A quantum-theoretic argument for timelessness

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If you want to understand the nature of time, you might get some help by looking at our most fundamental physical theories. There are two such: general relativity, and quantum field theory. Any accurate description of the universe — and in particular, of the nature of time — must somehow combine the insights of these two theories.

Some experts (e.g. Earman 2002, Rovelli 1991) argue that a universe that is both relativistic and quantum-theoretic must, of necessity, be an unchanging, essentially timeless universe. But the arguments for this claim often depend on intricacies of the the Hamiltonian formulation of Einstein's field equations.

In this note, I provide a more elementary argument that any quantum theory of spacetime will dispense with the traditional notion of the passage of time. In particular, I prove that quantum theory rules out the possibility of any quantity that one might call "the time interval between two events."

The mathematical fact on which my philosophical argument is based has long been known (see e.g. Pauli 1933), although I have given a more concise and transparent proof. However, philosophers seem to have been largely ignorant of this fact. I hypothesize that philosophers have felt entitled to ignore the matheamtical fact because it was usually interpreted as showing that, "time is not an *observable*." But that is to wildly understate the strength of the result! The result shows that, insofar as quantities are represented by operators, time is not a quantity — not even an unobservable quantity.

Assume then that we have a quantum theory whose state space is a vector space H with inner-product, and whose time evolution is represented by a group $\{u_t = e^{ith} : t \in \mathbb{R}\}$ of symmetries of H, where h is an operator that has spectrum bounded from below. In other words, we assume that there is a lower bound on energy.

Suppose now for reductio ad absurdum that for any interval (a, b) of real numbers, there is a subspace $s(a, b)$ of states that come about during that interval. Let $e(a, b)$ to represent the projection onto the subspace $s(a, b)$.

For any state v, applying the time-evolution operator u_t to v evolves the state forward by t (in whatever units of time we are using). Thus, if a state v is in the subspace $s(a, b)$, the evolved state $u_t v$ should be in $s(a+t, b+t)$. But a unitary operator u maps a subspace s onto a subspace s', i.e. $u(s) = s'$, if and only if $u^*(s') = s$. Hence, we should have $u_t e(a, b) = e(a+t, b+t)u_t$ for all a, b, t in $\mathbb{R}$. We are now ready to derive a contradiction from the assumption that there is a quantity called "time," whose value changes.

Lemma (Hegerfeldt 1994)**.** *Suppose that $u_t = e^{ith}$, where h is a half-bounded operator. Let e be a projection onto a subspace, and let $f(t) = \langle u_t v, e u_t v\rangle$. Then either $f(t) \neq 0$ on a dense open set, or $f(t) = 0$ for all t.*

Theorem. *Suppose that there is an assignment $(a, b) \mapsto s(a, b)$ of temporal intervals to subspaces of state space H such that:*

1. *$u_t s(a, b) = s(a+t, b+t)u_t$ for all $t \in \mathbb{R}$, and*
2. *$s(a, b)$ is orthogonal to $s(c, d)$ when (a, b) is disjoint from (c, d).*

Then $s(a, b) = 0$ for all (a, b).

Proof. Let v be a vector in $s(a, b)$. We will show that $v = 0$, in particular, $s(a, b)$ contains no unit vectors. Consider the function defined by

$$f(t) = \langle u_t v, e(a, b)u_t v\rangle = \langle v, e(a+t, b+t)v\rangle, \qquad (t \in \mathbb{R}).$$

Clearly f satisfies the hypotheses of Hegerfeldt's lemma. Furthermore, for all $t > |b - a|$,

$$f(t) = \langle v, e(a+t, b+t)v\rangle = \langle e(a, b)v, e(a+t, b+t)v\rangle = 0,$$

since the subspaces $s(a, b)$ and $s(a+t, b+t)$ are orthogonal. In particular, $f(t) = 0$ on an open set, and Hegerfeldt's lemma entails that $f(t) = 0$ for all $t \in \mathbb{R}$. In particular,

$$0 = f(0) = \langle v, e(a, b)v\rangle = \langle v, v\rangle,$$

hence $v = 0$. $\square$

Some have already grappled with the implications of this result (see Hilgevoord 2001). A common response is to claim that time *is* a quantity in quantum theory, but that it is represented by a parameter (c-number) rather than by an operator. But that distinction is merely verbal, and does nothing to help us understand the special role of time in quantum theory. What is the difference between quantities that can be represented by operators, and those — such as 'amount of time' — that cannot? And why is time the only such parametric quantity? What is special about time?

References

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