effectively used to test dynamical theories of the ocean, such as thermocline theory?* and, conversely, *can dynamical theories be put into a form that can be subject to observational tests?* Although we would like to answer both questions in the affirmative, both issues appear to be more difficult and ambiguous than one would like.

Much of our theoretical understanding of the large-scale circulation of the ocean comes by way of using simplified dynamical equations, in particular (and especially for the large-scale circulation) the *planetary-geostrophic equations*, or PGE. These equations have in turn been used to construct various models of how an idealized ocean might work — simple theories of the wind-driven circulation, thermocline theories and so forth. Thus, it seems that a worthwhile project would be to try to use observational data to test the validity of the planetary-geostrophic equations, and some of their consequences — in particular various models of the thermocline.

2 Dynamical Theories

2.1 The planetary-geostrophic equations

The planetary-geostrophic equations follow from the primitive equations simply by neglecting the total time derivative in the momentum equations. The scaling assumptions that must be satisfied to justify this are (c.f., Pedlosky 1987): (i) Small Rossby number; (ii) Length scales larger than the deformation radius; (iii) The Coriolis parameter must vary by O(1), else the resulting lowest-order dynamics are trivial. [On Earth, (ii) is implied by (iii).] The second of these conditions is particularly problematic, for it explicitly excludes there being a continuous spectrum of interacting scales from the planetary scale to the mesoscale. In the Boussinesq approximation, and neglecting saline effects, the equations that result are:

$$fu = -\frac{\partial \phi}{\partial y}, \qquad fv = \frac{\partial \phi}{\partial x}, \qquad \frac{\partial \phi}{\partial z} = b, \qquad \nabla_3 \cdot \mathbf{v} = 0, \qquad \frac{Db}{Dt} = \frac{\partial}{\partial z}\kappa \frac{\partial b}{\partial z}. \tag{1}$$

Here, b is buoyancy (or temperature), ϕ is pressure divided by density, and other notation is standard. These equations were first derived as a large-scale approximation for ocean circulation by Robinson and Stommel (1959) and Welander (1959). Burger (1958) had independently developed a similar approximation for large-scale atmospheric motions, and Phillips (1963) put them in the context of other approximations to the primitive equations.

The diffusive term is not part of the planetary-geostrophic approximation *per se.* Rather, one is additionally *assuming* that small-scale turbulent motions act diffusively on the large-scale. In particular, one is implicitly assuming that small-scale turbulence has an effect on the large-scale flow, but that mesoscale turbulence does not. Unfortunately, we have few dynamical arguments to indicate whether this is really valid, or what precise value of κ is appropriate.

2.2 Theory of the thermocline

Because of the uncertainty in the approximations leading to the planetary-geostrophic approximations, and at the same time the ubiquity and importance of these equations in our theories, plainly we should seek to test their validity. We come back to this more in the next section, but for now we note that one of the consequences of the planetary-geostrophic approximation has been our current understanding of the thermocline. Historically, there have been two approaches, diffusive theories and adiabatic theories. A number of early approaches concentrated on the former, for example assuming a balance in the thermodynamic equation of

$$w\frac{\partial b}{\partial z} \approx \kappa \frac{\partial^2 b}{\partial z^2}. \tag{2}$$

Similarity theories quantified this approach, leading to a picture of the thermocline as an *internal boundary layer* (Stommel and Webster 1962; Salmon 1990; Young and Ierley 1986). In these models, the thermocline is *irreducibly* diffusive; no matter how small the value of κ is, diffusion is always important. Typically, the thickness of the thermocline varies as some fractional power of κ like $\kappa^{1/2}$, the thermocline itself becoming a discontinuous front in the limit $\kappa \to 0$.

In some contrast, ideal theories assumed that the balance in the thermodynamic equation was essentially adiabatic ($\mathbf{v} \cdot \nabla b = 0$). This approach, really begun by Welander (1959), was reinvigorated by the 'LPS' theory of Luyten et al (1983). This theory provided solutions of the equivalent layered system that combined essentially arbitrary density and (downward) Ekman pumping conditions at the upper surface with a quiescent abyss. After some years of struggle, the continuously stratified equations also finally yielded to an analogous approach (Huang 1988). In these models, κ plays no role; the meridional profile of temperature is mapped *continuously* to a vertical profile, even in the limit of $\kappa = 0$. In subsequent developments, the roles of seasonal variations in surface density and mixed-layer thickness were recognized. The term "subduction," describing the theoretical vision of subtropical isopycnal layers sliding beneath one another, downward and equatorward in vast fluid sheets, has now become as common in our field as it is in plate tectonics. However, the ideal theories made no real explicit connection to the abyss.

