place to start is with G. E. Hughes & M. J. Cresswell, AN INTRODUCTION TO MODAL LOGIC (London, 1968). For tense logic, see A. N. Prior, TIME AND MODALITY (Oxford, 1957) and PAST, PRESENT AND FUTURE (Oxford, 1967); for epistemic logic, J. Hintikka, KNOWLEDGE AND BELIEF: AN INTRODUCTION TO THE LOGIC OF THE TWO NOTIONS (Ithaca, 1962); and for deontic logic, DEONTIC LOGIC: INTRODUCTORY AND SYSTEMATIC READINGS, ed. by R. Hilpinen (Dordrecht, 1971), especially the essays by D. Føllesdal & R. Hilpinen and J. Hintikka. Or, for a more elementary treatment of these topics, see Chapter Ten of R. L. Purtill, LOGIC: ARGUMENT, REFUTATION, AND PROOF (New York, 1979).

There are many other things one could do as well. What about imperatives and questions? Do they have "logics," and are they amenable to the same sort of formal treatment as are declarative sentences? And what about adverbs? From the truth of 'John walks slowly', for example, it follows that John walks, and our analysis of the logical behavior of adverbs ought to help explain this fact. But these questions must remain only questions for now. In the meantime, try your hand at the following exercises.

1 Prove the following sequents are sound patterns of argument using □E, RN and NC, together with the rules for PC.

(a) $\vdash P \rightarrow -\Box -P$

(b) $\Box P \vee \Box Q \vdash \Box(P \vee Q)$

(c) $\Box(P \,\&\, Q) \dashv\vdash \Box P \,\&\, \Box Q$

(d) $\Box(P \rightarrow Q), -\Box -P \vdash -\Box -Q$

(e) $\vdash -\Box -(P \rightarrow \Box P)$

(f) $-\Box -(P \vee Q) \dashv\vdash -\Box -P \vee -\Box -Q$

(g) $-\Box -(P \,\&\, Q) \vdash -\Box -P \,\&\, -\Box -Q$

2 Which of the rules □E, RN, NC, BP and □I are sound when '□' is read 'it is always the case that...'? As 'it is and always will be the case that...'? As 'it was the case that...'? As 'Smith knows that...'? As 'Smith believes that...'? As 'it is obligatory that...'? Are there any additonal rules governing '□' you would suggest on any of these readings?



## II. SOLUTIONS TO SELECTED EXERCISES FROM THE TEXT

Chapter 1, sec. 2

1 (d)

| | | | |
|---|---|---|---|
| 1 | (1) | $--Q \rightarrow P$ | A |
| 2 | (2) | $-P$ | A |
| 1, 2 | (3) | $---Q$ | 1, 2 MTT |
| 1, 2 | (4) | $-Q$ | 3 DN |

(g)

| | | | |
|---|---|---|---|
| 1 | (1) | $-P \rightarrow Q$ | A |
| 2 | (2) | $-Q$ | A |
| 1, 2 | (3) | $--P$ | 1, 2 MTT |
| 1, 2 | (4) | $P$ | 3 DN |
| 1 | (5) | $-Q \rightarrow P$ | 2, 4 CP |

(j)

| | | | |
|---|---|---|---|
| 1 | (1) | $P \rightarrow (Q \rightarrow R)$ | A |
| 2 | (2) | $P \rightarrow Q$ | A |
| 3 | (3) | $P$ | A |
| 1, 3 | (4) | $Q \rightarrow R$ | 1, 3 MPP |
| 2, 3 | (5) | $Q$ | 2, 3 MPP |
| 1, 2, 3 | (6) | $R$ | 4, 5 MPP |
| 1, 2 | (7) | $P \rightarrow R$ | 3, 6 CP |
| 1 | (8) | $(P \rightarrow Q) \rightarrow (P \rightarrow R)$ | 2, 7 CP |

(n)

| | | | |
|---|---|---|---|
| 1 | (1) | $P$ | A |
| 2 | (2) | $-(Q \rightarrow R) \rightarrow -P$ | A |
| 3 | (3) | $-R$ | A |
| 1 | (4) | $--P$ | 1 DN |
| 1, 2 | (5) | $--(Q \rightarrow R)$ | 2, 4 MTT |
| 1, 2 | (6) | $Q \rightarrow R$ | 5 DN |

