

Some Useful Validities and Equivalences.

1. Tautology Introduction

$$\vdash A \vee \neg A$$

2. Disjunctive Syllogism

$$A \vee B, \neg A \vdash B$$

$$A \vee B, \neg B \vdash A$$

3. Material Implication

$$A \rightarrow B \dashv\vdash \neg A \vee B$$

4. Negated Material Implication

$$\neg(A \rightarrow B) \dashv\vdash A \& \neg B$$

5. Ex Falso Quodlibet

$$A, \neg A \vdash B$$

Note: You can easily derive the following by using Ex Falso and CP:

$$\neg A \vdash A \rightarrow B$$

6. Irrelevant Antecedent

$$B \vdash A \rightarrow B$$

7. DeMorgan's Laws

$$\neg(A \vee B) \dashv\vdash \neg A \& \neg B$$

$$\neg(A \& B) \dashv\vdash \neg A \vee \neg B$$

8. Distributive Laws

$$A \& (B \vee C) \dashv\vdash (A \& B) \vee (A \& C)$$

$$A \vee (B \& C) \dashv\vdash (A \vee B) \& (A \vee C)$$

9. Transposition

$$A \rightarrow B \dashv\vdash \neg B \rightarrow \neg A$$

10. Hypothetical Syllogism

$$A \rightarrow B, B \rightarrow C \vdash A \rightarrow C$$

11. Exportation

$$(A \& B) \rightarrow C \dashv\vdash A \rightarrow (B \rightarrow C)$$

12. Consequentia Mirabilis

$$A \rightarrow \neg A \dashv\vdash \neg A$$

13. Modus Ponendo Tollens

$$\neg(A \& B), A \vdash \neg B$$

14. Redundancy

$$A \vee A \dashv\vdash A$$

$$A \& A \dashv\vdash A$$