Quantum Gravity in a Model Universe

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Abstract

This report describes a hypothetical universe termed Simple Absorption (SA). SA is based on one underlying principle: a specific definition of absorption. Elements in SA can absorb each other according to a well defined process. As a result of this definition of absorption, SA contains a surprisingly rich physics including both general relativity and quantum mechanics.

1. INTRODUCTION

I would like to start with a disclaimer. I am not a physicist. I am a neuroscientist with a background in physics. A few years ago, as a hobby, I defined a simple, hypothetical universe on the basis of one underlying principle: a specific definition of absorption. I called that universe Simple Absorption (SA) and explored some of its physical properties. The universe turned out to have an unexpectedly rich physics. Elements in SA could be shown to act as though they had momentum and energy, and to interact gravitationally via the equations of general relativity. Moreover, elements in SA behaved in a quantum-mechanical fashion and formed a many-worlds structure. I frankly do not know if there is any relationship between SA and the real world. But SA was such an interesting place for me to inhabit intellectually, and provided such a novel way to think about both general relativity and quantum mechanics as secondary manifestations of a simpler underlying process, that I thought it might be interesting to others as well. The purpose of this report is to describe SA and some of its physical properties.

SA grew out of a consideration of absorption and emission. Consider first a naive physical picture of how particles might absorb each other in a simple, hypothetical universe.
Imagine that two point particles, A and B, move along definite trajectories, bump into each other, absorb each other, and produce a resultant particle C. The absorption itself could be described as follows: at a certain point in space-time, particles A and B disappear and particle C appears. The process is discontinuous. The discontinuity is a result of the nature of point particles. A point particle can be described as being “present” at some locations in space-time and “absent” at other locations. At each point in space-time its value is either 0 or 1. Knowing nothing about the masses or energies or physical properties or equations of motion of these point particles, it is still possible to make the (seemingly trivial) statement that at the single space-time point at which the absorption occurs, the “value” of A experiences a sink, dropping from 1 to 0, the “value” of B experiences a similar sink, and the “value” of C experiences a source, increasing from 0 to 1.

Now consider a case that is only slightly more complex. Suppose that the hypothetical elements are not point particles but instead distributed entities for which the process of absorption may be spread out over space and time. Consider two “fogs”, fog1 and fog2, that absorb each other and thereby create a third entity, fog3. Is there a simple way to describe this process?

As a first intuitively reasonable guess, in a non-relativistic universe, one might describe a fog as having a density at each location in space-time. It is thicker here and thinner there. At any point in space-time, the extent of overlap between fog1 and fog2 can be quantified as the product of their densities. Suppose that where fog1 and fog2 overlap in space-time, they absorb each other and produce some quantity of fog3. The greater the extent of overlap between fog1 and fog2 at a point in space-time, the more absorption occurs there. As they absorb each other, some amount of fog1 and fog2 disappears (fog1 and fog2 experience sinks) and a corresponding amount of fog3 appears (fog3 experiences a source). All that has been done here is to outline a graded version of the discontinuous, point-particle process described above.

In the real world, one might think of fog1 as, for example, oxygen gas, fog2 as nitrogen gas, and fog3 as nitrous oxide. Where both oxygen and nitrogen are dense in the same place at the same time, they are more likely to combine and produce nitrous oxide. In this example of real-world gases, the gases are statistical descriptions of large numbers of particles. The combining of
oxygen with nitrogen is, microscopically, a discontinuous process. In the hypothetical universe being built up here, however, a fog is not a collection of particles. It is a hypothetical, single, continuous entity defined by means of a density that varies in space-time.

In a relativistic universe, density cannot by itself be an adequate description of a fog. Density is not a scalar quantity, but is instead one component of a vector, the flux-density four-vector. The time component of the vector is the density and the spatial components of the vector are the three spatial fluxes. At each point in space-time, the extent of overlap between fog1 and fog2 is not the product of their densities, but instead the dot product of their flux-density vectors. Therefore if one is to build the intuitively reasonable and simple description of absorption that is outlined above, but ensure that it is correct in relativistic space-time, one needs the following prescription: in any infinitesimal volume of space-time, the amount that fog1 and fog2 absorb each other, and therefore the sink in fog1 and fog2, should be proportional to the dot product of their flux-density vectors. The source in fog3 should be the sum of the sinks in fog1 and fog2. The equations for the three fogs should therefore be:

$$\text{(for fog1)} \quad \text{div}\vec{F}_1 = -k\vec{F}_1 \cdot \vec{F}_2$$ (1)

$$\text{(for fog2)} \quad \text{div}\vec{F}_2 = -k\vec{F}_1 \cdot \vec{F}_2$$ (2)

$$\text{(for fog3)} \quad \text{div}\vec{F}_3 = 2k\vec{F}_1 \cdot \vec{F}_2$$ (3)

where $\text{div}$ refers to a four-dimensional divergence that represents the sources and sinks of a fog, $\vec{F}$ refers to the flux-density vector of a fog, and $k$ is an absorption constant.

The Simple Absorption universe, SA, is built on the above description of absorption in a relativistic space-time. The only process defined in SA is the absorption between two vector fields called fogs. In exploring SA, this article asks: what does the process of absorption imply about the physics within SA? It turns out that the definition of absorption implies that objects in SA behave as though they have a four-momentum; momentum is conserved; momentum flux results in gravity consistent with general relativity; and SA is a probabilistic, quantum-mechanical universe with a many-worlds structure. SA is of interest because starting only from a geometric description of absorption – how two overlapping vector fields disappear while a third one is generated – a vast range of physics is automatically provided.
SA is defined by means of the following three rules:

1. SA contains a Lorentzian space-time manifold, including 4 dimensions and the metric tensor:

\[
\eta_{\alpha\beta} =
\begin{array}{cccc}
-1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}
\]

2. SA contains elements termed “fogs.” Each fog is described by a vector field, the flux-density four-vector \( \mathbf{F} \) that varies in space-time.

