Homework 8.

1. Paraphrase into quantificational notation. For (c)–(f), you may take the domain of discourse to be persons (so you don’t have to add an extra predicate symbol for “x is a person”).

(a) There is a river in America that is longer than any river in Europe.
(b) Not every river in America is longer than every river in Europe.
(c) Nobody likes all of the people she knows.
(d) Everyone knows someone to whom she is unknown.
(e) There is someone who helps all those who help themselves.
(f) There is someone who helps only those who help themselves.

2. Symbolize, taking the domain of discourse to be the class of persons, and using:

\( Sx \equiv x \) is a soprano. \( Tx \equiv x \) is a tenor.
\( Lxy \equiv x \) is louder than \( y \). \( Rxy \equiv x \) respects \( y \).

(a) A soprano who respects all tenors fails to respect herself.
(b) A tenor who is louder than all sopranos is respected by all sopranos.
(c) No tenor who is louder than all sopranos respects any soprano.
(d) A tenor who is louder than some soprano is also louder than some tenor.
(e) There are sopranos who respect only those tenors who are louder than they.
(f) If a tenor respects all sopranos who respect him, then that tenor is respected by all sopranos.

3. With domain of discourse and vocabulary as in the previous problem, translate into clear, idiomatic English:

(a) \( (\exists x)(\exists y)(Tx \& Rxy \& Sy) \rightarrow (\exists y)(Ty \& (x)(Sx \rightarrow Rxy)) \)
(b) \( (\exists x)(Tx \& (y)(Sy \rightarrow ((\exists z)(Tz \& Ryz) \rightarrow Ryx))) \)
(c) \( (x)(Sx \rightarrow ((y)(Sy \rightarrow Ryx) \rightarrow (y)(Ty \rightarrow Ryx))) \)