Phil 201
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Handout 4

Proof Strategies

Type-specific strategies:

To prove a…	**you could…**
Conditional	Assume the antecedent, derive the consequent, use CP.
Disjunction	Prove one disjunct, use vI.
Conjunction	Prove each conjunct, use &I.
Negation	Assume the un-negated sentence, derive a contradiction, use RAA.

Type-general strategies:

To prove…	**you could…**
any sentence φ	assume $-\varphi$, derive a contradiction, use RAA.
any sentence φ	derive a conditional of the form $\psi \rightarrow \varphi$, derive ψ, use MPP.
any sentence φ	derive a conditional of the form $-\varphi \rightarrow \psi$, derive $-\psi$, use MTT and DN.
any sentence φ from a disjunction	assume each disjunct, prove φ from each disjunct, use vE.

Questions to ask when doing a proof:

1) What type of sentence is the conclusion? Which type-specific strategies could I use? Which type-general strategies could I use?

2) Which sentences do I have at my disposal? What could I prove from them now? What would be useful to have?

3)Have I used all of the given sentences?

4)Which sentences are equivalent to the sentence I need to prove? Is there an easy way to get from what I've already got to the equivalent sentence? Is there an easy way to get from the equivalent sentence to the conclusion? Some helpful equivalences to keep in mind: $(\varphi \rightarrow \psi) = (-\psi \rightarrow -\varphi)$, $-(\varphi \& \psi) = (-\varphi \vee -\psi)$,
$-(\varphi \vee \psi) = (-\varphi \& -\psi)$

5)Do I have any assumptions that I need to discharge? How should I discharge them? Of the rules that allow me to discharge assumptions (RAA, CP, vE), which are available to me?

Miscellaneous tips:

If you have a contradiction, use it. Remember that you can use RAA to prove the negation of any assumption at all, once you've got a contradiction. Look at all of the assumptions you've got. See what you can prove. If you need to prove something specific, just assume its negation.

If you need to prove a disjunction and it's the last problem on the exam, there's a good chance that you'll not be able to prove one disjunct and use vI, since that's an easy way to get a disjunction. You'll probably need to use one of the type-general strategies. This goes for other types of sentences as well.

Whenever you see a disjunction, especially a disjunction embedded in a more complex sentence like $-(\varphi \vee \psi)$ or $(P \rightarrow (Q \rightarrow (\varphi \vee \psi)))$, remember that you can assume one of the disjuncts and use vI to get the full disjunction. That might give you something useful to work with.

Whenever you make an assumption, you'll need to discharge it. At the point that you introduce the assumption, you should have a plan for how you're going to discharge it. Will you use CP, RAA, or vE?

If you need to get a conditional at some point in a proof, make a list of all of the possible conditionals you could get using CP. Remember that any assumption you have could be the antecedent, and any line at all—assumption or derivation—could be the consequent.

If you're deep into a long proof, where you've introduced lots of assumptions and subsequently discharged them, it's sometimes a good idea to write down on scrap paper all of the steps in the proof that you still have at your disposal—that is, all the ones with the dependency numbers that you haven't gotten rid of yet. This will make it easier to see what you've got to work with.