3. Fogs interact by absorption, defined as follows: Absorption in SA is when two fogs combine to produce a new fog. Fog1 and fog2 absorb each other to produce fog3. In this process, where fog1 and fog2 overlap in space-time, they experience a sink and fog3 experiences a corresponding source. The sinks and sources obey equations 1-3. For simplicity, in SA, the absorption constant \( k=1 \).

There are several key points to note about SA. First, although all three fogs can overlap in space-time, fog1 and fog2 do not absorb fog3 to produce yet other fogs. This constraint is implicit in equations 1-3 in that they lack sink terms corresponding to absorptions between fog1 and fog3 or between fog2 and fog3. If SA is “seeded” with two fogs, they will absorb each other, produce fog3, and no other interactions will be possible. All behavior in the universe is confined to three fogs and to equations 1-3. Most of the analysis below is limited to this extremely simple case of a three-fog universe. However, the same rules can in principle be extended to a more complicated case in which \( N \) fogs are present (\( N>3 \)) absorbing each other. These \( N \)-fog cases will also be analyzed in some of the following sections.

A second key point concerns emissions in SA. How can emissions be incorporated into SA, or is it a universe solely of absorptions? In SA, absorption and emission are simply different names for the same thing. Consider again the case of fog1 and fog2 absorbing each other to produce fog3. All three fogs overlap in space and time. There is no definite time and place at which the absorption occurs; there is no distinct “before the absorption” or “after the absorption.” Instead the absorption is a process that takes place incrementally over all of space and time. Because of the spatial-temporal overlap, one could just as well say that fog3 emitted
fog1 and fog2. Whether an interaction “looks” more like an emission or an absorption depends on the particular spatial-temporal properties of the fogs involved. In the following sections the term absorption is used for simplicity and consistency, but in SA there is no conceptual difference between emission and absorption.

2. CURVED SPACE IN SA

The present section describes how the fundamental equations of SA (equations 1-3) can be re-written as wave equations within a curved space-time. Subsequent sections describe the relationship between that curved space-time and general relativity.

Equations 1-3 can be re-written in a more convenient way. Let $\Psi$ be a scalar field such that $F_\alpha = \Psi_{,\alpha}$ or (using the Einstein summation convention) $F^\alpha = \eta^{\alpha\beta} \Psi_{,\beta}$. Equations 1-3 become:

$$
(\eta^{\alpha\mu} \Psi_{1,\mu})_{,\alpha} = - (\eta^{\beta\mu} \Psi_{1,\mu}) \Psi_{2,\beta} \quad (4)
$$

$$
(\eta^{\alpha\mu} \Psi_{2,\mu})_{,\alpha} = - (\eta^{\beta\mu} \Psi_{1,\mu}) \Psi_{2,\beta} \quad (5)
$$

$$
(\eta^{\alpha\mu} \Psi_{3,\mu})_{,\alpha} = 2 (\eta^{\beta\mu} \Psi_{1,\mu}) \Psi_{2,\beta} \quad (6)
$$

First consider equation 4, the equation for fog1. The left hand side is a four-dimensional divergence. The right hand side is a more complicated expression. This expression on the right hand side can be made to disappear if the equation is re-written in curved space-time coordinates. Equation 4 is equivalent to the equation:

$$
(g^{\alpha\beta} \Psi_{1,\beta})_{,\alpha} = 0 \quad (7)
$$
in a space-time in which the metric tensor is:

$$
g^{\alpha\beta} = e^{-2\eta} \eta^{\alpha\beta} \cdot \quad (8)
$$

Equations 7 and 8 indicate that, in the presence of fog2, fog1 behaves as though it is following a simple wave equation but in a curved space-time.

Equations 7 and 8 can be verified in the following manner. First, to manipulate derivatives correctly in curved space, it is necessary to compute the Christoffel components, or $\Gamma^\alpha_{\beta\gamma}$, associated with that curvature. The metric tensor in equation 8 can be used to compute the Christoffel components using the standard equation:
\[ \Gamma^\gamma_{\beta\mu} = \frac{1}{2} g^{\alpha\gamma} (g_{\alpha\beta,\mu} + g_{\alpha\mu,\beta} - g_{\mu\beta,\alpha}). \] (9)

The 64 Christoffel components, computed according to equation 9, are given in Appendix A.

The covariant derivatives in equation 7, indicated by a semicolon, can be expressed by means of Christoffel components as follows:

\[ 0 = (g^{\mu\nu}\Psi_{1,\mu})_{,\alpha} = (g^{\mu\nu}\Psi_{1,\mu})_{,\alpha} + (g^{\mu\nu}\Psi_{1,\mu})\Gamma^\alpha_{\beta\alpha}. \] (10)

Inserting the expression for \( g \) from equation 8 yields:

\[ 0 = (e^{-\Psi^2}\eta^{\mu\nu}\Psi_{1,\mu})_{,\alpha} + (e^{-\Psi^2}\eta^{\mu\nu}\Psi_{1,\mu})\Gamma^\alpha_{\beta\alpha}. \] (11)

Using the expressions for the Christoffel components from Appendix A, equation 11 reduces to:

\[ (\eta^{\mu\nu}\Psi_{1,\mu})_{,\alpha} = -(\eta^{\mu\nu}\Psi_{1,\mu})\Psi_{2,\beta}. \]

which is the same as equation 4. In this way it is confirmed that equation 4 is equivalent to equation 7. The sink in fog1, created as it is absorbed by fog2, is mathematically equivalent to fog1 behaving according to a wave equation (equation 7) in a curved space-time. The metric tensor of this curved space-time (equation 8) depends on \( \Psi \). As the magnitude of \( \Psi \) approaches zero in any region of space-time, the metric tensor approaches the metric for flat space-time, and fog1 follows the simple homogeneous wave equation \( (\eta^{\mu\nu}\Psi_{1,\mu})_{,\alpha} = 0 \). As \( \Psi \) becomes non-negligible, fog1 behaves as if it were following the same wave equation but in a curved space-time with metric tensor \( g \). The larger \( \Psi \), the greater the curvature. Fog1 is essentially living in a space-time created by fog2. The nature of this curved space-time is explored further in the following two sections.