1, 2, 3 (7) -Q    3, 6 MTT
1, 2 (8) -R → -Q    3, 7 CP
1 (9) (-(Q → R) → -P) → (-R → -Q)    2, 8 CP

Chapter 1, sec. 3

1 (c)
1 (1) (P → Q) & (P → R)    A
1 (2) P → Q    1 &E
1 (3) P → R    1 &E
4 (4) P    A
1, 4 (5) Q    2, 4 MPP
1, 4 (6) R    3, 4 MPP
1, 4 (7) Q & R    5, 6 &I
1 (8) P → (Q & R)    4, 7 CP

(f)
1 (1) (P → R) & (Q → R)    A
1 (2) P → R    1 &E
1 (3) Q → R    1 &E
4 (4) P v Q    A
5 (5) P    A
1, 5 (6) R    2, 5 MPP
7 (7) Q    A
1, 7 (8) R    3, 7 MPP
1, 4 (9) R    4, 5, 6, 7, 8 vE
1 (10) (P v Q) → R    4, 9 CP

(j)
1 (1) -P → P    A
2 (2) -P    A
1, 2 (3) P    1, 2 MPP
1, 2 (4) P & -P    2, 3 &I
1 (5) --P    2, 4 RAA
1 (6) P    5 DN

Chapter 1, sec. 4

1 (d)
1 (1) -P ↔ -Q    A



1 (2) (-P → -Q) & (-Q → -P)    1 Df. ↔
1 (3) -P → -Q    2 &E
4 (4) Q    A
4 (5) --Q    4 DN
1, 4 (6) --P    3, 5 MTT
1, 4 (7) P    6 DN
1 (8) Q → P    4, 7 CP
1 (9) -Q → -P    2 &E
10 (10) P    A
10 (11) --P    10 DN
1, 10 (12) --Q    9, 11 MTT
1, 10 (13) Q    12 DN
1 (14) P → Q    10, 13 CP
1 (15) (P → Q) & (Q → P)    8, 14 &I
1 (16) P ↔ Q    15 Df. ↔

(f)
1 (1) P ↔ -Q    A
2 (2) Q ↔ -R    A
1 (3) (P → -Q) & (-Q → P)    1 Df. ↔
2 (4) (Q → -R) & (-R → Q)    2 Df. ↔
1 (5) P → -Q    3 &E
6 (6) P    A
1, 6 (7) -Q    5, 6 MPP
2 (8) -R → Q    4 &E
1, 2, 6 (9) --R    7, 8 MTT
1, 2, 6 (10) R    9 DN
1, 2 (11) P → R    6, 10 CP
12 (12) R    A
2 (13) Q → -R    4 &E
12 (14) --R    12 DN
2, 12 (15) -Q    13, 14 MTT
1 (16) -Q → P    3 &E
1, 2, 12 (17) P    15, 16 MPP
1, 2 (18) R → P    12, 17 CP
1, 2 (19) (P → R) & (R → P)    11, 18 &I
1, 2 (20) P ↔ R    19 Df. ↔

| | | | |
|---|---|---|---|
| 2 (b) | 1 (1) | P * Q | A |
| | 2 (2) | P * R | A |
| | 1 (3) | -P → Q | 1 Df. * |
| | 2 (4) | -P → R | 2 Df. * |
| | 5 (5) | -P | A |
| | 1, 5 (6) | Q | 3, 5 MPP |
| | 2, 5 (7) | R | 4, 5 MPP |
| | 1, 2, 5 (8) | Q & R | 6, 7 &I |
| | 1, 2 (9) | -P → (Q & R) | 5, 8 CP |
| | 1, 2 (10) | P * (Q & R) | 9 Df. * |

| | | | |
|---|---|---|---|
| (e) | 1 (1) | -P * R | A |
| | 2 (2) | -Q * R | A |
| | 3 (3) | P v Q | A |
| | 1 (4) | --P → R | 1 Df. * |
| | 2 (5) | --Q → R | 2 Df. * |
| | 6 (6) | P | A |
| | 6 (7) | --P | 6 DN |
| | 1, 6 (8) | R | 4, 7 MPP |
| | 9 (9) | Q | A |
| | 9 (10) | --Q | 9 DN |
| | 2, 9 (11) | R | 5, 10 MPP |
| | 1, 2, 3 (12) | R | 3, 6, 8, 9, 11 vE |