Samelson and Vallis (1997; henceforth SV) computed numerical solutions of a planetary-geostrophic model, and found that these could be understood as a combination of the diffusive and ideal theories. In their single-basin model, the main thermocline has two dynamical regimes: an upper adiabatic region in which the dynamics are essentially those of LPS, and a lower intrinsically diffusive part, which in the limit of small κ forms an internal boundary layer (see figure 1.) The upper, ventilated thermocline is, just as in LPS theory, a continuous mapping of a surface meridional temperature profile, but only the temperature profile across the subtropical gyre is so mapped. The temperature profile across the subpolar gyre maps to a diffusive, internal boundary layer as in the similarity theories. Furthermore, and perhaps importantly for the earth's climate, only the temperature gradient across the subpolar gyre is effective in driving the thermohaline circulation, because this circulation is buffered from the temperature gradient across the subtropical gyre by the isothermal layer at the base of the ventilated thermocline.

Arguably, SV's model can be seen as a direct consequence of the planetary-geostrophic equations, with small but non-zero vertical diffusion, convective adjustment, minimal parameterizations of lateral diffusion and friction, and certain boundary conditions of temperature and stress. An interesting question is, to what extent can the present theory of the thermocline be regarded quantitatively as the planetary-geostrophic approximation plus these or similar boundary conditions? Put another way, is the phenomenology simply the result of *calculations* from a planetary-geostrophic base, with the latter being the underlying theory?

3 Testing Dynamical Theories

As the planetary-geostrophic equations themselves are the foundation for our dynamical models of the large scale, it seems that they should be the starting point for observational tests. However, an objective, quantitative test of the planetary-geostrophic equations as an approximation for large-scale ocean circulation has not been achieved, forty years after their first derivation. The possible role of eddy heat and vorticity fluxes (which are excluded from the planetary-geostrophic equations) in setting the stratification of the thermocline remains a subject of active debate. Will the WOCE observations prove sufficient to settle this issue? If so, the result would be a fundamental advance in our physical understanding of the ocean, with critical implications for future ocean climate observations and modelling programs.

How can such a result be achieved? The most direct approach is to test the balances in the equations themselves. Some of the various approaches to inverse modeling (e.g,, Bennett 1992; Wunsch 1996) offer the opportunity to carry out quantitative statistical hypothesis testing on the PGE approximations in this direct manner. However, the nonlinearity of the PGE makes this a daunting project.

Is there an alternative to this direct approach? If so, it seems clear that it must be more phenomenologically oriented. As an analogy, consider for example the Schrödinger equation of quantum mechanics. This equation cannot be tested directly, in the manner of a weak-constraint inverse formulation, because the quantum mechanical wave-functions are not directly observable. Nonetheless, it has been tested stringently as a physical model through careful measurements of observable quantities — phenomena — that can be related to it or derived from it, in one way or another. To the extent that PGE theories predict specific, recognizable features in the ocean thermocline — such as the ventilated thermocline or the internal boundary layer — that can be compared to observations, they can and should be viewed in a similar fashion (and as the results of a similar scientific philosophy). The direct test of the dynamical equations may be as unachievable in practice for the planetary-geostrophic equations as it is in principle for the Schrödinger equation. The challenge for the practical theoretician is to quantify the planetary-geostrophic phenomenology in a way that exposes the approximations to quantitative observational tests.

Thus, we circle back to the phenomenology of the thermocline. Can the various models (ventilated, diffusive, 'two-thermocline' etc.) in and of themselves be subjected to such tests? To the extent that these theories follow from from the planetary-geostrophic equations, a test of these thermocline theories is a test of the adequacy of these equations and, implicitly, the lack of importance of eddies. Thus, we would ideally like a quantitative test of the phenonema predicted by the models, but this has not yet been achieved. Of course, some progress has been made in testing the theories. For example, the relative freedom with which surface boundary conditions could be specified in the LPS theory resulted immediately in calculations that could be compared quantitatively with observations, first for the subtropical North Atlantic (Luyten et al themselves), and, soon after, for the subtropical Pacific (Talley 1985). Some preliminary modelling studies of the thermocline have also been made with the primitive equations (e.g., Cox 1985) that do, in principle, allow for a continuum of scales down from the gyres to the first deformation radius. Will WOCE data allow us to do more? The two-thermocline model has a relatively rich dynamical structure that seems as well suited as anything to observational testing. For example, predictions include:

1. The formation of a thick, mode-water like, thermostad that outcrops at approximately the latitude of the mean zero wind-stress curl, separating two regions of larger vertical temperature gradient.

2. The vertical velocity should pass through zero in the lower region, that is in the internal boundary layer.

3. The heat budget should be approximately closed by the large-scale fields themselves. That is, eddies should have only a small contribution. Similarly, available potential energy should dominate the energy budget. (These are related more to the adequacy of the PGE than to thermocline models *per se*.)