### 3. CURVED SPACE WITH N FOGS

Consider a more general case: suppose there are \( N \) fogs such that fog1 can be absorbed by fogs 2-N. Each absorption results in a sink in fog1. Let \( \Psi_1 = \sum_{n=2}^{N} \Psi_n \). One can read \( \Psi_1 \) as “the sum of the scalar fields of all fogs that can absorb fog1.” The same logic used in section 1 to describe absorption implies:

\[ (\eta^{\mu\nu}\Psi_{1,\mu})_{,\alpha} = -(\eta^{\mu\nu}\Psi_{1,\mu})\Psi_{1,\beta}. \] (12)
Equation 4 is simply a limiting case of equation 12. Equation 12 indicates that the total sink in fog1 is a sum of sinks caused by the absorption of fog1 by a set of fogs. For the same reasons outlined in section 2, equation 12 is equivalent to the equation \((g^{\alpha\beta}\Psi_{1,\beta}) = 0\) in a curved space-time in which the metric tensor is:

\[ g_{\alpha\beta} = e^{\Psi_{1}}\eta_{\alpha\beta}. \]  

(13)

Equation 13 implies that fogs 2-N collectively create a curved space-time for fog1. By the same logic, just as fog1 behaves as if it is in a curved space-time, so does each of the N fogs. Note that each fog “sees” a slightly different universe (a different set of fogs by which it can be absorbed) and therefore experiences a slightly different curved space-time. For example, in the case of fog1 and fog2 absorbing each other, fog2 creates a curved space-time in which fog1 moves, and fog1 creates a different curved space-time in which fog2 moves. Each fog sees a different universe, and therefore a different curvature. In the case of N fogs, where N is large, each fog will see almost the same universe, and the curvature will be nearly the same for all N fogs.

Consider now the most general case in SA, a fog that has both a source and a sink. Suppose fog1 and fog2 absorb each other to produce fog3; but the triplet is embedded in a larger universe that includes other fogs that can absorb fog3. Let \(\Psi_{3}\) be the sum of the scalar fields of all fogs that can absorb fog3. In this case, fog3 has both a source (caused by \(\Psi_{1}\) and \(\Psi_{2}\)) and a sink (caused by \(\Psi_{3}\)). Its equation is therefore:

\[
(\eta^{\alpha\mu}\Psi_{3,\mu})_{,\alpha} = 2(\eta^{\beta\mu}\Psi_{1,\mu})\Psi_{2,\beta} - (\eta^{\beta\mu}\Psi_{3,\mu})\hat{\Psi}_{3,\beta}.
\]

(14)

By a similar derivation as in the previous sections, equation 14 is equivalent to the equation:

\[
(g^{\alpha\mu}\Psi_{3,\mu})_{,\alpha} = 2(g^{\beta\mu}\Psi_{1,\mu})\Psi_{2,\beta}
\]

in a curved space in which \(g_{\alpha\beta} = e^{\Psi_{3}}\eta_{\alpha\beta}\). In equation 15, the sink term has disappeared, subsumed into the space-time curvature, and the source term remains valid in that curved space-time. Equation 15 is the most general equation for a fog in SA. In essence, in SA, fogs can experience sinks and sources as a result of interacting. The sinks are equivalent to a curved space-time in which the fogs move. The sinks ultimately are responsible for gravity in SA. The source term in equation 15 will turn out to supply a version of quantum mechanics and the conservation of momentum.
SA can be understood from two different equivalent perspectives. In one perspective, expressed explicitly in the three defining rules, SA can be understood as containing a Lorentzian space-time (with the metric tensor of flat space) in which fogs experience absorption sinks and sources as they interact. In this view, flat space seems to play a special role in SA. However, that special role is illusory. In the second perspective, in which the absorption sink is handled mathematically by means of curvature, there is no such thing as flat space in SA. Instead, a fog in SA moves in a curved space-time created by the set of fogs with which it can interact.

If fogQ has no other fogs with which to interact (that is, if SA is seeded with a single fog), then the equations for fogQ are undefined. There is no basis for constructing the equation of motion for fogQ. All the equations in SA derive from the process of absorption. There is therefore no definable thing as a “free” fog moving in a flat background space. If, however, other fogs do exist that can interact with fogQ, not only does fogQ gain an equation of motion but it also gains a space-time in which to move. In some regions of space-time, these other fogs might have flux-density vectors that are approximately zero, thereby creating an approximately flat region of space-time in which fogQ can move. But that almost-flat space-time has no special role for fogQ. Whether curved or approximately flat, every region of space-time in which fogQ moves is created by the fogs with which it can interact.

4. STRESS-ENERGY TENSOR AND FOUR-MOMENTUM VECTOR IN SA

The previous sections described the curvature of space-time in SA. Yet the theory of general relativity is obviously more specific than merely a curvature of space-time. The central tensor in general relativity is the stress-energy tensor $T$. Each component of the stress-energy tensor, $T^{\alpha\beta}$, is the flux in the $\beta$ direction of the four-momentum in the $\alpha$ direction. The stress-energy tensor can therefore also be thought of as the momentum-flux tensor and is sometimes referred to in that way in this report. This flux of the four-momentum creates or specifies a curvature of space-time. The central equation of general relativity, equating momentum-flux with curvature, is $G=KT$, where $G$ is the Einstein curvature tensor and $K$ is a constant. From $G$, it is theoretically possible to compute the metric tensor $g$ and thereby gain some ability to compute the trajectories of objects in a gravitational field.