Chapter 1, sec. 5

| | | | |
|---|---|---|---|
| 1 (d) | 1 (1) | P v (Q & R) | A |
| | 2 (2) | P | A |
| | 2 (3) | P v Q | 2 vI |
| | 2 (4) | P v R | 2 vI |
| | 2 (5) | (P v Q) & (P v R) | 3, 4 &I |
| | 6 (6) | Q & R | A |
| | 6 (7) | Q | 6 &E |
| | 6 (8) | P v Q | 7 vI |
| | 6 (9) | R | 6 &E |



| | | |
|---|---|---|
| 6 (10) | P v R | 9 vI |
| 6 (11) | (P v Q) & (P v R) | 8, 10 &I |
| 1 (12) | (P v Q) & (P v R) | 1, 2, 5, 6, 11 vE |

| | | |
|---|---|---|
| 1 (1) | (P v Q) & (P v R) | A |
| 1 (2) | P v Q | 1 &E |
| 1 (3) | P v R | 1 &E |
| 4 (4) | P | A |
| 4 (5) | P v (Q & R) | 4 vI |
| 6 (6) | Q | A |
| 7 (7) | R | A |
| 6, 7 (8) | Q & R | 6, 7 &I |
| 6, 7 (9) | P v (Q & R) | 8 vI |
| 1, 6 (10) | P v (Q & R) | 3, 4, 5, 7, 9 vE |
| 1 (11) | P v (Q & R) | 2, 4, 5, 6, 10 vE |

| | | | |
|---|---|---|---|
| (f) | 1 (1) | -(P v Q) | A |
| | 2 (2) | P | A |
| | 2 (3) | P v Q | 2 vI |
| | 1, 2 (4) | (P v Q) & -(P v Q) | 1, 3 &I |
| | 1 (5) | -P | 2, 4 RAA |
| | 6 (6) | Q | A |
| | 6 (7) | P v Q | 6 vI |
| | 1, 6 (8) | (P v Q) & -(P v Q) | 1, 7 &I |
| | 1 (9) | -Q | 6, 8 RAA |
| | 1 (10) | -P & -Q | 5, 9 &I |

| | | |
|---|---|---|
| 1 (1) | -P & -Q | A |
| 2 (2) | P v Q | A |
| 3 (3) | P | A |
| 1 (4) | -P | 1 &E |
| 1, 3 (5) | P & -P | 3, 4 &I |
| 3 (6) | -(-P & -Q) | 1, 5 RAA |
| 7 (7) | Q | A |

| | | | |
|---|---|---|---|
| 1 | (8) | -Q | 1 &E |
| 1, 7 | (9) | Q & -Q | 7, 8 &I |
| 7 | (10) | -(-P & -Q) | 1, 9 RAA |
| 2 | (11) | -(-P & -Q) | 2, 3, 6, 7, 10 vE |
| 1, 2 | (12) | (-P & -Q) & -(-P & -Q) | 1, 11 &I |
| 1 | (13) | -(P v Q) | 2, 12 RAA |

<u>Chapter 2, sec. 2</u>

2 (a) 1, (c) 13, (e) 51

5 (b)

| | | | |
|---|---|---|---|
| | (1) | Q v -Q | TI(S) 44 |
| 2 | (2) | Q | A |
| 2 | (3) | P → Q | 2 SI(S) 50 |
| 2 | (4) | (P → Q) v (Q → R) | 3 vI |
| 5 | (5) | -Q | A |
| 5 | (6) | Q → R | 5 SI(S) 51 |
| 5 | (7) | (P → Q) v (Q → R) | 6 vI |
| | (8) | (P → Q) v (Q → R) | 1, 2, 4, 5, 7 vE |

(f)

| | | | |
|---|---|---|---|
| 1 | (1) | P v Q | A |
| 2 | (2) | -P | A |
| 1, 2 | (3) | Q | 1, 2 SI 52 |
| 1 | (4) | -P → Q | 2, 3 CP |

(i)

| | | | |
|---|---|---|---|
| 1 | (1) | P → (Q v R) | A |
| 1 | (2) | -P v (Q v R) | 1 SI(S) Ex. 1.5.1(i) |
| 3 | (3) | -P | A |
| 3 | (4) | P → Q | 3 SI 51 |
| 3 | (5) | (P → Q) v (P → R) | 4 vI |
| 6 | (6) | Q v R | A |
| 7 | (7) | Q | A |



| | | | |
|---|---|---|---|
| 7 | (8) | P → Q | 7 SI(S) 50 |
| 7 | (9) | (P → Q) v (P → R) | 8 vI |
| 10 | (10) | R | A |
| 10 | (11) | P → R | 10 SI(S) 50 |
| 10 | (12) | (P → Q) v (P → R) | 11 vI |
| 6 | (13) | (P → Q) v (P → R) | 6, 7, 9, 10, 12 vE |
| 1 | (14) | (P → Q) v (P → R) | 2, 3, 5, 6, 13 vE |

