Homework 3.

1. Prove that the following arguments are valid. You may use any of the Stage 1 rules of inference (MPP, MTT, DN, &I, &E, ∨I), plus the Rule of Assumptions (A) and Reductio ad Absurdum (RAA).

   (a) (1) \(-(P & Q)\) \hspace{1cm} // \hspace{1cm} -P \lor -Q

   (b) (1) \(-P \rightarrow Q\) \hspace{1cm} // \hspace{1cm} P \lor Q

2. Prove that the following arguments are valid. You may use any of the rules of inference that we have learned.

   (a) (1) \((P \rightarrow Q) \lor (P \rightarrow R)\) \hspace{1cm} // \hspace{1cm} P \rightarrow (Q \lor R)

   (b) (1) \((P \rightarrow Q) \rightarrow Q\) \hspace{1cm} // \hspace{1cm} P \lor Q

   (c) (1) \(-P \lor Q\) \hspace{1cm} // \hspace{1cm} P \rightarrow Q

3. Prove the following theorem. You may use of any the rules of inference that we have learned.

   // \hspace{1cm} (P \rightarrow Q) \lor (Q \rightarrow P)

4. Write out a full truth table following sentence. Highlight in some way (e.g., draw a circle around) the column under the major operator of the sentence.

   \(-(P \lor R) \& (-Q \rightarrow (P \& R))\)
5. Determine whether the following arguments are valid. If an argument isn’t valid, give a truth-assignment that witnesses this fact.

(a) (1) \((P \rightarrow Q) \lor (Q \rightarrow R)\)
(2) \(-R \rightarrow -(P \land Q)\)  // \(Q \rightarrow -P\)

(b) (1) \((P \lor Q) \rightarrow (R \lor S)\)
(2) \(P \leftrightarrow -(R \& S)\)
(3) \(Q \leftrightarrow -(P \& R)\)  // \((S \& P) \rightarrow -(P \lor R)\)

6. Determine whether each of the following sentences is consistent. If a sentence is consistent, give an assignment of truth values to its elementary sentences relative to which the sentence is true.

(a) \((P \lor -Q) \rightarrow (P \leftrightarrow (Q \& R))\)
(b) \((-P \lor (-Q \rightarrow R)) \rightarrow ((P \& R) \rightarrow -Q)\)

7. For each of the following pairs of sentences, determine whether the first sentence implies the second. If the implication fails to hold, give a truth-assignment that witnesses this fact.

(a) \((P \& Q) \leftrightarrow (Q \& R)\) \(P \leftrightarrow Q\)
(b) \(P \leftrightarrow (Q \lor R)\) \(-P \rightarrow (Q \leftrightarrow R)\)

8. Show that for any sentences \(\phi, \psi\), the sentence \(- (\phi \rightarrow \psi)\) is logically equivalent to the sentence \(\phi \& - \psi\).

9. Is logical implication symmetric? That is, if \(\phi\) implies \(\psi\) then does \(\psi\) imply \(\phi\)? Explain your answer.