In the present section, the logic of general relativity is applied to SA but traced in reverse. According to equation 8, there is a metric tensor $g$ that describes the space-time curvature
produced by a fog. Given the metric tensor $g$, it is possible to obtain the tensor $G$ that describes that space-time curvature produced by that fog. Given $G$, it is possible to obtain the momentum-flux tensor $T$ that would be needed, in general relativity, to produce that space-time curvature. Given the momentum-flux $T$, and given that flux is already defined in SA, it is possible to obtain an expression for the four-momentum of a fog. Four-momentum is not defined at the outset in SA. Instead, it is derived. In essence, the logic is that a fog creates a curvature of space-time; in general relativity, that curvature is possible only if the fog has an associated four-momentum; therefore one can calculate the effective four-momentum of the fog. The relevant question is whether that expression for the four-momentum, derived using general relativity, in any way resembles the usual definition of four-momentum in the real universe. If so, such a result would imply that the usual concepts of four-momentum and of general relativity provide a valid framework for understanding the physics within SA. As will be seen, the expression for four-momentum in SA, derived using general relativity, matches the quantum-mechanical definition of energy and momentum in the real universe.

To simplify the situation, consider again a universe containing only three fogs, fog1 and fog2 absorbing each other to produce fog3. Moreover, in this example, suppose that the vector fields do not have equal magnitude: $|\vec{F}_1| \ll |\vec{F}_2|$ in the region of space-time that is of interest. The effect of fog1 on fog2 is therefore negligible and the equation for fog2 (from equation 5) can be approximated as:

$$\left(\eta^{\alpha\beta}\Psi_{2,\beta}\right)_\alpha = 0.$$  \hspace{1cm} (16)

The effect of fog2 on fog1 however is not negligible. The effect produced by fog2 on fog1 is equivalent to fog1 moving in a curved space-time for which the metric tensor is (from equation 8):

$$g_{\alpha\beta} = e^{\Psi_2}\eta_{\alpha\beta}.$$  \hspace{1cm} (17)

Fog2 therefore produces a curvature of space-time described by equation 17. The associated Einstein tensor $G$, which also describes the curvature of space-time produced by fog2, can be constructed from the metric tensor $g$ according to standard equations. The 16 components of $G$ are given in Appendix B. The expressions for the components of $G$ can be simplified by using equation 16. For example, in $G_{00}$, the expression $-\Psi_{2,1,1} - \Psi_{2,2,2} - \Psi_{2,3,3}$ can be replaced by $-\Psi_{2,0,0}$. With this simplification, $G$ can be expressed in covariant form as:
\[ G_{\alpha\beta} = \frac{1}{2} \Psi_{2,\alpha} \Psi_{2,\beta} - \Psi_{2,\alpha,\beta} + \frac{1}{4} \Psi_{2,\mu} \Psi_{2,\alpha} \eta^{\mu\alpha} \eta_{\alpha\beta}. \]  

Since fog2 produces a curvature of space-time described by equation 18, one can determine the stress-energy tensor \( T \), associated with fog2, that would be necessary in general relativity to produce that curvature. The relationship between \( T \) and \( G \) is the Einstein equation \( G = K T \). For the moment, for convenience, let \( K = \frac{1}{2} \) such that:

\[ T_{\alpha\beta} = \Psi_{2,\alpha} \Psi_{2,\beta} - 2 \Psi_{2,\alpha,\beta} + \frac{1}{2} \Psi_{2,\mu} \Psi_{2,\alpha} \eta^{\mu\alpha} \eta_{\alpha\beta}. \]  

The present choice for \( K \) is an arbitrary definition and is meant only for convenience. Any value would be conceptually equivalent. (Indeed there is really only one free variable in SA, the absorption constant \( k \) in equations 1-3, which was arbitrarily set to 1. Setting \( k \) to other values leads to some minor variations in SA that will not be discussed here.)

Equation 19 supplies the stress-energy tensor associated with fog2. From this equation one can see that \( \Psi_{2,\alpha} \) can serve as the four-momentum of fog2 in the \( \alpha \) direction. The reason is as follows. Consider the first term in equation 19: \( \Psi_{2,\alpha} \Psi_{2,\beta} \). Since \( \Psi_{2,\beta} \) is by definition the covariant form of the flux of fog2 in the \( \beta \) direction, the term \( \Psi_{2,\alpha} \Psi_{2,\beta} \) refers to a direct transport of \( \Psi_{2,\alpha} \) in the \( \beta \) direction. The second term \( -2 \Psi_{2,\alpha,\beta} \) acts as a transport of \( \Psi_{2,\alpha} \) in the \( \beta \) direction through viscosity. The third term \( \frac{1}{2} \Psi_{2,\mu} \Psi_{2,\alpha} \eta^{\mu\alpha} \eta_{\alpha\beta} \) acts as a pressure term, present only on the diagonal elements of the tensor. Equation 19 therefore is the covariant form of a stress-energy tensor for a substance with the following properties: its flux is represented by \( F_{2,\alpha} = \Psi_{2,\alpha} \), its four-momentum is represented by \( P_{2,\alpha} = \Psi_{2,\alpha} \), and the substance acts as a fluid with viscosity and pressure.

