Practice Final Exam

Please make sure that you have all three pages of the exam. Write your name, your preceptor's name, and your pledge on your exam booklet. You have three hours to complete the exam.

Short answer

1. State the Existential Elimination (EE) rule, along with each of its restrictions.
2. True or False (explain your answer): If $\phi_1, \ldots, \phi_n$ are inconsistent predicate calculus sentences, then there is a correctly written proof whose premises are $\phi_1, \ldots, \phi_n$ and whose conclusion is $P \,\&\, -P$.
3. Complete the following sentence: Predicate logic sentences ϕ and ψ are inconsistent just in case . . . (Please give the *semantic* definition that uses the concept of an "interpretation.")
4. Grade the following proof.

1	(1)	$P \vee Q$	A
2	(2)	P	A
3	(3)	Q	A
2,3	(4)	$P \,\&\, Q$	2,3 &I
2,3	(5)	P	4 &E
1	(6)	P	1,2,2,3,5 $\vee$E

5. Grade the following proof.

1	(1) $-P$	A
2	(2) $(\exists x)(Fx \,\&\, -Fx)$	A
2	(3) $- - P$	1,2 RAA
2	(4) P	3 DN
	(5) $(\exists x)(Fx \,\&\, -Fx) \rightarrow P$	2,4 CP

6. A "bad line" in a proof is a line where the sentence on the right is not a logical consequence of its dependencies. Identify all the bad lines in the previous two proofs.

Translation

Translate the following sentences into predicate calculus notation. Use the following dictionary.

$Mx \equiv x$ is male $\quad Pxy \equiv x$ is a parent of $y \quad Axy \equiv x$ adores y
$Ixy \equiv x$ is identical to $y \quad b =$ Bob

(The domain of discourse is persons — you do not need a predicate symbol for "is a person." For the purposes of this problem, a "child" is anyone who has a parent.)

1. Every man who has a son adores him.
2. Every man who has a daughter adores his daughter's mother.
3. Everybody adores their own grandchildren.
4. Every woman adores her brothers' children.
5. No man adores children unless he has his own.
6. Bob has at most three children.

Proofs and Counterexamples

1. Prove the following tautology of the propositional calculus:

$$-(P \to Q) \leftrightarrow (P \,\&\, -Q).$$

2. Prove the validity of the following argument. You may use any of the rules of inference.

   1. $(\exists x)(Fx \,\&\, (y)(Gy \to Rxy))$
   2. $(x)(Fx \to (y)(Hy \to -Rxy)) \qquad // \; (x)(Gx \to -Hx)$

3. Consider the sentence "$(x)(y)[Qxy \leftrightarrow (z)(Rzx \to Rzy)]$".
   (a) Show by giving a proof that this sentence implies "$(x)Qxx$".
   (b) Give an interpretation that shows that the sentence does not imply "$(x)(y)(Qxy \to Qyx)$".
   (c) The sentence implies one of (i) and (ii) but not the other; give a proof to show the implication in the one case, and give an interpretation to show the lack of implication in the other:
      (i) $(\exists y)(x)Rxy \to (\exists y)(x)Qxy$
      (ii) $(\exists y)(x)Qxy \to (\exists y)(x)Rxy$

4. Use some reliable method (e.g., Algorithm C) to determine whether or not the following argument is valid. Display your reasoning; and if the argument is invalid, give a witnessing interpretation.

   1. $(\exists x)Gx \vee -(x)Fx$
   2. $-(x) - Fx \rightarrow -(x)Fx \qquad // \ (\exists x)Fx \rightarrow (\exists x)Gx$

Metatheory

For the following problems, please give rigorous (but informal) arguments.

1. Use "proof by induction" to show that the connective "$\rightarrow$" is not by itself truth-functionally complete.

2. State precisely what it means to say that the predicate calculus is "sound" and "complete." (i.e., state the soundness and completeness theorems for the predicate calculus.) Prove the soundness of the Disjunction Elimination ($\vee$E) rule.

3. Let's say that a "schminterpretation" of a predicate calculus sentence is an interpretation whose domain has *at most two elements*; and let's say that a "schmautology" is a sentence that is true relative to all schminterpretations. Give an example of a schmautology that is not a tautology. [Hint: Do *not* introduce a symbol for "x is not equal to y," because predicate calculus relation symbols can always be reinterpreted.]

Extra Credit

1. Prove that there is no pure monadic sentence that is true relative to all and only those interpretations whose domains have exactly n elements. [Hint: Show that if a pure monadic sentence ϕ is true relative to an interpretation of size n (i.e., an interpretation whose domain has n elements), and $n < m$, then ϕ is true relative to an interpretation of size m.]
2. Prove that if a simple monadic sentence is consistent, then it is true relative to some interpretation whose domain has a *finite* number of elements. [Hint: You may assume that Algorithm B is a reliable test for consistency.]