Homework 3.

1. Prove that the following arguments are valid. You may use any of the Stage 1 rules of inference (MPP, MTT, DN, &I, &E, $\vee$I), plus the Rule of Assumptions (A) and Reductio ad Absurdum (RAA).

   (a) (1) $-(P \,\&\, Q)$ $\quad$ $// \; -P \vee -Q$

   (b) (1) $-P \rightarrow Q$ $\quad$ $// \; P \vee Q$

2. Prove that the following arguments are valid. You may use any of the rules of inference that we have learned.

   (a) (1) $(P \rightarrow Q) \vee (P \rightarrow R)$ $\quad$ $// \; P \rightarrow (Q \vee R)$

   (b) (1) $(P \rightarrow Q) \rightarrow Q$ $\quad$ $// \; P \vee Q$

   (c) (1) $-P \vee Q$ $\quad$ $// \; P \rightarrow Q$

3. Prove the following theorem. You may use of any the rules of inference that we have learned.

   $// \quad (P \rightarrow Q) \vee (Q \rightarrow P)$

4. Write out a full truth table following sentence. Highlight in some way (e.g., draw a circle around) the column under the major operator of the sentence.

   $-(P \vee R) \,\&\, (-Q \rightarrow (P \,\&\, R))$

5. Determine whether the following arguments are valid. If an argument isn't valid, give a truth-assignment that witnesses this fact.

   (a) (1) $(P \to Q) \vee (Q \to R)$
   (2) $-R \to -(P \,\&\, Q)$ $\quad // \; Q \to -P$

   (b) (1) $(P \vee Q) \to (R \vee S)$
   (2) $P \leftrightarrow -(R \,\&\, S)$
   (3) $Q \leftrightarrow -(P \,\&\, R)$ $\quad // \; (S \,\&\, P) \to -(P \vee R)$

6. Determine whether each of the following sentences is consistent. If a sentence is consistent, give an assignment of truth values to its elementary sentences relative to which the sentence is true.

   (a) $(P \vee -Q) \to (P \leftrightarrow (Q \,\&\, R))$
   (b) $(-P \vee (-Q \to R)) \to ((P \,\&\, R) \to -Q)$

7. For each of the following pairs of sentences, determine whether the first sentence implies the second. If the implication fails to hold, give a truth-assignment that witnesses this fact.

   (a) $(P \,\&\, Q) \leftrightarrow (Q \,\&\, R)$ $\quad P \leftrightarrow Q$
   (b) $P \leftrightarrow (Q \vee R)$ $\quad -P \to (Q \leftrightarrow R)$

8. Show that for any sentences ϕ, ψ, the sentence $-(\phi \to \psi)$ is logically equivalent to the sentence $\phi \,\& \, -\psi$.

9. Is logical implication symmetric? That is, if ϕ implies ψ then does ψ imply ϕ? Explain your answer.