The current approximation, in which \( |\vec{F}_1| \ll |\vec{F}_2| \) and therefore the effect of fog1 on fog2 is neglected, is critical to the present logic. In the situation analyzed here, fog2 is effectively in isolation. It creates a curvature of space-time. That curvature is confirmable by analyzing the behavior of fog1, which has such a small magnitude that it effectively does not alter the isolation of fog2. Like monitoring a tiny chip of wood to measure the movement of ocean waves, here the behavior of fog1 is analyzed to understand the curvature of space-time created by fog2. In this manner it is possible to analyze the effective four-momentum attributable specifically and only
to fog2. It is seen that fog2 can be treated mathematically as though it were a substance with a viscosity term, a pressure term, and a four-momentum \( P_{2\alpha} = \Psi_{2\alpha} \). The resulting stress-energy tensor of fog2 can be used in the Einstein equation for gravity, \( G = KT \). This equation correctly describes the curvature of space-time that is generated by fog2. Fog1 experiences that curvature of space-time. In this way, fog2 affects fog1 gravitationally in a manner that follows general relativity, and fog2 has a gravitationally-defined four-momentum. Without the approximation, when fog1 and fog2 are allowed to affect each other, the math becomes much more complicated and, when computing a stress-energy tensor, it is no longer possible to clearly attribute that tensor to fog1, to fog2, or to the gravitational interaction between them. Isolating the four-momentum of a single fog becomes impossible. The approximation, therefore, allows one to define the four-momentum of a single fog.

A similarity exists between the four-momentum of a fog in SA and the four-momentum of a particle in quantum physics. The quantum-mechanical expression for the four-momentum of a particle is \( P_{\alpha} = \hbar \Psi_{\alpha} \), where \( \Psi \) is the state vector of the particle. (The typical sign reversal on the time component of the momentum comes from raising the index and expressing the four-momentum as a vector rather than as a one form.) In SA, the expression for the four-momentum of a fog was identified above as \( P_{\alpha} = \Psi_{\alpha} \) where \( \Psi \) is the scalar field of the fog. These expressions differ merely by a constant. It would have been equally valid in SA, if inelegant, to define \( \Psi \) as a scalar field such that \( \Psi_{\alpha\alpha} = \frac{i}{\hbar} F_{\alpha} \). In that case, the resulting expression for the four-momentum in SA would have exactly matched the quantum mechanical expression.

How meaningful is this similarity of the four-momentum of a fog in SA to the four-momentum of a particle in the real world? For the four-momentum in SA to resemble the four-momentum in the real world in any meaningful sense, two basic properties need to be established. First, one would need to show that the scalar field \( \Psi \) of a fog in SA resembles the state vector \( \Psi \) of a particle in quantum mechanics. In particular, one would need to show that a fog in SA somehow acts like a particle with undetermined position, with each possible position in space weighted by the scalar field \( \Psi \), such that the squared amplitude of \( \Psi \) integrated over a volume of space provides the probability of experimentally finding the particle in that volume of
space. This task sounds somewhat daunting, given that there is so far no provision for probabilities, particles, or experiments in SA.

A second property that one would need to show is that when fog1 and fog2 absorb each other to produce fog3, the four-momentum of fog3 is the vector sum of the four-momenta of fog1 and fog2. That is, one would need to show the conservation of momentum in SA, in order for it to play a similar role in SA as it does in the real world.

All of these properties do actually pertain to SA and are elaborated in sections 5 and 6.

5. ABSORPTION SOURCES

The above sections focus on the sinks in SA. When fog1 and fog2 absorb each other to produce fog3, fog1 and fog2 experience sinks. Yet SA also includes absorption sources. The present section explores an approximation in which the sinks, which are responsible for the gravitational effects in SA, are ignored and only the sources are considered. In working out classical quantum mechanics in the real world, physicists worked with the assumption that gravity was so weak at a small scale that its effects could be ignored; or, alternatively, that space-time was locally flat and therefore curvature could be ignored in considering the quantum domain. What would happen if the same approximation were made in SA? If the absorption sinks (that result in curvature) are put aside and instead a version of SA is examined in which fogs experience only absorption sources, what physical properties will result and will they in any way resemble the classical properties of quantum mechanics?

Consider again the simple three-fog universe, in which fog1 and fog2 absorb each other to produce fog3. In the present approximation, gravitational effects are ignored. The equations for fog1 and fog2 (from equations 4 and 5) become:

\[
(\eta^{\mu\nu}\Psi_1)_{,\alpha} = (\eta^{\mu\nu}\Psi_2)_{,\alpha} = 0. \tag{20}
\]

The equation for fog3 (from equation 6) is:

\[
(\eta^{\mu\nu}\Psi_3)_{,\alpha} = 2(\eta^{\beta\mu}\Psi_1)_{,\beta} \Psi_2. \tag{21}
\]

A solution for \(\Psi_1\) (or \(\Psi_2\)) that satisfies equation 20 is a sum of terms for which each term has the form:

\[
\Phi = A e^{i p_{\mu} x^\mu}. \tag{22}
\]
In this expression, $A$ is the amplitude of the term relative to other terms in the series. The momentum operator was defined gravitationally in the previous sections as $P_\alpha(\Phi) = \Phi_{-\alpha}$. In equation 22, $ip_\alpha$ is the eigenvalue for the momentum operator. Effectively, $p_\alpha$ is the real value of the momentum of this particular term in the $\alpha$ direction.