(l)

| | | | |
|---|---|---|---|
| 1 | (1) | -Q | A |
| 1 | (2) | Q → P | 1 SI(S) 51 |
| 1 | (3) | P v Q ↔ P | 2 SI Ex. 2.2.3(b) |

6 (i) Suppose A ⊢ B is provable. Then:

| | | | |
|---|---|---|---|
| 1 | (1) | A | A |
| 1 | (2) | B | 1 SI |
| | (3) | A → B | 1, 2 CP |

Hence ⊢ A → B is provable. [The other half of 6(i) is similar.]

7 See the solution to <u>Chapter 2, sec. 2</u>, Ex. 3 of Part I.

<u>Chapter 2, sec. 3</u>

1 (ii) (i) is contingent. '((P → P) & (P → P) → (P → P)) → ((P → P) → (P → P)) & ((P → P) → (P → P))' is tautologous and '((P → P) & -(P → P) → -(P → P)) → ((P → P) → -(P → P)) & (-(P → P) → -(P → P))' is inconsistent.

2 Define '→' in terms of '&' and '-' (or in terms of 'v' and '-') and then use the expressions (a)-(p) given by Lemmon on pp. 71-2.

5 Suppose A has the value T. Then, if A and B are contraries, B has the value F and -B has the value T. Hence A implies -B.

Likewise, B implies -A.

If A and B cannot both have the value T, then -A and -B cannot both have the value F. Hence -A and -B are subcontraries if A and B are contraries.

6 (i)

| A | a | b | c | d |
|---|---|---|---|---|
| T | T | T | F | F |
| F | T | F | T | F |

(a) A → A
(b) -(A → A)
(c) A → -A
(d) -(A → -A)

(ii) 256, $2^{2^n}$

Chapter 3, sec. 1

1 (m) (∃x)Kxx

(p) [This one is ambiguous.]
There is some poor victim who was gotten by everyone.
(∃x)(y)Kxy
Everyone is a killer.
(x)(∃y)Kxy

(s) (∃x)(Gx & (y)(Sy → Lxy))

(v) [This one is ambiguous.]
There is at least one individualistic sport which Tom likes.
(∃x)(Ix & Sx & Lmx)
Tom likes all individualistic sports.
(x)(Ix & Sx → Lmx)



(x) [This one is triply ambiguous!]
Some boys like very few things--only fast-moving sports.
(∃x)(Bx & (y)(Lxy → Fy & Sy))
Some boys are such that the only sports they like are fast-moving ones.
(∃x)(Bx & (y)(Sy & Lxy → Fy))
Some boys are such that the only fast-moving things they like are sports.
(∃x)(Bx & (y)(Fy & Lxy → Sy))

Chapter 3, sec. 2

1 (e) (x)(Fx & -Px → Kx), Fm, -Km ⊢ Pm

| | | | |
|---|---|---|---|
| 1 | (1) | (x)(Fx & -Px → Kx) | A |
| 2 | (2) | Fm | A |
| 3 | (3) | -Km | A |
| 1 | (4) | Fm & -Pm → Km | 1 UE |
| 5 | (5) | -Pm | A |
| 2, 5 | (6) | Fm & -Pm | 2, 5 &I |
| 1, 2, 5 | (7) | Km | 4, 6 MPP |
| 1, 2, 3, 5 | (8) | Km & -Km | 3, 7 &I |
| 1, 2, 3 | (9) | --Pm | 5, 8 RAA |
| 1, 2, 3 | (10) | Pm | 9 DN |

2 (i)(c)

| | | | |
|---|---|---|---|
| 1 | (1) | (x)(Fx → Gx) | A |
| 2 | (2) | (x)(Hx → -Gx) | A |
| 3 | (3) | Fa | A |
| 1 | (4) | Fa → Ga | 1 UE |
| 2 | (5) | Ha → -Ga | 2 UE |
| 1, 3 | (6) | Ga | 3, 4 MPP |
| 1, 3 | (7) | --Ga | 6 DN |
| 1, 2, 3 | (8) | -Ha | 5, 7 MTT |
| 1, 2 | (9) | Fa → -Ha | 3, 8 CP |