Homework 6

A. Show that the following arguments are valid by constructing formal proofs. You may use any of the inference rules — i.e., the propositional calculus rules, plus EE, EI, UE, UI. (Note: "P" stands for an atomic sentence.)

1. (1) $(x)(Fx \rightarrow -Gx)$ // $-(\exists x)(Fx \,\&\, Gx)$

2. (1) $(x)((Fx \,\&\, Gx) \rightarrow Hx)$
   (2) $-(\exists x)Hx$ // $(x)(Fx \rightarrow -Gx)$

3. (1) $(x)(P \vee Fx)$ // $P \vee (x)Fx$

4. (1) $(x)Fx \rightarrow P$ // $(\exists x)(Fx \rightarrow P)$

5. (1) $(x)((\exists y)Fxy \rightarrow Gx)$ // $(x)(y)(Fxy \rightarrow Gx)$

B. Prove the following theorems of the predicate calculus.

1. $\vdash (x)(Fx \vee -Fx)$
2. $\vdash (\exists x)(Fx \rightarrow (y)Fy)$
3. $\vdash (x)((\exists y)Fxy \rightarrow (\exists z)Gxz) \leftrightarrow (x)(y)(\exists z)(Fxy \rightarrow Gxz)$