Homework 5

1. Translate the following sentences into predicate logic notation. You may use the suggested upper case letters for predicates, and lower case letters for names.
   (a) Some people born in Ontario are US citizens. \((O, U)\)
   (b) All people born in Alaska are US citizens. \((A, U)\)
   (c) No person born in Ontario is eligible to become the US President. \((O, E)\)
   (d) Some Princeton students are not US Citizens. \((P, U)\)
   (e) Rob is a tall Canadian. \((r, T, C)\)
   (f) Only Princeton students may attend this lecture. \((L, P)\)
   (g) Some things are round and some things are square, but there are no round squares. \((R, S)\)
   (h) Dogs and cats are good housepets. \((D, C, G)\)

2. Show that the following arguments are valid by constructing formal proofs. You may use any of the inference rules from the first half of the semester, as well as Universal Elimination (UE) and Universal Introduction (UI).
   (a) (1) \((x)(Fx \rightarrow Gx)\)  
        (2) \((x)(Hx \rightarrow \neg Gx)\)  // \((x)(Fx \rightarrow \neg Hx)\)
   (b) (1) \((x)((Fx \lor Gx) \rightarrow Hx)\)  
        (2) \((x)\neg Hx\)  // \((x)\neg Fx\)
   (c) (1) \((x)(Fx \rightarrow Gx)\)  // \((x)Fx \rightarrow (x)Gx\)
   (d) (1) \((x)\neg Fx\)  // \(\neg (x)Fx\)
3. Show that the following arguments are valid by constructing formal proofs. You may use any of the inference rules we have learned to date (including Existential Introduction (EI) and Existential Elimination (EE)).

(a) (1) \((x)(Fx \to (Gx \& Hx))\)  
(2) \((\exists x)Fx \quad // \quad (\exists x)Hx\)

(b) (1) \((x)(Fx \to Gx)\)  
(2) \((\exists x) \neg Gx \quad // \quad (\exists x) \neg Fx\)

(c) (1) \((x) (Gx \to \neg Hx)\)  
(2) \((\exists x) (Fx \& Gx) \quad // \quad (\exists x) (Fx \& \neg Hx)\)

(d) (1) \((\neg)Fx \quad // \quad (\exists x) \neg Fx\)