A solution that satisfies equation 21 is:

$$\Psi_3 = \Psi_1\Psi_2.$$  

This solution can easily be confirmed by plugging it into equation 21 and differentiating. Let one particular term in $\Psi_1$ be represented by $\Phi_1 = A_1e^{i[p_1\alpha x^\alpha]}$. Likewise let one particular term in $\Psi_2$ be represented by $\Phi_2 = A_2e^{i[p_2\alpha x^\alpha]}$. By equation 23, $\Psi_3$ must contain the term:

$$\Phi_3 = A_3e^{i[p_3\alpha x^\alpha]} = A_1A_2e^{i[(p_1\alpha + p_2\alpha) x^\alpha]}.$$  

In this equation, $p_{3\alpha} = p_{1\alpha} + p_{2\alpha}$ and $A_3 = A_1A_2$.

These results share a certain connection to classical quantum mechanics. The discussion so far has not yet involved probability in SA, or the meaning of a “possible state” for a fog, but for the moment bear with a quantum-like description of SA to see where it leads. Let $\Phi_1$ represent one possible state of fog1, with a probability amplitude of $A_1A_1^*$, and $\Phi_2$ represent one possible state of fog2, with a probability amplitude of $A_2A_2^*$. If one analyzes the possible outcome that those two states absorb each other to produce a resultant state $\Phi_3$, the probability amplitude of $\Phi_3$ is a product of the probabilities of $\Phi_1$ and $\Phi_2$, following the correct multiplicative rule for probability. The momentum of $\Phi_3$ is a sum of the momenta of $\Phi_1$ and $\Phi_2$, obeying the conservation of momentum. The treatment of the absorption source in SA therefore leads to some basic quantum-mechanical properties, and to the conservation of momentum.

Yet how does probability enter into SA? Nothing thus far has suggested that SA is a probabilistic universe. The next section describes how SA has a many-worlds structure that would result in the appearance of probability to any observers doing experiments in SA.

6. MANY WORLDS AND PROBABILITY IN SA

In SA, not all pairs of fogs can absorb each other. As stated in the definition of absorption in section 1, in the case of fog1 and fog2 absorbing each other to produce fog3, neither fog1 nor
fog2 can absorb fog3. A fog cannot absorb its own “offspring.” One way to think of this “no incest” constraint is that it creates a many-worlds structure. In the case of the three-fog universe, there are two worlds. Fog1 and fog2 belong together in world A, the pre-absorption world (or “parent” world). Fog3 belongs to world B, the post-absorption world (or “offspring” world). At any point in space-time, it is simultaneously true that the absorption has not occurred (world A) and that the absorption has occurred (world B). The worlds are separated from each other by means of the no-incest constraint. Fog1 and fog2, in world A, are barred from ever absorbing fog3 in world B.

The constraint can be generalized to the case of many fogs. Consider SA seeded with three unrelated fogs: fog1, fog2, and fog3. Each can absorb the others. As fog1 absorbs fog2, it produces a new fog that will be labeled here fog[1,2]: As fog1 absorbs fog3, it produces fog[1,3]: As fog2 absorbs fog3, it produces fog[2,3]. Respecting the no-incest constraint, fog1 can absorb fog[2,3] to produce fog[1,[2,3]]: fog2 can absorb fog[1,3] to produce fog[2,[1,3]]: fog3 can absorb fog[1,2] to produce fog[3,[1,2]]. These nine fogs complete the set. No other fogs can be generated given a seed of three unrelated fogs. (A seed of 4 unrelated fogs generates a set of 40.)

Consider a case in which SA is seeded with trillions of fogs and therefore develops an extreme complexity. Suppose a particular fog is absorbed by fogA and by fogB. The ramifying consequences of absorption A, however vast and however many fogs are involved, are barred by the no-incest constraint from directly interacting with the ramifying consequences of absorption B. One can consider future A and future B to be separate worlds in the sense that fogs in world A can never be absorbed by fogs in world B. Intelligent creatures living in SA (if such a thing were possible) would therefore experience probability. If three amplitude units of Michael flow down future A for every one that flows down future B, and if a similar event is repeated many times, then in accumulating data over many events, each Michael would conclude that each time either A or B occurs, never both, and that A is three times more likely than B. Yet from the outside looking in on SA, one sees that probability is an illusion created by the many-worlds structure.

Michael of SA could construct the following interpretation to explain the behavior of fogs: Fog1 is not a spread-out entity (a vector field $\vec{F}_1$ or a scalar field $\Psi_1$), but a particle with an uncertain position. The probability of it being in any small volume of space-time is given by $\Psi_1\Psi_1^*$ integrated over that volume. Likewise, fog2 is a particle whose probability amplitude is
represented by $\Psi_2\Psi_2^*$. The likelihood that particle 1 will be absorbed by particle 2 is proportional to the product of the two probabilities, $(\Psi_1\Psi_1^*)(\Psi_2\Psi_2^*)$, integrated over the relevant volume of space-time. Michael finds that this compounding of probabilities is indeed correct (given equation 23), appearing to confirm his interpretation. Michael is therefore led to a Copenhagen interpretation of SA. In this interpretation, his universe is made of particles, each particle can be described by a state vector, and the squared amplitude of that state vector provides the correct probability function.

Michael is certain that fog1 is actually a particle for two reasons. First, its position can apparently be measured. If particle 2 has a sharply peaked $\Psi_2$ at some location in space, and if particle 1 is absorbed by particle 2, then Michael presumes he has measured the position of particle 1.

The second reason is that, when particle 1 is absorbed by particle 2, no other possible absorptions of particle 1 can be detected. Particle 1 is apparently no longer available to be absorbed by anything else. This particle-like behavior arises because, although fog1 is actually absorbed by many fogs, and each absorption produces a consequent fog, these consequences are isolated from each other into separate, non-interacting worlds. Each Michael can measure only one of them.

As fog1 is absorbed by other fogs, it experiences absorption sinks and therefore its behavior is altered. Michael of SA can measure that change in fog1, but rather than interpret it as a consequence of many different absorptions occurring simultaneously, Michael of SA interprets that behavior as a gravitational effect of other objects on particle 1. To an observer in SA, the alternate worlds in the many-worlds structure are therefore inextricably bound to the phenomenon of gravity.

