

**Homework 2.**

1. Prove that the following arguments are valid. You may use only the following rules: MPP, MTT, DN, &I, &E,  $\forall$ I, the Rule of Assumptions (A), and Conditional Proof (CP). [None of these proofs requires Reductio ad Absurdum. You will get no points for a proof that uses RAA.] *You must list dependency numbers for each line of your proof.*

$$\begin{array}{l} \text{(a) (1) } (D \& E) \rightarrow \neg F \\ \quad (2) \ D \rightarrow E \quad // \ D \rightarrow \neg F \end{array}$$

$$\begin{array}{l} \text{(b) (1) } E \rightarrow (F \rightarrow G) \\ \quad (2) \ H \rightarrow (G \rightarrow I) \\ \quad (3) \ (F \rightarrow I) \rightarrow (H \rightarrow J) \quad // \ (E \& H) \rightarrow J \end{array}$$

$$\text{(c) (1) } \neg(A \& B) \rightarrow \neg(C \vee D) \quad // \ C \rightarrow A$$

$$\text{(d) (1) } \neg(P \vee Q) \quad // \ \neg P$$

$$\text{(e) (1) } \neg P \quad // \ P \rightarrow Q$$

$$\begin{array}{l} \text{(f) } \textit{Extra Credit} \\ \quad (1) \ P \rightarrow \neg P \quad // \ \neg P \end{array}$$

2. Give an informal counterexample to the following invalid argument form.

$$(1) \ P \rightarrow Q \quad // \ Q \rightarrow P$$

3. Is the following argument valid or invalid? If it is valid, prove it. If it is invalid, give an informal counterexample.

$$(1) \quad (P \rightarrow R) \vee (Q \rightarrow R) \quad // \quad (P \vee Q) \rightarrow R$$

4. Prove that the following arguments are valid. You may use any of the Stage 1 rules of inference plus the Rule of Assumptions (A) and  $\vee$ -Elimination ( $\vee$ E). You must list dependency numbers for each line of your proof.

$$\begin{aligned} \text{(a)} \quad (1) \quad & R \vee S \\ (2) \quad & \neg P \rightarrow \neg R \\ (3) \quad & S \rightarrow Q \quad // \quad Q \vee P \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad (1) \quad & P \vee Q \\ (2) \quad & P \vee R \quad // \quad P \vee (Q \& R) \end{aligned}$$