4. Specifically, in the upper thermocline (isotherms that outcrop in regions of Ekman downwelling) the heat budget should (below the surface) close on the basis of an advective balance, whereas below the region of mode water the balance is advective-diffusive.

5. In so far as the upper part of the thermocline is a ventilated thermocline, the phenomena associated with that should be present there.

Some of these predictions may be impossible to directly test; for example, it is very difficult to directly measure vertical velocity, although tracer release experiments (as in Ledwell et al 1993) may provide a means. Others are rather qualitative and do not directly or quantitatively test thermocline models. For example, mode water (item 1) has been the object of many observational studies (e.g., McCartney and Talley 1982) and, in part because of strong seasonal effects, it conceivably cannot be uniquely associated with a two-thermocline model. Similarly, heat transport estimates (e.g., Hall and Bryden 1982) indeed appear to show only small eddy effects (item 3), but this does not discount the possibility that eddies might have a cumulative influence on the large-scale fields that is impossible to detect in such diagnostics. And the specific heat budgets of item 4 may be very hard to quantitatively test, especially without exact knowledge of the variability of the vertical diffusivity.

Some of these observational tasks may ultimately be made easier by assimilating data into a primitive equation ocean model, and then testing the dynamical balances in that model. However, this may be ambiguous on various counts. In particular, such 're-analysis' products (to use a phrase common in the atmospheric sciences) unavoidably incorporate the strengths and weaknesses of the underlying numerical model, especially if observations are sparse, and so are not a direct observational test of the phenomenology. Relatedly, the primitive equation model should certainly be eddy resolving, else it is really just solving the planetary-geostrophic equations and then the thermocline solution is assured. But observations will typically not resolve the eddies, except perhaps altimeter observations of the surface only, and thus the test may end up being mainly a test of the planetary-geostrophic equation thermocline theory against another numerical model.

4 Summary

Although one of the goals of observations should presumably be to test dynamical theories, the nonlinearity of the equations, the lack of knowledge of the value of key parameters and the sparsity of data make this difficult. Similarly, theorists should presumably seek to formulate hypotheses that are observationally testable. But this too is easier said than done. Even in the atmosphere, where the data are so much more plentiful, unresolved questions remain about such first-order questions as what determines the height of the tropopause or the latitudinal extent of the Hadley Cell. In the ocean we are both unsure about the basic level of approximation (e.g., the appropriateness of the planetary-geostrophic equations), and the dynamics that result from such equations. Because it may be near-impossible to ever directly test the dynamical balances in the equations of motion themselves, we may seek to make unambiguous phenomenological predictions from the equations of motion that in turn can be subject to observational tests. The theory of the structure of the thermocline may be a good starting point. It is then a challenging and important task for both observers and theoreticians to come up with tests that are both severe and feasible.

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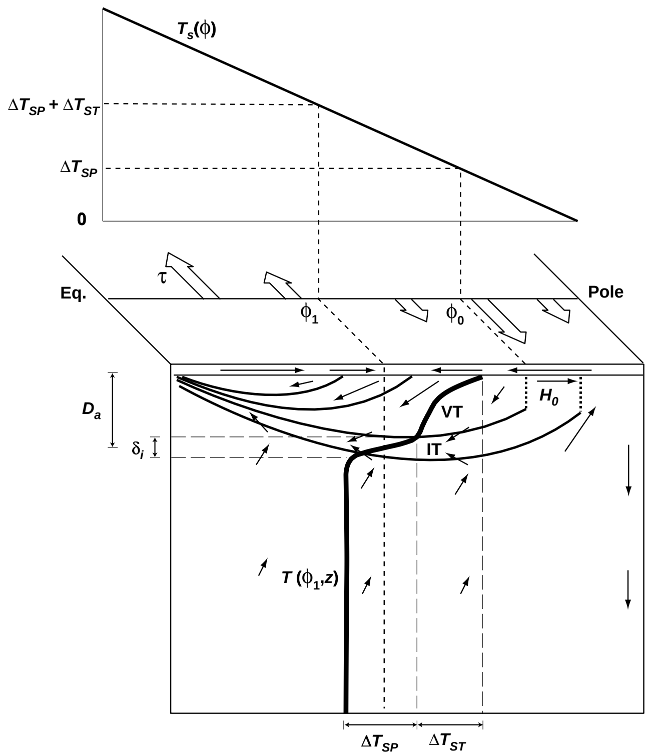

Figure 1: A schematic picture of the large-scale circulation and the thermocline regimes in a simply-connected, single-hemisphere ocean driven by wind-stress and differential heating. The idealized meridional section below is at a mid-basin longitude. The main thermocline is composed of an upper, ventilated thermocline and a lower internal boundary layer, separated by a region of mode water. From Samelson and Vallis 1997.