Michael’s interpretation of SA, that it is made of tiny moving particles with uncertain position, is an alternative if inelegant way to describe SA. A more elegant description of SA is that its fundamental objects are not particles, but vector fields; the vector fields absorb each other in a rule-based manner; and there is no true probability, but instead a many-worlds type structure.
7. SUMMARY AND FURTHER DIRECTIONS

SA is defined on the basis of a single underlying description of how one vector field absorbs another in a Lorentzian space-time. Fog1 and fog2 absorb each other to produce fog3. The absorption includes two related processes. In the first process, fog1 and fog2 experience sinks as they absorb each other. These sinks are mathematically equivalent to a curvature of space-time that follows general relativity. As a consequence, it is possible to define a gravitational four-momentum for each fog. In the second process, fog3 experiences a source (it gains what is lost by fog1 and fog2). This source supplies the math behind the conservation of momentum. Fog3 gains the combined momentum of fog1 and fog2. The source term also supplies the math behind the quantum mechanical properties of SA. The rules of absorption result in a many-worlds structure in which some sets of fogs are barred from directly interacting with other sets of fogs. As a result of these many worlds that cannot directly interact with each other, SA is pseudo-probabilistic. Measurements made within SA would reveal an apparent probability that follows the rules of classical quantum mechanics. The most surprising property of SA is that a range of physics emerges simply from defining absorption. SA is nothing more than a precise definition of how two distributed entities absorb each other in space-time.

The properties of SA described in the present report are not exhaustive. Many more physical properties that may be of interest are not pursued here. In the examples provided above, each fog in SA is described by a wave equation, sometimes in a curved space, with a wave speed of 1. In this sense a fog is similar to a zero-rest-mass particle. Its wave speed is fixed. However, it is possible to construct an object in SA that effectively has a rest mass. Consider again the case of two fogs, fog1 and fog2, that absorb each other to produce fog3. The interaction between fog1 and fog2 can be described as a gravitational interaction, spelled out in section 4. Suppose that the scalar field of fog1 is spatially peaked in its magnitude. In principle, it could be so sharply peaked as to create a local, gravitational event horizon from which other fogs cannot escape. Suppose that fog2 is also so sharply peaked spatially as to produce a gravitational event horizon around its peak. Now consider the interaction of the two fogs. These two peaks could in principle capture each other gravitationally and create a single, highly localized object whose speed in any arbitrary coordinate system can be less than 1. The object contains the combined four-momenta of fog1 and fog2, incorporates complicated spin properties, and can be at rest in some coordinate frame. It is an exotic particle constructed out of two fogs. Other particles could be constructed in
similar ways. The equations to describe such objects are not pursued here. The point of this example is that a complicated chemistry can exist in SA. The properties of SA go beyond the basic descriptions provided here.

**Appendix A.** Table for $\Gamma$ given $g_{\alpha \beta} = e^{\psi} \eta_{\alpha \beta}$. $\Gamma$ is computed from the metric tensor based on standard formulas from general relativity.

\[
\begin{align*}
  \Gamma^0_{00} &= \frac{1}{2} \psi_{2,0}^0, & \Gamma^0_{01} &= \frac{1}{2} \psi_{2,1}^0, & \Gamma^0_{02} &= \frac{1}{2} \psi_{2,2}^0, & \Gamma^0_{03} &= \frac{1}{2} \psi_{2,3}^0, \\
  \Gamma^0_{10} &= \frac{1}{2} \psi_{2,0}^0, & \Gamma^0_{11} &= \frac{1}{2} \psi_{2,1}^0, & \Gamma^0_{12} &= 0, & \Gamma^0_{13} &= 0, \\
  \Gamma^0_{20} &= \frac{1}{2} \psi_{2,2}^0, & \Gamma^0_{21} &= 0, & \Gamma^0_{22} &= \frac{1}{2} \psi_{2,0}^0, & \Gamma^0_{23} &= 0, \\
  \Gamma^0_{30} &= \frac{1}{2} \psi_{2,3}^0, & \Gamma^0_{31} &= 0, & \Gamma^0_{32} &= 0, & \Gamma^0_{33} &= \frac{1}{2} \psi_{2,0}^0, \\
  \Gamma^1_{00} &= \frac{1}{2} \psi_{2,0}^1, & \Gamma^1_{01} &= \frac{1}{2} \psi_{2,1}^0, & \Gamma^1_{02} &= 0, & \Gamma^1_{03} &= 0, \\
  \Gamma^1_{10} &= \frac{1}{2} \psi_{2,0}^1, & \Gamma^1_{11} &= \frac{1}{2} \psi_{2,1}^1, & \Gamma^1_{12} &= \frac{1}{2} \psi_{2,2}^1, & \Gamma^1_{13} &= \frac{1}{2} \psi_{2,3}^1, \\
  \Gamma^1_{20} &= 0, & \Gamma^1_{21} &= \frac{1}{2} \psi_{2,2}^1, & \Gamma^1_{22} &= -\frac{1}{2} \psi_{2,1}^1, & \Gamma^1_{23} &= 0, \\
  \Gamma^1_{30} &= 0, & \Gamma^1_{31} &= \frac{1}{2} \psi_{2,3}^1, & \Gamma^1_{32} &= 0, & \Gamma^1_{33} &= -\frac{1}{2} \psi_{2,1}^1, \\
  \Gamma^2_{00} &= \frac{1}{2} \psi_{2,0}^2, & \Gamma^2_{01} &= 0, & \Gamma^2_{02} &= \frac{1}{2} \psi_{2,0}^2, & \Gamma^2_{03} &= 0, \\
  \Gamma^2_{10} &= 0, & \Gamma^2_{11} &= -\frac{1}{2} \psi_{2,2}^2, & \Gamma^2_{12} &= \frac{1}{2} \psi_{2,1}^1, & \Gamma^2_{13} &= 0, \\
  \Gamma^2_{20} &= \frac{1}{2} \psi_{2,0}^2, & \Gamma^2_{21} &= \frac{1}{2} \psi_{2,1}^0, & \Gamma^2_{22} &= \frac{1}{2} \psi_{2,2}^2, & \Gamma^2_{23} &= \frac{1}{2} \psi_{2,3}^2, \\
  \Gamma^2_{30} &= 0, & \Gamma^2_{31} &= 0, & \Gamma^2_{32} &= \frac{1}{2} \psi_{2,3}^0, & \Gamma^2_{33} &= -\frac{1}{2} \psi_{2,2}^2, \\
  \Gamma^3_{00} &= \frac{1}{2} \psi_{2,3}^0, & \Gamma^3_{01} &= 0, & \Gamma^3_{02} &= 0, & \Gamma^3_{03} &= \frac{1}{2} \psi_{2,0}^0, \\
  \Gamma^3_{10} &= 0, & \Gamma^3_{11} &= \frac{1}{2} \psi_{2,3}^0, & \Gamma^3_{12} &= 0, & \Gamma^3_{13} &= \frac{1}{2} \psi_{2,1}^0.
\end{align*}
\]
\[ \Gamma^3_{20} = 0 \quad \Gamma^3_{21} = 0 \quad \Gamma^3_{22} = -\frac{1}{2} \psi^{2,2} \quad \Gamma^3_{23} = \frac{1}{2} \psi^{2,3} \]

