Final Exam Practice Problems.

Short answer

1. What is included in a predicate calculus interpretation of a collection $A_1, \ldots, A_n$ of sentences?
2. Complete the following sentences:
   (a) A universal sentence "$(x)\phi$" true in an interpretation $\mathcal{I}$ just in case ...
   (b) Predicate logic sentences A and B are inconsistent just in case ...
3. Grade the following "proof."

1	(1)	$P \vee Q$	A
2	(2)	P	A
3	(3)	Q	A
2,3	(4)	$P \& Q$	2,3 &I
2,3	(5)	P	4 &E
1	(6)	P	1,2,2,3,5 $\vee$E

Translation

Translate the following sentences into predicate calculus notation. Use the following dictionary.

$Gxy \equiv x$ gets y $\quad$ $Wxy \equiv x$ wants y
$Px \equiv x$ is a person

1. If everyone wants something, then no one gets it.
2. Some people want only things they don't get.
3. There is something that nobody wants.

Proofs

1. Prove the following. You may use any of the rules of inference.

(1) $(\exists x)(Fx \,\&\, (y)(Gy \to Rxy))$
(2) $(x)(Fx \to (y)(Hy \to -Rxy))$ $\qquad$ $/ \ (x)(Gx \to -Hx)$

2. Prove the following theorem of the predicate calculus. You may use any of the rules of inference.

$$// \qquad ((\exists x)Fx \to P) \to (x)(Fx \to P)$$

Semantics

1. Use the algorithm for pure monadic sentences ("Algorithm C") to determine whether or not the following argument is valid. Show all steps. If the argument is invalid, give an interpretation that makes the premises true and conclusion false.

   (1) $(\exists x)Gx \vee -(x)Fx$

   (2) $-(x)(Fx \to -Fx) \qquad / \ (\exists x)Fx \to (\exists x)Gx$

2. Give an interpretation that shows that the following argument is invalid.

   (1) $(x)(Fx \to (\exists y)Rxy) \qquad / \ (\exists y)(x)(Fx \to Rxy)$

Reflection

For the following problems, you will give rigorous (but informal) arguments.

1. Show that logical implication (as defined by predicate calculus interpretations) is transitive. i.e., if A logically implies B, and B logically implies C, then A logically implies C.

2. State precisely what it means to say that the Predicate Calculus is "sound" and "complete." (i.e., state the Soundness and Completeness Theorems for the Predicate Calculus.) Prove the soundness of the rule Conditional Proof (CP).