Homework 7 (Revised Version)

1. Let $\mathscr{I}$ denote the interpretation given by:

   $X = \{1, 2, 3, 4, 5\}$
   $\mathrm{Ref}(m) = 2 \qquad \mathrm{Ref}(n) = 4$
   $\mathrm{Ext}(Fx) = \{1, 2, 3\} \qquad \mathrm{Ext}(Gx) = \{4\} \qquad \mathrm{Ext}(Hx) = \{5\}$

   Determine if the following sentences are true or false relative to $\mathscr{I}$.

   (a) $(x)(-Fx \vee -Gx)$
   (b) $(x)((Gx \,\&\, Fx) \rightarrow Hx)$
   (c) $-(x)Fx \rightarrow (\exists x)(Fx \,\&\, Gx)$
   (d) $-Gm \,\&\, (\exists x)(Fx \,\&\, Gn)$
   (e) $(\exists x)(-Fx \,\&\, -Gx)$
   (f) $(\exists x)(Fx \rightarrow Gx)$

2. Give *formal* counterexamples to the following argument forms. For discovery of the counterexample, you may use any of the methods that we have learned (e.g., "Algorithm C" or the "Small Domain Method"), or your own self-invented method.

   (a)
   (1) $(x)(Fx \rightarrow Hx) \qquad /\!/\, (x)((Fx \vee Gx) \rightarrow Hx)$

   (b)
   (1) $(\exists x)(Fx \rightarrow Gx)$
   (2) $(\exists x)Fx \vee (\exists x)Gx \qquad /\!/\, (\exists x)Fx \rightarrow (\exists x)Gx$

   (c)
   (1) $(x)Fx \leftrightarrow (x)Gx \qquad /\!/\, (x)(Fx \leftrightarrow Gx)$

3. Transform the following sentence into Disjunctive Normal Form. (Confused? See Lemmon, Appendix A.) If the sentence is consistent, give a witnessing truth assignment.

   $((-P \,\&\, -Q) \rightarrow R) \leftrightarrow -(Q \vee -R)$