\[ \Gamma^3_{30} = \frac{1}{2} \psi^{2,0} \quad \Gamma^3_{31} = \frac{1}{2} \psi^{2,1} \quad \Gamma^3_{32} = \frac{1}{2} \psi^{2,2} \quad \Gamma^3_{33} = \frac{1}{2} \psi^{2,3} \]

**Appendix B.** Table for Einstein tensor \( G_{\alpha\beta} \) given the metric tensor \( g_{\alpha\beta} = e^{\psi x} \eta_{\alpha\beta} \). \( G \) is computed from the metric tensor using standard formulas from general relativity.

\[ G_{00} = \frac{1}{2} \psi_{2,0} \psi_{2,0} - \psi_{2,1}, - \psi_{2,2} - \psi_{2,3,3} + \frac{1}{4} \psi_{2,0} \psi_{2,0} - \frac{1}{4} \psi_{2,1} \psi_{2,1} - \frac{1}{4} \psi_{2,2} \psi_{2,2} - \frac{1}{4} \psi_{2,3} \psi_{2,3} \]

\[ G_{01} = \frac{1}{2} \psi_{2,0} \psi_{2,1} - \psi_{2,0,1} \]

\[ G_{02} = \frac{1}{2} \psi_{2,0} \psi_{2,2} - \psi_{2,0,2} \]

\[ G_{03} = \frac{1}{2} \psi_{2,0} \psi_{2,3} - \psi_{2,0,3} \]

\[ G_{10} = \frac{1}{2} \psi_{2,1} \psi_{2,0} - \psi_{2,1,0} \]

\[ G_{11} = \frac{1}{2} \psi_{2,1} \psi_{2,1} - \psi_{2,0,0} + \psi_{2,2,2} + \psi_{2,3,3} - \frac{1}{4} \psi_{2,0} \psi_{2,0} + \frac{1}{4} \psi_{2,1} \psi_{2,1} + \frac{1}{4} \psi_{2,2} \psi_{2,2} + \frac{1}{4} \psi_{2,3} \psi_{2,3} \]

\[ G_{12} = \frac{1}{2} \psi_{2,1} \psi_{2,2} - \psi_{2,1,2} \]

\[ G_{13} = \frac{1}{2} \psi_{2,1} \psi_{2,3} - \psi_{2,1,3} \]

\[ G_{20} = \frac{1}{2} \psi_{2,2} \psi_{2,0} - \psi_{2,2,0} \]

\[ G_{21} = \frac{1}{2} \psi_{2,2} \psi_{2,1} - \psi_{2,2,1} \]
\[ G_{22} = \frac{1}{2} \psi_{2,2} \psi_{2,2} - \psi_{2,0,0} + \psi_{2,1,1} + \psi_{2,3,3} - \frac{1}{4} \psi_{2,0} \psi_{2,0} + \frac{1}{4} \psi_{2,1} \psi_{2,1} + \frac{1}{4} \psi_{2,2} \psi_{2,2} + \frac{1}{4} \psi_{2,3} \psi_{2,3} \]

\[ G_{23} = \frac{1}{2} \psi_{2,2} \psi_{2,3} - \psi_{2,2,3} \]

\[ G_{30} = \frac{1}{2} \psi_{2,3} \psi_{2,0} - \psi_{2,3,0} \]

\[ G_{31} = \frac{1}{2} \psi_{2,3} \psi_{2,1} - \psi_{2,3,1} \]

\[ G_{32} = \frac{1}{2} \psi_{2,3} \psi_{2,2} - \psi_{2,3,2} \]

\[ G_{33} = \frac{1}{2} \psi_{2,3} \psi_{2,3} - \psi_{2,0,0} + \psi_{2,1,1} + \psi_{2,2,2} - \frac{1}{4} \psi_{2,0} \psi_{2,0} + \frac{1}{4} \psi_{2,1} \psi_{2,1} + \frac{1}{4} \psi_{2,2} \psi_{2,2} + \frac{1}{4} \psi_{2,3} \psi_{2,3} \]